# Correlation matrix approach to synchronisation of burst events in multivariate electroencephalograph

Siddharth Arora [a], M. S. Santhanam [a,*], J. C. Parikh [a], R. Pratap [b]

[a] *Physical Research Laboratory, Ahmedabad 380 009, India.*

[b] *Ilya Prigogine Centre for Nonlinear System Studies, International School of Photonics, Cochin University of Science & Technology, Cochin 682 022, India.*

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**Abstract**

The existence of synchronization in neuronal firings is by now well established. Occasionally, as recorded by the multichannel electroencephalograph (EEG), most of the channels covering the entire scalp burst in a synchronous manner. They are called synchronized burst events. Using the measured multichannel EEG data with high temporal resolution, we show that the synchronized burst events can be identified using the correlation matrix formalism. The eigenvectors capture not only the synchronised burst events but also the complete synchronization among small clusters of channels. This method allows us to detect all the major synchronized activity in the EEG records and can provide information about the functional connectivity in the brain from the EEG perspective.

*Key words:* Synchronisation, synchronized burst event, electroencephalograph
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## 1 Introduction

Many physiological processes display synchronization [1]. The most easily recognizable phenomenon of this class in our daily life is the wake-sleep rhythm that is synchronous with the light-dark cycle [1]. Physiological functions in biological systems result from complex interactions among various sub-units and

* Corresponding author. *E-mail address :* santh@prl.res.in

they also respond to external stimuli. Another instance of synchronization is in the electroencephalograph (EEG) signals from the brain, which represents the electrical signal from a collection of neurons in the brain. These signals display synchrony in individual neuronal spikes as well as the burst events [2]. For instance, it is now mostly agreed that epilepsy in humans is associated with pathological synchronization of neurons [3]. Hence it is important to quantify and understand synchronization in collective neuronal firings and EEG.

At the neuronal level, many mathematical models and even experimental results are available for collective burst events of neurons called synchronized burst events (SBE). Considerable progress has been made in the last decade in understanding the dynamics of neuronal events and their synchrony [4]. Experimental approach using lithographically prepared *In Vitro* networks have shown spontaneous and stimulated events both of which display SBE and they are long range correlated [5]. However, most of the work dealing with synchronization in brain at the level of EEG deal with a single or few scalar time series of EEG signals [6,7]. On the other hand, it is also known that certain activities of the cerebral cortex take place due to interaction of assembly of neurons from different parts of the brain [8]. A related and important question is of the networks and connectivity; how various parts of the brain get connected in response to certain stimulus [9].

Therefore, to obtain a complete picture, it is important to take a holistic approach to the question of synchronization and burst events in EEG signals. In this paper, we propose a method based on correlation matrix spectra to identify synchronized burst events in EEG signals. We also study the signatures of SBEs in spectra of correlation matrix for subjects under two different states of brain, namely, eyes closed (EC) and eyes open (EO). In the light of results in Ref. [5], a related question is if the properties of SBE at the neuronal level are related to that of the EEG signals. While a complete answer to this question is not straightforward, we hope this work will also shed light on this possible connection.

In the next section, we discuss the measured EEG data and define the synchronized burst event that forms the basis for this study. In section III, we introduce the correlation matrix and its spectra and demonstrate how it can detect synchronized burst event.

## 2 EEG signals and synchronized burst events

### 2.1 EEG signals and data

Electroencephalograph (EEG) signals measured on the scalp record the resultant electrical activity due to millions of neurons in the brain. Since EEG signals are simultaneously recorded using large number of channels or electrodes on the scalp, the correlations that exist between various channels can be used to study the connectivity and the collective emergent behaviour of neural clusters. In the last few years, high space and time resolution EEG recordings have become possible and they offer a deeper insight into such processes and connectivity in the brain.

In this work, we study scalp EEG signals from subjects in their eyes closed and eyes open condition. The data was recorded on a 128 channel high resolution [10] Electric Geodesic systems at 200 Hz referenced to a vertex electrode. The positions of the electrodes on the scalp is shown in Fig 1. The data was processed through a band pass filter operating in the range 0.1 to 70 Hz and its artifacts removed. This data had earlier been used to study the coordination in the dynamics of brain and also for complexity analysis [11]. We study a two minute trace of the EEG time series and identify the synchronized burst events in them using the correlation matrix formalism.

