An uncountable structure is **quasiminimal** if every definable subset is countable or else has countable complement. This paper considers the problem of finding quasiminimal models of a given first-order theory, and obtains various results under various assumptions.

It was shown in [**?**] that the generic type $p_M = \{\theta(x) : |\theta(M)| > \aleph_0\} \in S_1(M)$ of a well-behaved quasiminimal structure M has global heir p which is definable and **strongly regular**: each $p{\restriction}_B$ is orthogonal to any other $q \in S_1(B)$.

This paper conversely considers a first order theory T with a definable strongly regular type p and builds quasiminimal models M for which $p{\restriction}_M$ is the generic type. It first observes, using Vaught two-cardinal techniques, that such a model of cardinality $\aleph_1$ always exists. Under further hypotheses, it finds arbitrarily large models. The first such hypothesis is that T admits definable Skolem functions, and p is symmetric. The second is that T admits a quasiminimal model of cardinality $\geq \beth_{\omega_1}$ with generic type p, since this suffices for an Erdős-Rado argument; hence the Hanf number for existence of quasiminimal models is at most $\beth_{\omega_1}$.

The third is the case that T is stable. The proof here goes via the technology of l-isolation and independent systems, building a "highly l-atomic" model over a Morley sequence in p. Finally, the paper considers the case of ω-stable T and shows that in this case the class of prime models over Morley sequences in p is moreover a "quasiminimal excellent class", using their characterisation obtained in [**?**].

References