

Geometric methods in  $p$ -adic representation theory  
Workshop at Trinity College Dublin  
July 29th - August 2nd 2019

Programme

Monday 29th July

**9.15-10.15: Christophe Breuil**

*Locally analytic  $\text{Ext}^1$  for  $\text{GL}_n(\mathbb{Q}_p)$  and  $(\varphi, \Gamma)$ -modules.*

Let  $n \geq 2$  and  $p$  a prime number. Using (patched) completed cohomology, one can attach locally analytic representations of  $\text{GL}_n(\mathbb{Q}_p)$  to (many)  $n$ -dimensional de Rham representations of  $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$  with distinct Hodge-Tate weights. Apart from their subspaces of locally algebraic vectors, these locally analytic representations of  $\text{GL}_n(\mathbb{Q}_p)$  are mysterious when  $n > 2$ . I will state a conjecture, in terms of  $(\varphi, \Gamma)$ -modules over the Robba ring, on what should happen “right after” the locally algebraic vectors. I will then give several results towards this conjecture. This is joint work with Yiwen Ding.

**10.30-11.30: Wiesława Nizioł**

*$p$ -adic cohomologies of the Drinfeld symmetric space.*

I will discuss a joint work with Pierre Colmez and Gabriel Dospinescu in which we compute the integral  $p$ -adic étale cohomology of the Drinfeld half-space of any dimension. In an earlier work we have computed the integral de Rham cohomology using the fact that the standard semistable integral model of the Drinfeld half-space is pro-ordinary. The integral  $p$ -adic étale cohomology is obtained from that via the integral comparison theorems of Bhatt-Morrow-Scholze.

**12.00-13.00: Andrea Pulita**

*Recent developments on  $p$ -adic differential equations.*

In the first part of the talk, I will provide an introductory and understandable talk on recent developments on the topic of  $p$ -adic differential equations over  $p$ -adic curves such as finite dimensionality of the de Rham cohomology, its relations with the behaviour of the radii of convergence of the solutions, some classification results. In the second part, I will report on some more technical foundational aspects related to the extension of these results to the case of varieties of higher dimension.

**15.00-16.00: Rachel Ollivier**

*A derived Hecke algebra in the context of the mod  $p$  Langlands program.*

Given a  $p$ -adic reductive group  $G$  and its (pro- $p$ ) Iwahori-Hecke algebra  $H$ , we are interested in the link between the category of smooth representations of  $G$  and the category of  $H$ -modules. When the field of coefficients has characteristic zero this link is well understood by work of Bernstein and Borel. In characteristic  $p$  things are still poorly understood. In this case the role of the pro- $p$  Iwahori-Hecke algebra  $H$  is played by a differential graded Hecke algebra. In particular, by work of Peter Schneider, the module category over the d.g. Hecke algebra is equivalent to the derived category of smooth representations of  $G$ . Unlike in the case of  $H$ , we know little about the structure of this d.g. Hecke algebra. In this talk I will report on joint work with Peter Schneider where we take the first steps in this direction by studying the cohomology of the d.g. Hecke algebra.

**16.30-17.30: Matthias Strauch**

*Restricting Banach space representations of  $\mathrm{GL}(2, \mathbb{Q}_p)$  to  $\mathrm{SL}(2, \mathbb{Q}_p)$ .*

Let  $V$  be a 2-dimensional absolutely irreducible cristabelline or semistable representation of the absolute Galois group of  $\mathbb{Q}_p$  with distinct Hodge-Tate weights, and let  $\Pi(V)$  be the unitary Banach space representation of  $\mathrm{GL}(2, \mathbb{Q}_p)$  associated to  $V$  by Colmez'  $p$ -adic Langlands correspondence. We show that the restriction of  $\Pi(V)$  to  $\mathrm{SL}(2, \mathbb{Q}_p)$  is reducible if and only if the locally algebraic subrepresentation  $\Pi(V)_{\mathrm{alg}}$  is irreducible but its restriction to  $\mathrm{SL}(2, \mathbb{Q}_p)$  is reducible. In the latter case the restriction of  $\Pi(V)_{\mathrm{alg}}$  to  $\mathrm{SL}(2, \mathbb{Q}_p)$  is a direct sum of two irreducible representations, and the same is true for the restriction of  $\Pi(V)$  to  $\mathrm{SL}(2, \mathbb{Q}_p)$ . The explicit description of the locally analytic representation  $\Pi(V)_{\mathrm{an}}$  by Liu-Xie-Zhang and Colmez is a key result that we use. One can also characterize this situation in terms of projective Galois and associated projective Weil group representations. This is joint work in progress with Dubravka Ban.

## Tuesday 30th July

**9.15-10.15: Arthur-César Le Bras**

*Geometrization of the local Langlands correspondence (after Fargues and Scholze).*

There has been recently spectacular progress on the understanding of the geometry underlying the local Langlands correspondence, mostly due to Fargues and Scholze. In this survey talk, I will try to explain some of the key features of this geometric reformulation.