### 2.2 Synchronized Burst Events

In Fig 2(a,b) we show a sample of the EEG record from one particular electrode for subjects in eyes closed and eyes open condition. Let $x_i(t)$ represent the EEG signal recorded by $i$th channel ($1 \leq i \leq 128$) at time $t$. Here time is discrete in units of 1/200 second; $t = 1, 2, ....T$. A sample of EEG signal recorded at one particular electrode for the subject in eyes closed (Fig 2(a)) and eyes open (Fig 2(b)) condition is displayed. Following Segev [5], we create a new time series $y_i$ by integrating the absolute of EEG values within $M$ time bins.

$$y_i(\tau) = \sum_{t=M(\tau-1)+1}^{M\tau} |x_i(t)| \qquad \tau = 1, 2, 3.......\tau_{max}, \tag{1}$$

where $\tau_{max} = [T/M]$ and $[z]$ represents the integer part of $z$. For the results presented in this paper, we have taken $M = 25$. We also define the mean $\langle y \rangle$

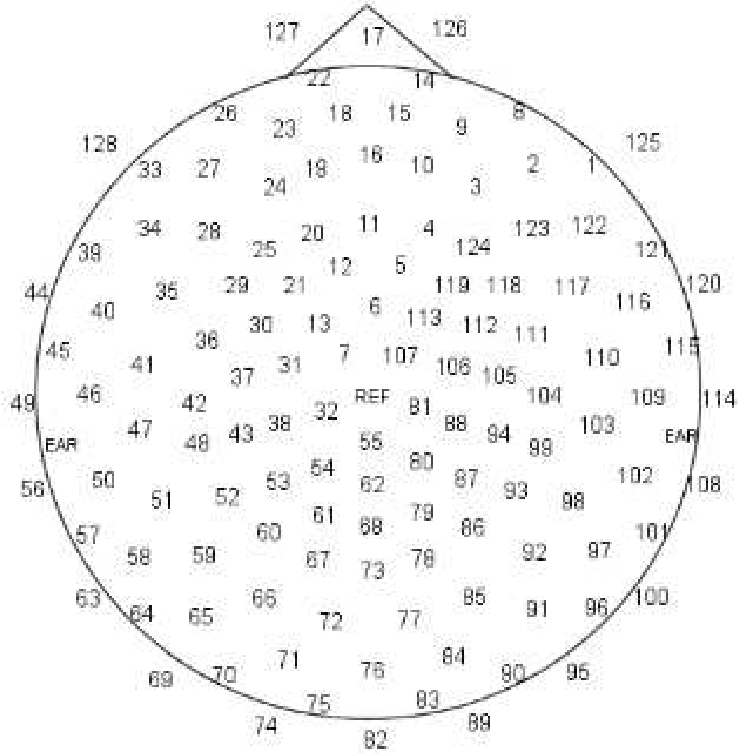


Fig. 1. The positions of the electrode for EEG measurement on the scalp of the subject is shown.

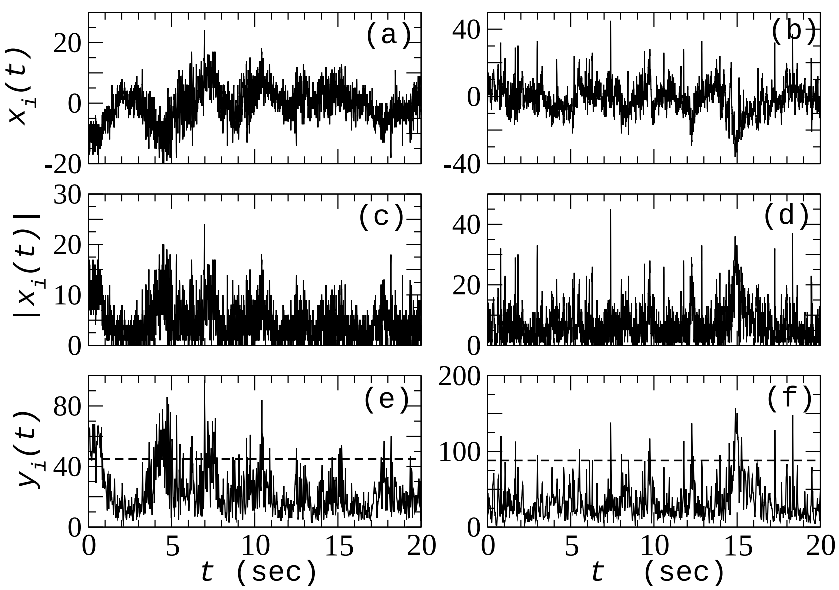


Fig. 2. (a,b) Sample EEG record, (c,d) Absolute of EEG values and (e,f) $y_i(t)$ as defined by Eq. 1. The figures (a,c,e) is for eyes closed and (b,d,f) for eyes open case. The dashed line in (c) and (f) denote the threshold given by $\langle y \rangle$. All the events that lie above this line are the burst events.

