Morning lectures

Jean-Marc Azaïs.
Title: On the asymptotic variance of the number of real roots of random polynomial systems

Abstract: We obtain the asymptotic variance, as the degree goes to infinity, of the normalized number of real roots of a square Kostlan-Shub-Smale random polynomial system of any size. Our main tools are the Kac-Rice formula for the second factorial moment of the number of roots and a Hermite expansion of this random variable. Joint work with D. Armentano, F. Dalmao & J. León.

Dmitry Beliaev.
Title: Gaussian Fields and Percolation

Abstract: It was conjectured by Bogomolny and Schmit that the nodal domains of the Random Plane Wave could be described by the critical bond percolation on the square lattice. There is strong evidence that this conjecture is not true in this form, but RPW are closely related and it seems that they both belong to the same universality class. In particular, it is believed that the nodal lines have conformally invariant scaling limit which is described by Schramm-Loewner Evolution. In the recent years it become evident that this is a part of a rather general phenomenon and the same connection should hold for a wide class of smooth Gaussian fields.

In this talk I am going to describe these conjectures and give a brief survey of the recent results.

Yaiza Canzani.
Title: Local universality for zeros and critical points of monochromatic random waves

Abstract: In this talk we will discuss the asymptotic behavior of zeros and critical points for monochromatic random waves on compact, smooth, Riemannian manifolds, as the energy of the waves grow to infinity. This is joint work with Boris Hanin.

Anne Estrade.
Title: Anisotropic random wave model

Abstract: Let d be an integer greater or equal to 2 and let k be a d-dimensional random vector. We call random wave model with random wavevector k any stationary random field defined on R^d with covariance function $t \mapsto E[\cos(k.t)]$. The purpose of my talk will be to link properties that concern the geometry and the anisotropy of the random wave with the distribution of the random wavevector. For instance, when k almost surely belongs to the unit sphere in R^2 and the random wave model is nothing but the anisotropic version of Berry's planar waves, we prove that the expected length of the nodal lines is decreasing as the anisotropy of the random wavevector is increasing. Joint work with Julie Fournier (MAP5, Paris Descartes University).

Damien Gayet.
Title: Percolation and random nodal lines

Abstract: If a real smooth function is given at random on the plane, what is the probability that its vanishing locus has a large connected component? I will explain some answers we obtained with Vincent Beffara to this question, for some natural models coming from algebraic geometry and statistical physics.

Marie Kratz.

Title: Level Functionals for Gaussian Fields and Applications to Oceanography

Abstract: We illustrate the theory of level crossings or more generally of excursion sets by stationary Gaussian fields, with some applications in sea modelling. The main tools are the generalizations of the well known Rice formula and a general method developed with J. Leon to obtain representations into the Ito-Wiener Chaos and asymptotic behaviors for level functionals.

Manjunath Krishnapur.

Title: A question on discrete nodal length

Abstract: A quantity that we call the discrete nodal length, for eigenfunctions of the Laplacian on a finite graph, is introduced. Simulations and simple examples show specific behaviour of this quantity as a function of the eigenvalue. This leads us to a conjecture that (in a crude sense) extends the erstwhile conjecture of Yau for nodal length in the continuum setting.

Pär Kurlberg.

Title: Level repulsion for arithmetic toral point scatterers

Abstract: The Seba billiard was introduced to study the transition between integrability and chaos in quantum systems. The model seem to exhibit intermediate level statistics with strong repulsion between nearby eigenvalues (consistent with random matrix theory predictions for spectra of chaotic systems), whereas large gaps seem to have “Poisson tails” (as for spectra of integrable systems.)

We investigate the closely related “toral point scatterer”-model, i.e., the Laplacian perturbed by a delta-potential, on 3D tori of the form $\mathbb{R}^3/\mathbb{Z}^3$. This gives a rank one perturbation of the original Laplacian, and it is natural to split the spectrum/eigenspaces into two parts: the ”old” (unperturbed) one spanned by eigenfunctions vanishing at the scatterer location, and the ”new” part (spanned by Green’s functions). We show that there is strong repulsion between the new set of eigenvalues.

Steve Lester.

Title: Scars for wave functions of a point scatterer on the torus

Abstract: A fundamental problem in Quantum Chaos is to understand the distribution of mass of Laplace eigenfunctions on a smooth Riemannian manifold in the limit as the eigenvalue tends to infinity. I will consider a Laplace operator perturbed by a delta potential (point scatterer) on the torus and describe the distribution of mass of the eigenfunctions of this operator. In this setting, the distribution of mass of the eigenfunctions is related to properties of integers which are representable as a sum of two squares. I will indicate how tools from analytic number theory can be applied to study the properties of these integers. This is joint work with Pär Kurlberg and Lior Rosenzweig.

Domenico Marinucci.

Title: The Asymptotic Equivalence of the Sample Trispectrum and the Nodal Length for Random Spherical Harmonics

Abstract: We study the asymptotic behaviour of the nodal length of random 2d-spherical harmonics f_ℓ of high degree $\ell \to \infty$, i.e. the length of their zero set $f_\ell^{-1}(0)$. It is found that the nodal lengths are asymptotically equivalent, in the L^2-sense, to the “sample trispectrum”, i.e., the integral of $H_4(f_\ell(x))$, the fourth-order Hermite polynomial of the values of f_ℓ. An important

by-product of this is a Quantitative Central Limit Theorem (in Wasserstein distance) for the nodal length, in the high energy limit.

