NIP formulas in continuous logic

__Setting__: M metric structure (in particular a complete bdd metric space), $G = \mathrm{Aut}(M)$.

* If M is separable, it is called __ω-categorical__ if unique separable model of $\mathrm{Th}(M)$ up to isomorphism.

Recall (continuous Ryll-Nardzewski):

M separable. TFAE

- M is ω-cat.
- $M^\omega /\!\!/ G$ is compact ($G \curvearrowright M$ approximately oligomorphic)
- All uniformly continuous G-invariant bdd functions $h: M^\omega \to \mathbb{R}$ are given by formulas.

__Terminology__: M __$\emptyset$-saturated__ if M realizes all $p \in S_\omega(\emptyset)$.

__Fact 1__: M ω-cat $\Rightarrow$ M $\emptyset$-saturated.

Fix countable ordinals α, β, variables $(x_i)_{i<\alpha} = x$ and $y = (y_i)_{i<\beta}$ and a formula $f(x;y): M^\alpha \times M^\beta \to \mathbb{R}$.

Moreover, let $A \subseteq M^\alpha$ and $B \subseteq M^\beta$ be $\emptyset$-(type)-definable sets. If $\tilde{M} \succ M$, we write $\tilde{f}$ and $\tilde{A}, \tilde{B}$ for the interpretations in $\tilde{M}$.