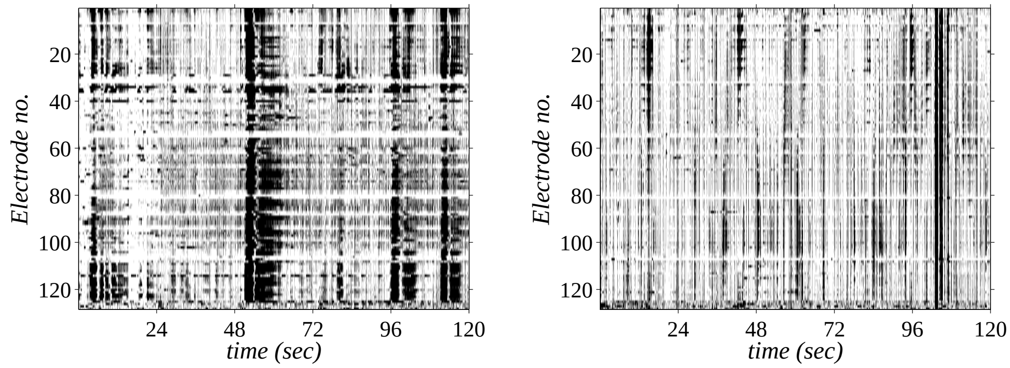


Fig. 3. The space-time plot of $y_i(t)$ for eyes closed (left) and eyes open (right) case. A black dot is placed at positions of the burst event. The dark, vertical lines, for instance at $t = 52$ in the left figure, indicate synchronized burst events.

to be,

$$\langle y \rangle = \frac{1}{128\ \tau_{max}} \sum_{\tau=1}^{\tau_{max}} \sum_{i=1}^{128} y_i(\tau). \tag{2}$$

A burst event at $t$th instant is one for which $y_i(\tau) > \langle y \rangle$. The EEG values that lie above the threshold marked by dashed line in Fig 2(c,f) are the burst events. If such burst events happen in at least 80% of the channels, we call it a synchronized burst event (SBE). In Fig 3, we show $y_i(t)$ as a space-time plot. Notice that the dark vertical lines extending through most of the electrodes denote the synchronized burst events. To see this more clearly, in Fig 4 we display the SBEs in the data using the above criteria. In this figure, every time an SBE occurs, we have marked it by a dark vertical line. With this criteria, the presence of such collective burst event implies that the neuronal firing pattern is mostly sporadic and occasionally they exhibit synchronous firing activity involving almost all the channels. The data analysis performed by varying time bin $M$ or the threshold criteria for the burst event indicate that they do not affect the results presented in this work.

At this point, it is worth pointing out that when one is interested in the correlations among the burst events, typically the bursts are converted in to binary signals and then further analysis is performed [5]. However, we dispense with this pre-processing since the main purpose is to identify the signatures of SBEs by directly applying our method on the raw data. We have also verified that for a given data set, whether the binary output is considered or the raw signal, the correlations that we compute and analyze in the next sections remain almost unchanged.

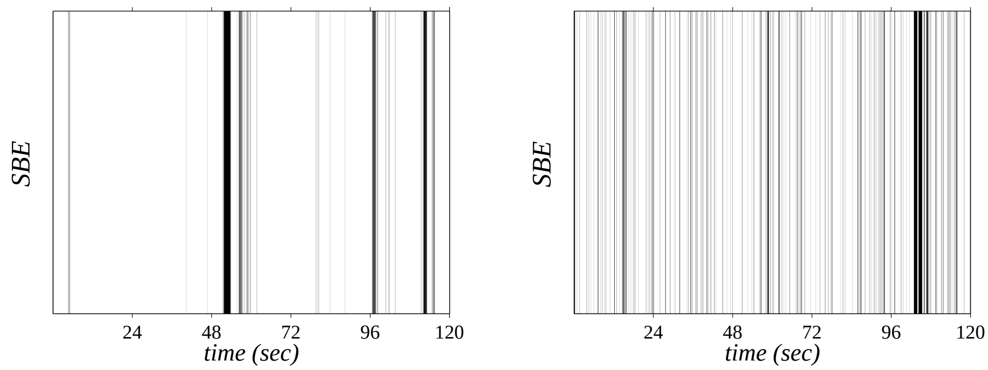


Fig. 4. The synchronised burst events for the cases shown in Fig 3 for EC (left) and EO (right) obtained by applying the criteria discussed above. If atleast 80% of the channels exhibit a burst event at any given instant, dark vertical lines are drawn at those times.

