Viscous-Inviscid Analysis of Transonic and Low Reynolds Number Airfoils

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A method of accurately calculating transonic and low Reynolds number airfoil flows, implemented in the viscous-inviscid design/analysis code ISES, is presented. The Euler equations are discretized on a conservative streamline grid and are strongly coupled to a two-equation integral boundary-layer formulation, using the displacement thickness concept. A transition prediction formulation of the e^9 type is derived and incorporated into the viscous formulation. The entire discrete equation set, including the viscous and transition formulations, is solved as a fully coupled nonlinear system by a global Newton method. This is a rapid and reliable method for dealing with strong viscous-inviscid interactions, which invariably occur in transonic and low Reynolds number airfoil flows. The results presented demonstrate the ability of the ISES code to predict transitioning separation bubbles and their associated losses. The rapid airfoil performance degradation with decreasing Reynolds number is thus accurately predicted. Also presented is a transonic airfoil calculation involving shock-induced separation, showing the robustness of the global Newton solution procedure. Good agreement with experiment is obtained, further demonstrating the performance of the present integral boundary-layer formulation.

Nomenclature

C_D = dissipation coefficient, $(1/\rho_e u_e^3)\int\tau(\partial u/\partial\eta)\,\mathrm{d}\eta$
C_f = skin-friction coefficient, $2\tau_{\mathrm{wall}}/\rho_e u_e^2$
C_τ = shear stress coefficient, $\tau_{\max}/\rho_e u_e^2$
h_0 = stagnation enthalpy
H = shape parameter, δ^*/θ
H^* = kinetic energy shape parameter, θ^*/θ
H^{**} = density shape parameter, δ^{**}/θ
H_k = kinematic shape parameter, $\int[1-(u/u_e)]\,\mathrm{d}\eta \div \int(u/u_e)[1-(u/u_e)]\,\mathrm{d}\eta$
M_e = boundary-layer edge Mach number
$\tilde{n}$ = transition disturbance amplification variable
Re_θ = momentum thickness Reynolds number, $\rho_e u_e \theta/\mu_e$
p = pressure
q = speed
u_e = boundary-layer edge velocity
u_τ = wall shear velocity, $\sqrt{\tau_{\mathrm{wall}}/\rho}$
δ^* = displacement thickness, $\int[1-(\rho u/\rho_e u_e)]\,\mathrm{d}\eta$
δ^{**} = density thickness, $\int(u/u_e)[1-(\rho/\rho_e)]\,\mathrm{d}\eta$
ξ,η = thin shear layer coordinates
θ = momentum thickness, $\int(\rho u/\rho_e u_e)[1-(u/u_e)]\,\mathrm{d}\eta$
θ^* = kinetic energy thickness, $\int(\rho u/\rho_e u_e)[1-(u^2/u_e^2)]\,\mathrm{d}\eta$
μ_e = boundary-layer edge viscosity
ρ = density
ρ_e = boundary-layer edge density
τ = shear stress

I. Introduction

EFFECTIVE airfoil design procedures require a fast, robust analysis method for on-design and off-design performance evaluation. For a given time and cost schedule, a fast analysis method obviously permits more detailed optimization than a slower method of comparable accuracy and thus results in a better final design.

The various airfoil analysis and/or design algorithms that have been developed in the past decade have employed one of two distinct approaches: the full Reynolds-averaged Navier-Stokes approach and the interacted viscous-inviscid zonal approach.

As a rule, the Navier-Stokes approach is too slow for routine design work and has not yet shown any accuracy advantages over the much faster zonal approaches. Typical zonal approaches, such as the GBK code of Garabedian, Bauer, Korn,[1] and the GRUMFOIL code of Melnik, Chow, and Mead,[2] use a full-potential formulation for the inviscid flow and an integral boundary-layer formulation for the boundary-layer and wake regions. The viscous and inviscid flows are strongly coupled, usually through a wall transpiration boundary condition on the inviscid flow. The interacted zonal approaches are reasonably fast and accurate for transonic flows and are generally preferred for transonic airfoil analysis.

The applicability of any interacted viscous-inviscid analysis method to low Reynolds number flows (chord $Re < 1$ million) critically depends on the boundary layer and transition prediction formulations employed in the method. Accurate representation of both laminar and turbulent separated flow is a must since transitional separation bubbles and their losses must be accurately calculated if accurate drag predictions are to be obtained. The transition prediction algorithm must likewise be reliable since it affects the termination point of any transitional separation bubble and hence determines the bubble's size and associated losses.

Transitional bubble calculations have previously been reported by several workers. Gleyzes, Cousteix, and Bonnet[3] employ an incompressible integral boundary-layer formulation with entrainment closure and couple this to some unspecified inviscid (presumably potential) solver for a model geometry. Vatsa and Carter[4] employ a localized approach to calculate the transitional bubbles near an airfoil leading edge. The bubble solution is treated as a perturbation on a base solution obtained from the GRUMFOIL code.

The present airfoil analysis formulation, implemented in the transonic airfoil/cascade analysis/design code ISES,[5-7] incorporates features aimed at computational economy, minimal user intervention, and good prediction accuracy for a wide range of Mach and Reynolds numbers. The steady Euler equa-

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