Pseudo-foo Manchester '18

$\mathbb{C}_{exp} := (\mathbb{C};+,\cdot,\exp)$

Conj^{re} [Zilber]: $\mathbb{C}_{exp}$ is quasiminimal:
every def^{ble} $X \subseteq \mathbb{C}$ is c^{ble} or coc^{ble}

Th^m [Zilber]: Can give unc^{bly} cat^{c} q-min^{l} ($L_{\omega_1,\omega}(Q)$) axiomatisation (u.c.q.) $\boxed{\text{unique model in each card}^{ty} \geq \aleph_1, \text{ which is q-min}^{l}}$
not known not to hold of $\mathbb{C}_{exp}$.

$\mathbb{C}^{cov}_{exp} := (\mathbb{C};+) \xrightarrow{exp} (\mathbb{C};+,\cdot)$

Th^m [Zilber]: $\mathbb{C}^{cov}_{exp}$ has unc^{bly} cat^{c} q-min^{l} ($L_{\omega_1,\omega}$) axiomatisation

u.c.q. $\Leftarrow$ QME $\approx$ ccl pregeometry (ccl(A) := union of c^{ble} orbits of Aut(M/A))
+ "ω-stability over ccl-closed sets"
+ "ω-stability over ind^{t} systems"

q-min^{l} excellence

$C := \bigcup_{X \subsetneq S} \operatorname{ccl}(AX)$ where S finite ccl-ind^{t} set / A, A c^{ble};
c^{bly} many realisable types / C.

// Process of consolidation & generalisation

Pseudo-$\wp$:

$E := \{y^2 = x^3 + ax + b\}$

$\exp_E : \mathbb{C} \twoheadrightarrow E(\mathbb{C})$
$z \mapsto (\wp(z), \wp'(z))$

$\mathbb{C}_{\exp_E} = (\mathbb{C};+,\cdot,\exp_E)$ q-min^{l}? cat^{c}? ($\Rightarrow (\mathbb{C};+,\cdot,\wp)$ q-min^{l})

Sim for $\exp_A : \mathbb{C}^g \to A(\mathbb{C})$, A ab^{n} var^{y}

$\mathbb{C}^{cov}_{\exp_A}$

- Z-Gavrilovich, B: $\mathbb{C}^{cov}_{\exp_E}$ u.c.q.
- B-Gavrilovich-Hils: ω-stability / models holds for fRM comm^{ve} div^{ble} gr^{ps}, e.g. $A(\mathbb{C})$

$(\operatorname{tp}^{ACF}(\exp(\bar a), \exp(\bar a/2), \exp(\bar a/3), \ldots / \bar{\mathbb{R}})$ ($\exp(\bar a)$ l.i. $/A(\bar{\mathbb{R}})$)
only c^{bly} many possibilities

- B-Hart-Pillay: In covers, ω-stability / models $\Rightarrow$ ω-stability / ind^{t} systems! (*)
Analogous to Shelah's NOTOP — suggested by H-i.
So: $\mathbb{C}^{cov}_{\exp_A}$ u.c.q. — Wrinkle: $\exp_A|_{\mathbb{Q}\text{-ker}\exp_A}$ not known (Galois representation on Tate modules)
Sol^{n}: cheat! Specify it in an axiom.
- B-Hart-Hyttinen-Kesälä-Kirby: (*) goes through in abstraction of QME! "QPS" $\approx$ ccl pregeometry + ω-stability / ccl-closed $\Rightarrow$ u.c.q.

B-Kirby:

Hr^{i} construction of pseudoexp

Structures: divisible tf^{n} subgroups $B \leq (\mathbb{G}_a \times \mathbb{G}_m)(\mathbb{C})$ w/ rel^{ns} for $V \in (\mathbb{G}_a \times \mathbb{G}_m)^n$ constructible / $\mathbb{Q}$ (so $A \hookrightarrow B$, $\mathbb{Q}(A) \hookrightarrow \mathbb{Q}(B)$)

(mutatis mutandis for $\mathbb{G}_a \times E$, $\mathbb{G}_a^g \times A$ etc, End(A)-action, A simple)