## 3 Correlation matrix

We assume that the measured strength of EEG signal denoted by $x_i(t)$, $i = 1, 2....N$ and $t = 1, 2...T$, has been standardised such that it has zero mean and unit variance. The distribution of EEG data is very close to a Gaussian and justifies such a standardisation. Let $\mathbf{Z}$ denote a $T \times N$ data matrix where each column contains standardised EEG time series from a given channel. Then, the correlation matrix $\mathbf{C}$ of order $N$ is,

$$\mathbf{C} = \mathbf{Z^T Z}, \tag{3}$$

where the superscript $.^{\mathbf{T}}$ represents matrix transposition. The elements $C_{ij}$ of $\mathbf{C}$ indicate the correlation between the EEG values generated at $i$th and $j$th channels. Hence every element of the matrix $\mathbf{C}$ indicates spatial correlations in the data. If the EEG series from these two channels are uncorrelated then, $|C_{ij}| = 0$ and if they are perfectly (anti)correlated then $|C_{ij}| = 1$. This is a linear measure of synchronization and is suitable for EEG time series where synchronization takes place within some frequency bandwidth.

The major features in the space-time plot of EEG profiles $x_i(t)$, from all the 128 channels for the subject in EC and EO condition, are similar to that depicted in Fig 3. As can be seen in this figure, some of the SBEs can be visually identified by inspection but most others require some quantitative tool to locate them. We will use the correlation matrix formalism to identify SBEs in this multivariate EEG data.

The correlation matrix computed using Eq. 3 for both the EC and EO cases is displayed in Fig 5. Based on this figure, qualitatively we can expect a lot more correlations and hence many more SBEs in the EO case than in EC case. It is

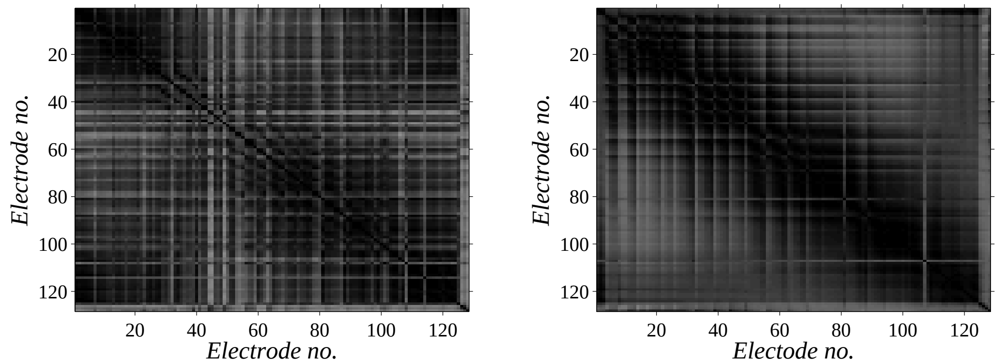


Fig. 5. The correlation matrix for eyes closed (left) and eyes open (right) case. The black indicates strong correlations and white indicates weak correlations.

also instructive to note that in EC case we see correlations among channels, for instance numbered 4 and 120, that are located far apart on the scalp of the subject. In the EO case, the diagonal dominance of the **C** shows that essentially only the contiguous channels are strongly correlated. Notice that even though burst events are defined through Eqns. (1,2) by taking absolute values, the correlation matrix is computed from the raw time series $x_i(t)$ after standardisation. Hence, we are attempting to obtain information about burst events and their synchrony by analysing the raw EEG data.

To determine the major modes of synchronized activity among the EEG channels, we diagonalize the correlation matrix **C** to solve the eigenvalue problem,

$$\mathbf{C}\ \mathbf{v}_k = \lambda_k \mathbf{v}_k, \tag{4}$$

where $\lambda_k$ $(k = 1, 2, \ldots, N)$ are the eigenvalues and $\mathbf{v}_k$ are the corresponding eigenvectors. Each eigenvector captures one mode of synchronous activity and $\phi_k = \lambda_k/N$ denotes the relative significance of the mode. Information about the correlation structure of the multivariate EEG data is now studied using the correlation matrix spectra.

Earlier, Kwapien et. al. [12] used correlation matrix to analyze magnetoencephalograph (MEG) signals to quantify the degree of collectivity during certain latency intervals. They also showed that the random matrix theory (RMT) can model the MEG correlation matrix. Sěba [13] showed that the EEG correlations can be studied using the RMT framework. This work also shows that visual stimulation in EEG records leads to deviation from RMT predictions. Recently, correlation matrix formalism has been used to detect and characterize changes in the degree of synchronization and correlations in a multivariate data set [14]. In the present work, we study the eigenmodes of the correlation matrix to determine the signature of collective behaviour like the SBEs in the multivariate EEG data.