**10.30-11.30: Sascha Orlik**

*The pro-étale cohomology of Drinfeld's upper half space.*

In my talk I want to present an approach for determining the geometric pro-étale cohomology of Drinfeld's upper half space  $X$  over a  $p$ -adic field  $K$ . The strategy is different from the one given by Colmez, Dospinescu and Nizioł. It uses the theory developed in a former paper of the speaker describing global sections of equivariant vector bundles on  $X$ .

**12.00-13.00: Tobias Schmidt**

*$\mathcal{D}$ -modules and locally analytic representations from the BGG category  $\mathcal{O}$ .*

Let  $G$  be a reductive  $p$ -adic group and let  $P$  be a parabolic subgroup of  $G$ . Orlik-Strauch have constructed a functor from the BGG category  $\mathcal{O}$  relative to  $P$  to locally analytic  $G$ -representations. In this talk, we will analyze the essential image of the functor geometrically on the rigid-analytic flag variety  $X$  of  $G$  in terms of classical Beilinson-Bernstein modules associated to category  $\mathcal{O}$  as well as a new theory of parabolic induction for coadmissible equivariant  $\mathcal{D}$ -modules on  $X$ . This is joint work with K. Ardakov.

**15.00-16.00: Matthew Emerton**

*Localizing  $\mathrm{GL}_2(\mathbb{Q}_p)$ -representations.*

We let  $Z$  denote a certain chain of projective lines over a field  $k$  of characteristic  $p$ , and we let  $A$  denote the category of smooth (but not necessarily admissible)  $\mathrm{GL}_2(\mathbb{Q}_p)$ -representations on  $k$ -vector spaces (with some fixed central character). Our goal in this talk is to explain how the category  $A$  may be “localized” over the scheme  $Z$ , and to explain various applications of this localization. This is work-in-progress with Andrea Dotto and Toby Gee.

**16.30-17.30: Jan Kohlhaase**

*Fourier analysis on  $p$ -adic division algebras.*

Let  $B$  be a finite dimensional division algebra over the  $p$ -adic numbers. We introduce a class of functions on  $B$  that we call locally  $B$ -analytic. This requires working with the large coefficient rings of  $p$ -adic Hodge theory. We study the corresponding distribution algebras through a version of Fourier theory. This is based on the universal formal cover of a  $p$ -divisible group as introduced by Fargues-Fontaine and Scholze-Weinstein.

## Wednesday 31th July

**9.15-10.15: Vytautas Paškūnas**

*On infinitesimal characters.*

We prove a quite general result about existence of infinitesimal characters in arithmetic families of locally analytic representations. As a consequence of this and dimension considerations, we deduce that certain Banach space representations occurring in completed cohomology are of finite length. (Joint with Gabriel Dospinescu and Benjamin Schraen).

**10.30-11.30: Pierre Colmez**

*On the étale cohomology of the Drinfeld tower.*

I will discuss a description *a la* Emerton of the étale cohomology of the Drinfeld tower over  $\mathbb{Q}_p$  in dimension 1 (joint work with G. Dospinescu and W. Nizioł).

**12.00-13.00: Simon Wadsley**

*Admissible representations arising from equivariant  $\mathcal{D}$ -modules on rigid spaces.*

Let  $p$  be a prime, let  $F$  be a  $p$ -adic local field and let  $G = GL_2(F)$ . The  $p$ -adic upper half plane  $\Omega$  is obtained from the projective line viewed as a rigid analytic variety by removing the  $F$ -rational points. It comes equipped with a natural action of  $G$ . Using Ardakov's theory of equivariant  $\mathcal{D}$ -modules on rigid spaces we can prove that the strong duals of the global sections of certain natural  $G$ -equivariant line bundles with flat connection on  $\Omega$  are irreducible admissible locally analytic representations of  $G$  in the sense of Schneider and Teitelbaum. These line bundles can be understood in terms of the first étale covering of  $\Omega$  introduced by Drinfeld. It is reasonable to hope that this strategy will ultimately generalise to other  $p$ -adic symmetric spaces and to higher coverings. Patel, Schmidt and Strauch have also given an argument for the admissibility of these representations; whilst the two approaches are similar in certain respects they are also significantly different. This is joint work with Konstantin Ardakov.

