http://ms.mcmaster.ca/~mbays/

$$\begin{array}{lll} & \mathrm{Lie}\ \mathbb{G}_m(\mathbb{C}) & \leftarrow \mathbb{Q}\text{-VS} \\ U_{\mathbb{G}_m}: & \downarrow \exp & \\ & \mathbb{G}_m(\mathbb{C}) \text{ in } \langle \mathbb{C};+,\cdot\rangle & \end{array}$$

Th^m [Zil] $U_{\mathbb{G}_m}$ determined up to IM by

- Lie $G_m(\mathbb{C}) \models \mathbb{Q}$-VS
- $\langle \mathbb{C};+,\cdot\rangle \models ACF_0$
- exp surj^{ve} HM
- $\ker \exp \cong \langle \mathbb{Z};+\rangle$
- $|U_{\mathbb{G}_m}| = 2^{\aleph_0}$

} axiomatise $\hat{T}_{\mathbb{G}_m} := Th(U_{\mathbb{G}_m})$

pf: Models of $\hat{T}_{\mathbb{G}_m} \cup \{\ker \cong \mathbb{Z}\}$ $\leftarrow L_{\omega_1,\omega}$ sentence form a "quasiminimal excellent class" $\Rightarrow$ uncountably cat^c $\square$

$$\begin{array}{ll} \mathrm{Lie}\ A(\mathbb{C}) & \leftarrow \mathrm{End}\ A \otimes \mathbb{Q}\text{-module} \\ \downarrow \exp & =: U_A \\ A(\mathbb{C}) & \leftarrow \text{in } \langle \mathbb{C};+,\cdot\rangle \end{array}$$

// Generalisations [Gavrilovich, B] to ab^n vars

// New best result;

Th^m [BHP]: IM-type of $\tilde{\mathbb{G}} \models \hat{T}_{\mathbb{G}_m^n \times A}$ determined by over $\wedge f$

IM-type of $(\ker \exp)^{\tilde{\mathbb{G}}}$

IM-type of $(\mathrm{im}\ \exp)^{\tilde{\mathbb{G}}}$ (i.e. tr.d(field)).

fRM

$\mathbb{G}$ finite Morley rank comm^{ve} group^{wAE} rigid: any $H \leq^{def} \mathbb{G}$ is $/acl^{eq}(\emptyset)$

e.g. $\mathbb{G}$ $S(\mathbb{C})$ with structure from $\langle \mathbb{C};+,\cdot\rangle$ assume $/\emptyset$

$T = T_{\mathbb{G}} = Th(\mathbb{G})$

$\hat{\mathbb{G}} := \varprojlim [n] : \mathbb{G} \to \mathbb{G}$

$= \{(x_i)_i \in \mathbb{N} \mid [n]x_{nm} = x_m\}$

$\rho_n((x_i)_i) = x_n \qquad \rho = \rho_1$

For $H \leq^{def} \mathbb{G}^n$, $\hat{H}^{\hat{\mathbb{G}}} := \{(x_i)_i \mid \forall n\ x_n \in H\}$

$\hat{T} = \hat{T}_{\mathbb{G}} := Th(\hat{\mathbb{G}}, \mathbb{G}, \rho_n, \hat{H})$ agrees with $\hat{T}$ $U_A \leq \widehat{A(\mathbb{C})}$

QE.

Th^{m}2 [BHP]: IM-type of $\tilde{\mathbb{G}} \models \hat{T}$ det^d by

- IM-type of $\mathbb{G} := \rho(\tilde{\mathbb{G}}) \models T$
- IM-type of $\tilde{\mathbb{G}}_0 := \rho^{-1}(\mathbb{G}_0)$ where $\mathbb{G}_0 \leq_{prime} \mathbb{G}$

(Th^{m}2 + Kummer $\Rightarrow$ Th^{m}1)

$\tilde{\mathbb{G}}_0$ atomic / ker

pf. $\mathbb{G}$ prime & min^l / $\mathbb{G}_0 B$ $\leftarrow$ bases for finitely many s.m. sets / M_0

we show $\tilde{\mathbb{G}}$ prime & min^l / $\tilde{\mathbb{G}}_0 B$

min^l $\Leftarrow$ $\mathbb{G}$ min^l / $\mathbb{G}_0 B$, $\ker \leq \tilde{\mathbb{G}}_0$.