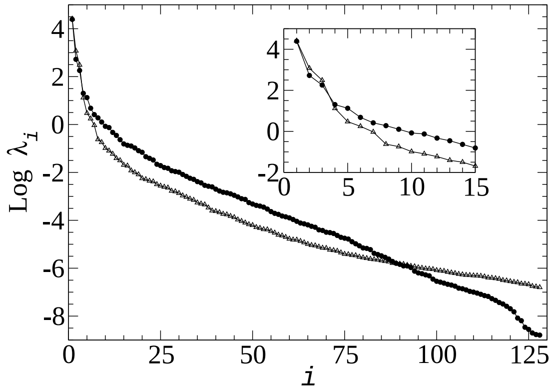


Fig. 6. The logarithm of the eigenvalues of the EEG correlation matrix for eyes closed (circles) and eyes open (triangles) cases are shown. The inset is the magnification of the 15 largest eigenvalues.

The eigenvalues of $\mathbf{C}$ are ordered such that $\lambda_1 > \lambda_2 > ... > \lambda_N$. In Fig 6, the eigenvalues for the EC and EO cases are shown. Except the top 5 eigenvalues, in a statistical test, the rest would be classified as being noisy. This tallies with the result of Sěba [13] that the dominant eigenvalues of EEG correlation matrix deviate from random matrix predictions. While these dominant eigenvalues contain information about SBEs, we will show later that the last few eigenmodes contain non-trivial information as well. The matrix formed by putting the columns of eigenvectors together is denoted by $\mathbf{E}$; *i.e.,* $\mathbf{E} = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3} \ldots \mathbf{v}_N\}$. Since $\mathbf{C}$ is a real symmetric matrix by construction, its eigenvector $\mathbf{E}$ is an orthogonal matrix. The data matrix $\mathbf{Z}$ can be written in terms of the eigenvectors of $\mathbf{C}$ as,

$$\mathbf{Z} = \mathbf{A}\ \mathbf{E}^{\mathbf{T}}, \tag{5}$$

where $\mathbf{A} = \mathbf{Z}\ \mathbf{E}$ is the matrix of principal components. This exact synthesis formula is the basis for many applications of correlation matrix formalism in image compression and data denoising [15] and clustering and pattern recognition [16] in many disciplines. We will use this formula to reconstruct EEG time series by selectively summing over certain suitable eigenvectors.

## 4 SBE from the eigenvectors

### *4.1 The dominant eigenvectors*

The modes of collective synchronized activity of the EEG signals are obtained from the eigenvectors of the correlation matrix. Note that the eigenvalues of $\mathbf{C}$ are positive semi-definite, i.e., $\lambda_i \geq 0$ for all $i$. In Fig 7(a,e), $|\mathbf{v}_1|^2$ corre-

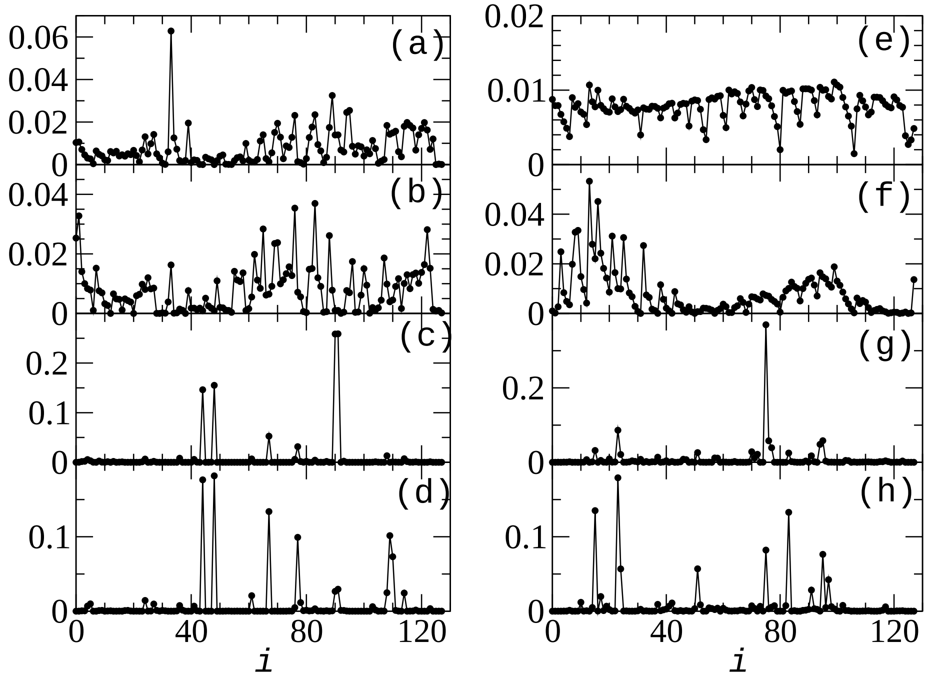


Fig. 7. The eigenvectors $|\mathbf{v}_i|^2$ for EC cases (a) $i = 1$, (b) $i = 2$, (c) $i = 127$ and (d) $i = 128$. For EO cases (e) $i = 1$, (f) $i = 2$, (g) $i = 127$ and (h) $i = 128$.