## Free afternoon

## Thursday 1st August

**9.15-10.15: Gergely Zábrádi**

*Multivariable  $(\varphi, \Gamma)$ -modules.*

The notion of cyclotomic multivariable  $(\varphi, \Gamma)$ -modules were introduced recently in order to generalize (parts of) Colmez's work on the  $p$ -adic Langlands programme from  $GL_2(\mathbb{Q}_p)$  to groups of higher rank. More specifically: there exists a functor with promising exactness and compatibility properties from the category of smooth mod  $p^n$  representations of the group  $G$  of  $\mathbb{Q}_p$ -points of a  $\mathbb{Q}_p$ -split reductive group with connected centre to  $d$ -variable  $(\varphi, \Gamma)$ -modules, where  $d$  is the number of simple roots of  $G$ . Further, there is a Fontaine-style equivalence of categories between these multivariable objects and  $p$ -adic representations of  $d$ -fold products of local Galois groups. There is a new proof of this fact using Drinfeld's lemma for perfectoid spaces (joint with Annie Carter and Kiran S. Kedlaya). Usual methods like overconvergence and Herr's complex computing cohomology generalize to this context, too. In part also joint work with Aprameyo Pal.

**10.30-11.30: Elmar Große-Klönne**

*On the structure of supersingular Hecke modules.*

I want to discuss a decomposition theorem for supersingular modules over the pro- $p$ -Iwahori Hecke algebra, with coefficients in a field of characteristic  $p$ , of a reductive  $p$ -adic group. On the one hand, it is an ingredient for the construction of certain functors to mod- $p$ -Galois representation. On the other hand, it seems to be related to rich combinatorics of affine Weyl groups, possibly to 'cells' in them.

**12.00-13.00: Andreas Bode**

*Connections and coadmissibility.*

We report on recent results explaining how vector bundles with flat connections fit into the theory of coadmissible  $\widehat{\mathcal{D}}$ -modules on smooth rigid analytic varieties. We give a weak non-Liouville condition familiar from  $p$ -adic differential equations which ensures that meromorphic connections can be extended to coadmissible  $\widehat{\mathcal{D}}$ -modules, and use this theory to show that the direct image of a connection along a Zariski open embedding  $j : U \rightarrow X$  need not be coadmissible. We then establish that such pathologies cannot occur when the module is the restriction of a connection on  $X$ , and indicate how our results might be used to obtain a satisfactory theory of holonomicity. This is partly joint work with T. Bitoun, partly joint with K. Ardakov and S. Wadsley.

**15.00-16.00: Claus Sorensen**

*Duality modulo  $p$  and Hecke DGAs.*

In the talk we will present two results relating Schneider's Hecke DGAs and Kohlhaase's higher duality functors  $S^i$ . If  $G$  is a  $p$ -adic Lie group, with a torsionfree pro- $p$  subgroup  $I$ , Schneider showed that  $\text{ind}_I^G(1)$  is a compact generator for  $D(G)$ ; the derived category of smooth mod  $p$  representations of  $G$ . By a general result of Keller  $D(G)$  is then equivalent to  $D(\mathcal{H}_I^\bullet)$  where  $\mathcal{H}_I^\bullet$  is a certain differential graded variant of the Hecke algebra for  $(G, I)$ . Our first result is for unramified  $p$ -adic reductive groups  $G$ . We show that  $S^i$  vanishes on the generator  $\text{ind}_I^G(1)$  in the range  $i > \dim(G/B)$ . This is due to Kohlhaase for  $\text{GL}_2(\mathbb{Q}_p)$ . Our proof uses Lazard's theory of  $p$ -valued groups, and a spectral sequence argument to pass to a graded setting. Our second result pertains to compact  $G$ . We use the  $S^i$  to show that  $\mathcal{H}_I^\bullet$  admits a dualizing module, which refines and answers a question of Harris in the compact case.

**16.30-17.30: Laurent Berger**

*Tensor products and trianguline representations.*

Two  $p$ -adic Galois representations whose tensor product is trianguline are themselves potentially trianguline. I will recall the definition of trianguline representations and explain the proof of the above statement.

**19.00: Dinner at Brasserie Sixty 6**

66-67 South Great George's Street, Dublin, D02 YD61

## Friday 2nd August

### 9.15-10.15: Benjamin Schraen

*On the density of automorphic points in global deformation spaces.*

I will discuss the problem of density of automorphic points in global polarized deformation spaces in arbitrary dimension. I will explain how to extend infinite fern argument of Gouvea-Mazur and Chenevier to the case of arbitrary dimension using existence of global companion points and the geometry of local deformation spaces of trianguline deformations. This is a joint work with Eugen Hellmann and Christophe Margerin.

### 10.30-11.30: Christine Huyghe

*Coadmissible modules over Zariski-Riemann spaces.*

This is joint work with Matthias Strauch and Tobias Schmidt. In this talk I will introduce the category of coadmissible modules over Zariski Riemann spaces (resp. equivariant) and give some examples and some applications of this theory.

### 12.00-13.00: Ruochuan Liu

*Towards  $p$ -adic  $\lambda$ -connections for rigid analytic varieties.*

Inspired by our previous works on the  $p$ -adic Simpson correspondence and the  $p$ -adic Riemann-Hilbert correspondence, we propose a  $p$ -adic analogue of Deligne's construction of  $\lambda$ -connections. Joint work with Xinwen Zhu.

## End of workshop