$A \underset{f.g.}{\leq} B$ if $\dim_{\mathbb{Q}}(B/A) < \aleph_0$

$A \trianglelefteq B$ if $\cdot \ker \pi_1|_A = \ker \pi_1|_B$ (no new $(0,y)$ or $(x,1)$)
$\cdot$ If $A \underset{f.g.}{\leq} C \leq B$
then $\delta(C/A) := \operatorname{trd}(C/A) - \dim_{\mathbb{Q}}(C/A) \geq 0$

$\Gamma_{base} := \Gamma_{exp}|_{\mathbb{Q}\text{-ker}\exp} = \{(2\pi i q, e^{2\pi i q}) \mid q \in \mathbb{Q}\}$ (graph of usual complex exp)

Schanuel Conj^{re} (SC): $\Gamma_{base} \trianglelefteq \Gamma_{exp}$

Given $A \trianglelefteq B$ $\operatorname{cl}^B(A) := \bigcup\{C \mid A \underset{f.g.}{\trianglelefteq} C \trianglelefteq B, \delta(C/A) = 0\}$
$A \underset{cl}{\trianglelefteq} B : \operatorname{cl}^B(A) = A$

Th^m [Z; BK]: Axioms are u.c.q.
$\cdot$ $\Gamma_{base} \trianglelefteq \Gamma$ (SC)
$\cdot$ Γ is saturated for $\underset{f.g.}{\trianglelefteq}$ over Γ_{base} (SEAC)
i.e. $\Gamma_{base} \underset{f.g.}{\trianglelefteq} A \trianglelefteq \Gamma$, $A \underset{f.g.}{\trianglelefteq} B$
$\cdot$ $|\operatorname{cl}^{\Gamma}(A)| \leq \aleph_0$ if $|A| \leq \aleph_0$ (CCP)
[This is in $\mathbb{G}_a \times \mathbb{G}_m$, or $\mathbb{G}_a^g \times A$] (etc)

Pf: $\cdot$ AP for $\underset{f.g.}{\trianglelefteq}$ over Γ_{base}
$\cdot$ $|\{X \mid \Gamma_{base} \underset{f.g.}{\trianglelefteq} X\}/_{\text{ISM}/\Gamma_{base}}| = \aleph_0$ $\leftarrow$ Kummer
} Fraïssé $\Rightarrow$ c^{ble} model $\Gamma_{\aleph_0}$
$\cdot$ $\Gamma_0 \underset{cl}{\trianglelefteq} \Gamma_{\aleph_0}$
$\Rightarrow |\{X \mid \Gamma_0 \underset{f.g.}{\trianglelefteq} X\}/_{\text{ISM}/\Gamma_0}| = \aleph_0$ $\leftarrow$ [BGH]
$\Rightarrow$ "ω-stab^{ty} / ccl-closed"
$\Rightarrow$ QPS $\square$

Remark: Ax $\Rightarrow$ $\Gamma_{exp} \models$ CCP

Generic pseudoexp

$\Gamma^G_{base} := \operatorname{ecl}^{\Gamma_{exp}}(\emptyset)$

(GSC) $\Gamma^G_{base} \underset{cl}{\trianglelefteq} \Gamma$

(GSEAC) $\Gamma^G_{base} \underset{f.g.}{\trianglelefteq} A \trianglelefteq \Gamma$, $A \underset{f.g.}{\trianglelefteq} B$, $\Gamma^G_{base} \underset{cl}{\underset{f.g.}{\trianglelefteq}} B$

Th^m [BK]: (GSC) + (GSEAC) + (CCP) is u.c.q.

- Ax $\Rightarrow$ $\Gamma_{exp} \models$ (GSC)

Th^m [BK]: (GSEAC) $\Leftrightarrow$ (GEAC) $\Leftarrow$ (EAC)

(EAC): V free & rotund & $\delta = 0$ $\Rightarrow V \cap \Gamma \neq \emptyset$

$\boxed{\begin{array}{l} V \subseteq (\mathbb{G}_a \times \mathbb{G}_m)^n \ \dim V = n \ \text{(e.g.)} \\ \pi_1 V \text{ in no proper alg subgroup} / \mathbb{C}, \ \text{coset} \\ \dim V/_{L \times M} \geq \dim H \ \forall H \leq \text{alg subgr}^{p} \end{array}}$

Cor^y: If $\mathbb{C}_{exp} \models$ (EAC) then $\mathbb{C}_{exp}$ is q-min^{l}.