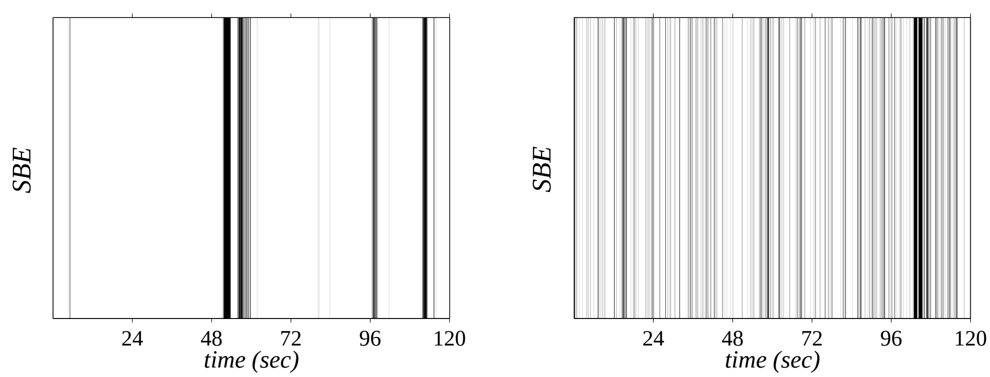


Fig. 8. The SBE reconstructed from the dominant eigenvectors of EEG correlation matrix for eyes closed (left) and eyes open (right) case. A black line is drawn across all electrodes ($y$-axis) if more than 80% of them display a burst event at any instant. This reproduces most of the SBEs seen qualitatively in Fig 4.

sponding to the eigenvalue with largest magnitude $\lambda_1$ is shown, for eyes closed and eyes open cases respectively. Almost all the channels contribute to these eigenvectors. The second dominant eigenvector corresponding to $\lambda_2$ is shown in Fig 7(b,f) and it has similar qualitative behaviour. Both these modes capture the major synchronized bursting events in the EEG signals from all the channels. To see this clearly, we reconstruct the EEG data using Eq. 5 and the eigenvectors with eigenvalues $\lambda_1$ and $\lambda_2$. The reconstructed data is again processed to obtain SBEs and is shown as space-time plot in Fig 8. The vertical lines in this figure are the location of SBEs.

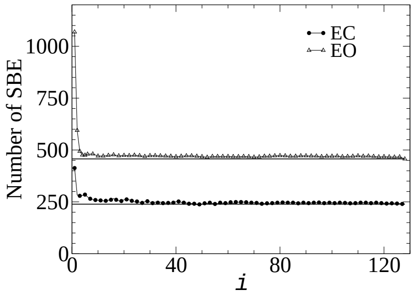


Fig. 9. The SBEs in the EEG data reconstructed (Eq 5) using successively more and more eigenvectors as a function of the eigenvalue index. The dark lines represent the SBE obtained from the original data.

The above results show that the signature of collective neural synchrony lies solely in the higher eigenvalues. The results obtained from empirical correlation matrices from stock market data also show that the dominant eigenvector, dubbed as market mode, has contributions from all the participating stocks [17]. The dominant eigenvectors shown here could be thought of as an equivalent of such market mode in EEG data. Note that the time at which the synchronized bursting events occur in the reconstructed data (Fig 8) is same as that in the original data (Figs 3,4). It is however interesting to note that the highest eigenvalue overestimates the number of synchronized bursting events present in the data. Such overestimation cannot be ruled out in general. For instance, in this case the most coherent representation of SBEs in the data requires a linear combination of at least five eigenvectors of the EEG correlation matrix. This is seen from the fact that the first few eigenvalues, say, about five of them, and their eigenvectors lead to proper cancellations which corrects this overestimation. The number of eigenvectors, about 5 in this case, needed for a reasonably good representation of the data is obtained by the following methods. We construct an ensemble of 100 correlation matrices, each of size 128 by 128, with random entries. The eigenvalues are computed for each of them, appropriately normaised and are averaged over the ensemble. Then, by comparing the the ensemble averaged eigenvalues with those generated from the EEG data, we arrive at the number of significant eigenvectors [18].