prime: we show constructible

Recall: $A \models M$ ... $\models A \leq M$ ... atomic / A ... M ... $A \leq M$ ... $\forall a$ ... $tp(a/A)$ isolated

First suppose B finite $B = b$

Lemma [B-Gavrilovich-Hils '11]

$\tilde{\mathbb{G}}$ is atomic / $\tilde{\mathbb{G}}_0 b$

hence constructible by ctblty of $\mathbb{G}$

Pf idea: Let $\tilde{c} \in \tilde{\mathbb{G}}$

... $\tilde{\mathbb{G}}$ ... $/\tilde{\mathbb{G}}_0$ ... $tp(\tilde{c}/\tilde{G}_0 b) \vdash tp(\tilde{c}/\tilde{G}_0 ...)$

(BGH pf cont^d) (RM $\mathbb{G}$)

Let $\bar{c} \models tp(c/\mathbb{G}_0)$

So $\Sigma\bar{c} \models p_{\mathbb{G}}|_{\mathbb{G}_0}$

so $(\Gamma_{\Sigma\bar{c}}, \Sigma\bar{c}) = \hat{\ker}$

so $\langle \Gamma_{\bar{c}}, \Sigma\bar{c}\rangle \leq_{op} \hat{\ker}$ (by finite mult^y of $tp(\bar{c}/\mathbb{G}_0 \Sigma\bar{c})$)

$\|$

$\Sigma \langle \Gamma_{\bar{c}_i}, c_i\rangle$

$\|$

$\langle \Gamma_c, c\rangle$

so $\langle \Gamma_{cb}, c\rangle \leq_{op} \hat{\ker}$.

so $tp(\tilde{c}/\tilde{\mathbb{G}}_0 b) \vdash tp(\rho_n \tilde{c}/\mathbb{G}_0 b)$ some n

isolated since $\rho_n c \in \mathbb{G}$ atomic/$\mathbb{G}_0 b$ $\square$

Infinite B: $\leftarrow$ prime model / $\mathbb{G}_0 B'$

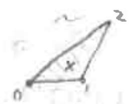
$\mathbb{G} = \bigcup_{B' \subseteq_{fin} B} \mathbb{G}_0(B')$

Let $\tilde{\mathbb{G}}_0(B') := \rho^{-1}(\mathbb{G}_0(B'))$

WTS $\tilde{\mathbb{G}}_0(B')$ atomic / $B' \cup \bigcup_{B'' \subsetneq B'} \tilde{\mathbb{G}}_0(B'')$ (holds downstairs) so $\tilde{\mathbb{G}} = \bigcup_{B' \subseteq_{fin} B} \tilde{\mathbb{G}}_0(B')$ gives construction sequence.

$|B'| = 1$ done above.

e.g. $|B'| = 3 \quad B' = b_0 b_1 b_2$



WTS $\tilde{\triangle}$ atomic / $\tilde{\triangle}$

Let $\hat{x} \in \tilde{\triangle}$; assume free / $\tilde{\triangle}$

so want $tp(\hat{x}/\triangle)$ isolated

say $tp(x/\triangle) \vdash tp(x/\triangle) \qquad x = \rho\hat{x}$

let $--- := acl^{eq}(\cdot\,\cdot\,\cdot)$

then $tp(\hat{x}/\triangle) \vdash tp(\hat{x}/\triangle)$

since $tp(\triangle/---)$ stationary

so $tp(\triangle/\triangle)$ stationary $(\triangle \perp \triangle)$

but $tp(\rho_n\hat{x}/\triangle)$ has no forking ext^n to $\triangle$

so unique ext^n. $n = 1$ – atomic $n > 1$ inductively

Now take independent copy of $\triangle$ over —.

fixed by σ pointwise

Claim: $tp(\hat{x}/\triangle, \sigma\hat{x}, \circ) \vdash tp(\hat{x}/\triangle)$

Pf: if $\models \phi(\rho_n\hat{x}, b, e)$ say $\theta(b) \vdash tp(b/\cdot\,\cdot\,\cdot)$

then $\psi(z) := \forall y.\ (\phi(z,y,e) \leftrightarrow \phi(\sigma\rho_n\hat{x}, y, \sigma e)$

$\models \phi(z, b, e)$ $\square$

but $tp(\hat{x}, \sigma\hat{x}, \circ/\triangle)$ isolated by IH