This can be visualized by plotting the number of eigenvalues and eigenvectors involved in data reconstruction using Eq 5 against the number of synchronized bursting events present in the reconstructed data. This is shown in Fig 9 and clearly summing over the first few eigenvectors in Eq. 5 is sufficient to reproduce nearly accurate SBE in the original EEG data.

The above results show that the detection and quantification of synchronization in voluminous multivariate EEG data can be done by correlation matrix formalism. Recently, a different approach based on eigenvalue decomposition of the mean phase coherence matrix has been used to identify synchronized clusters in cortical EEG recordings from epileptic subjects [19]. The main idea behind these techniques is to detect phase synchronization clusters in realistic systems.

### *4.2 Least dominant eigenvectors*

The intermediate eigenvalues mostly correspond to noisy behaviour. However, the eigenvector corresponding to the last few eigenvalues, say, $\lambda_{123}, ....\lambda_{128}$, display peaks that indicate presence of strongly pair-wise correlated channels on the scalp. In Fig 7(d,h), $|\mathbf{v}_{128}|^2$ for the eigenvalue with least magnitude is shown for EC and EO case respectively. In contrast to the case of dominant eigenvalues, there are a few peaks in $|\mathbf{v}_{128}|^2$ and the contribution from most of the channels is nearly zero. This scenario is true for the last few eigenvectors; the case for $|\mathbf{v}_{127}|^2$ in Fig 7(c,g) is for EC and EO respectively. We show that these pair-wise correlations correspond to nearly identical EEG signals, to within experimental errors, between the two channels. To see this, we plot the time series from the EEG channels which display peaks in Fig 7(c). For instance, consider the EEG series $x_{48}(t)$ and $x_{78}(t)$ (note the peaks at $i = 48$ and $i = 78$ in Fig 7(c)) chosen based on the peaks in Fig 7(c) for EC case. In Fig 10(c), the difference $\Delta = x_{48}(t) - x_{78}(t)$ is shown. Note that $\Delta \approx 0$ indicates strong correlation between these two channels. For EO case, Fig. 10(e,f) displays EEG series from channels $i = 76$ and $i = 96$ corresponding to the peaks in Fig 7(h). In fact, though we have chosen these two channels for illustration purposes, groups of peaks in Fig 7(h) display similar characteristic in their time series. From Fig. 10(f), we see that $\Delta = x_{76}(t) - x_{96}(t) \approx 0$, indicating strong correlations in the time series even though the electrodes are mostly not contiguous on the scalp. Note the electrode positions denoted by 76 and 96 in Fig 1 that shows the placement of electrodes. The position of eigenvector peaks is an indicator of the connectivity in the brain. Thus, this method reveals how brain is functionally connected from the EEG perspective. In fact, our results indicate that several peaks in Fig 7(c) are either pair wise correlated or anti-correlated. This is reminiscent of empirical financial correlation matrices whose lower eigenvectors were found to be pair wise anti-correlated [17]. This indicates that the organisational principles of complex systems such as the financial markets or the brain are similar in nature, at least when seen from the perspective of their internal correlations.

## 5 Discussion and Conclusions

Thus, using the eigenvectors of the EEG correlation matrix, it is possible to detect and segregate the synchronized burst events, a collective activity involving almost all the neurons. Synchronous bursts are spatially larger scale

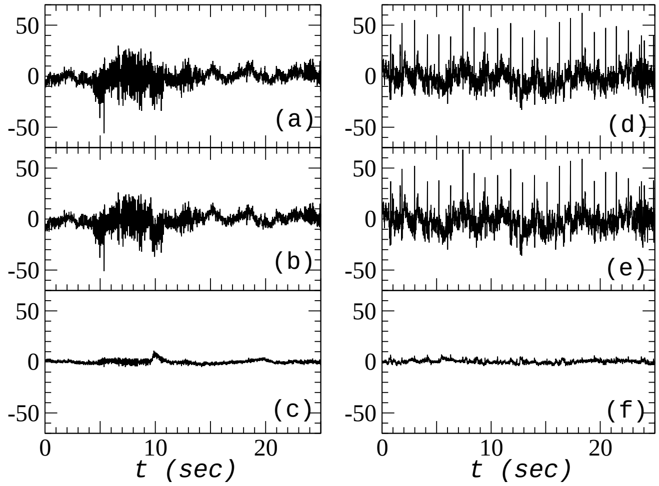


Fig. 10. The EEG time series from those channels that are peaked in Fig 7(c,d,g,h). For the EC case (left panel), (a) EEG series $x_{48}(t)$, (b) $x_{78}(t)$ and (c) $x_{48}(t) - x_{78}(t)$. For the EO case (right panel), (d) $x_{76}(t)$, (e) $x_{96}(t)$ and (f) $x_{76}(t) - x_{96}(t)$.

phenomena when compared with the complete synchronization in small groups of channels, which involve only a few EEG channels and affects the neurons on a smaller area on the scalp. We have shown this using measured EEG data from subjects in eyes closed and eyes open condition. We have performed this analysis on 10 subjects for both EC and EO case for which the data is available with us. The results are qualitatively the same as reported above. We also stress that our approach to handling the SBEs does not require the EEG signal to be in binary form. Infact, we have checked that almost all the results discussed above are independant of whether the signal is in binary or raw signal form. This is a useful feature in practice since it saves processing time and also the components that would convert signal to binary form will not be needed. Experiments to study the connectivity in the brain during various stimulus such as for example, visualisation, breathing, physical stress etc., attempt to identify regions of neurons that are involved in processing the stimulus [20]. We believe that the approach outlined above will provide a time averaged picture of the brain connectivity. This could be thought as the dominant mode of brain connectivity during the time of the experiment. If there are $N$ channels measuring the EEG activity, the largest correlation matrix encountered will be of order $N$. Current technology allows for $N = 128, 256$ or occasionally 512 and for these values eigenvalue problems can be solved in less than few seconds of CPU time of desk top computers. Hence, this method can be automated to be performed in near real-time for purposes of clinical diagnosis as well. Further, in this direction, it is necessary to study the physiological states that lead to large scale SBEs in EEG records.

To summarise, we identify the synchronized burst events in multichannel EEG records. To obtain a complete picture of the synchronized activity in the brain from the EEG, we apply the correlation matrix formalism to this multivariate data. We have shown that by using the eigenvectors of the correlation matrix, it is possible to separate the synchronized burst events from the complete synchronization in small clusters of channels. It would be interesting to see if the properties of eigenvalues and eigenvectors are sensitive to different physiological states of the subject and if they can be used to identify the source of classes of EEG signals.

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## References


[1] L. Glass, Nature **410** 277 (2001).

[2] B. W. Colder *et. al.*, J. Neurophysiology **75** 2496 (1996)

[3] J. Engel Jr. and T.A. Pedley, *Epilepsy: A Comprehensive Textbook*, (Lippincott-Raven, New York, 1997).

[4] Andreas V. M. Herz et. al., Science **314** 5796 (2006).

[5] Ronen Segev et. al. Phys. Rev. Lett. **88**, 118102 (2002).

[6] F. Mormann et. al., Physica D **144** 358 (2000).

[7] R. Q. Quiroga, A. Kraskov, T. Kreuz, and P . Grassberger, Phys. Rev. E. **65** 041903 (2002).

[8] P. R. Roelfsema et. al, Nature **385** 157 (1997).

[9] T. Koenig *et. al.*, Philos Trans R Soc Lond B Biol Sci. **360**, 1015 (2005).

[10] G. Edlinger *et. al.*, IEEE Trans. Biomedical Engg. **45**, 736 (1998).

[11] R. Pravitha *et. al.*, Int. J. Neurosci. **111**, 175 (2001); R. Pravitha and V. P. N. Nampoori, Int. J. Neurosci. **112** 1245 (2002); R. Pravitha, R. Sreenivasan and V. Nampoori, Int. J. Neurosci. **115**, 445 (2005).

[12] J. Kwapien, S. Drodz, and A. A. Ioannides, Phys. Rev. E **62** 5557 (2000).

[13] P. Sĕba, Phys. Rev. Lett. **91**, 198104 (2003).

[14] M. Müller, G. Baier , A. Galka, U. Stephani, and H. Muhle, Phys. Rev. E. **71**, 046116 (2005).

[15] R. N. Hoffmann and D. W. Johnson, IEEE Trans. Geosci. Remote Sens. **GE-32**, 25 (1994), and references therein.

[16] M. Turk and A. Pentland, *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition* 586 (1991); M. Turk and A. Pentland, J. Cognitive Neurosci. **3**, 71 (1991).

[17] P. Gopikrishnan, B. Rosenow, V. Plerou and H. E. Stanley, Phys. Rev. E **64**, 035106(R) (2001); V. Plerou *et. al.*, Phys. Rev. E **65**, 066126 (2002).

[18] R. W. Preisendorfer and C. D. Mobley, *Principal Component Analysis in Meteorology and Oceanography*, (Elsevier, Amsterdam, 1988).

[19] S. Bialonski and K. Lehnertz, Phys. Rev. E **74**, 051909 (2006).

[20] L. Lee, L. M. Harrison and A. Mechelli, Network **14**, R1 (2003).