Stephen Muirhead.
Title: The phase transition for level sets of smooth planar Gaussian fields

Abstract: In recent years the strong links between the geometry of smooth planar Gaussian fields and percolation have become increasingly apparent, and it is now believed that the connectivity of the level sets of a wide class of smooth, stationary planar Gaussian fields exhibits a sharp phase transition that is analogous to the phase transition in, for instance, Bernoulli percolation. In recent work we prove this conjecture under the assumptions that the field is (i) symmetric, (ii) positively correlated, and (iii) the covariance kernel decay sufficiently rapidly at infinity (roughly speaking, the integrability of the kernel is enough). Key to our proofs are (i) the white-noise representation of Gaussian fields, and (ii) the randomised algorithm approach to noise sensitivity. Joint work with Dmitry Beliaev, Hugo Vanneuville and Alejandro Rivera.

Giovanni Peccati.
Title: Second order results for nodal sets of random waves

Abstract: I will present some recent advances concerning limit theorems (central and non-central) for local functionals of nodal sets of random waves on two-dimensional manifolds. I will devote special attention to the planar and arithmetic cases. I will also present an application to the study of nodal lengths fluctuations for monochromatic waves on shrinking domains. (Partially based on joint works with F. Dalmao, G. Dierickx, D. Marinucci, I. Nourdin, M. Rossi and I. Wigman)

Guillaume Poly.
Title: Universality for roots of random trigonometric models

Abstract: In this talk, we will present some recent results around the dependence of the asymptotic behaviour of the roots of random trigonometric models with respect to the underlying randomness. We shall see that, contrarily to several well known families of random polynomials, second order statistics for roots of trigonometric models display a behaviour which is not universal and which strongly depends on the specific distribution of the chosen randomness. We shall also discuss universality questions for random singular functions and try to highlight the role played by the smoothness of the considered functions in these universality problems.

Maurizia Rossi.
Title: Asymptotic distribution of nodal intersections for arithmetic random waves

Abstract: In this talk we focus on the nodal intersections number of random Gaussian toral Laplace eigenfunctions ("arithmetic random waves") against a fixed smooth reference curve. The expected intersection number is proportional to the the square root of the eigenvalue times the length of curve, independent of its geometry. The asymptotic behaviour of the variance was addressed by Rudnick-Wigman; they found a precise asymptotic law for "generic" curves with nowhere vanishing curvature, depending on both its geometry and the angular distribution of lattice points lying on circles corresponding to the Laplace eigenvalue. They also discovered that there exist peculiar "static" curves, with variance of smaller order of magnitude, though did not prescribe what the true asymptotic behaviour is in this case. In this talk we investigate the finer aspects of the limit distribution of the nodal intersections number. For "generic" curves we prove the Central Limit Theorem (at least, for "most" of the energies). For the aforementioned static curves we establish a non-Gaussian limit theorem for the distribution of nodal intersections, and on the way find the true asymptotic behaviour of their fluctuations, under the well-separatedness assumption on the corresponding lattice points, satisfied by most of the eigenvalues. This talk is based on a joint work with Igor Wigman (King's College London).

Igor Wigman.
Title: Variation of the Nazarov-Sodin constant for random plane waves and arithmetic random waves

Abstract: This is joint work with Par Kurlberg. We prove that the Nazarov-Sodin constant, which up to a natural scaling gives the leading order growth for the expected number of nodal components of a random Gaussian field, genuinely depends on the field. We then infer the same for "arithmetic random waves", i.e. random toral Laplace eigenfunctions.

Afternoon lectures

Jacques Benatar.
Title: Quasi-correlations of lattice points on the sphere

Abstract: We will discuss some old and new results concerning (quasi) correlations of integer points on the d-dimensional sphere and explain their connection to arithmetic random waves. Given a large natural number n, we seek to understand 'almost' linear relations among lattice points lying on the sphere $S = S(0, \sqrt{n}) \subset \mathbb{R}^d$. More precisely, our goal is to estimate the size of the set

$$\mathcal{Q}(\ell, \delta) := \left\{ (\xi_1, ..., \xi_\ell) \in S^\ell : \left\| \sum_{i=1}^{\ell} \xi_i \right\| \leq n^{1/2-\delta} \right\}.$$

Valentina Cammarota.
Title: Two point function for critical points of a random plane wave.

Abstract: Random plane wave is conjectured to be a universal model for high-energy eigenfunctions of the Laplace operator on generic compact Riemanian manifolds. This is known to be true on average. In the present paper we discuss one of important geometric observable: critical points. We first compute one-point function for the critical point process, in particular we compute the expected number of critical points inside any open set. After that we compute the short-range asymptotic behaviour of the two-point function. This gives an unexpected result that the second factorial moment of the number of critical points in a small disc scales as the fourth power of the radius. Joint work with Dmitry Beliaev and Igor Wigman.

Federico Dalmao.
Title: CLT for Kostlan Shub Smale polynomial systems

Abstract: Kostlan Shub Smale random polynomial systems of equations were introduced in the early nineties and the expectation of the number of real roots in the square systems and of the volume of the zero set on the non-square case were computed. This was the first polynomial system to be studied and one of its main features is that its distribution is invariant under the action of the orthogonal group in the parameter space. We now present the Central Limit Theorem in both the square and the non-square case for the number of roots and the volume of the zero level set respectively as the degree tends to infinity. Our main tools are the chaotic expansions, the existence of a local limit process and a convenient partition of the sphere.

Naomi Feldheim.

Title: Exponential concentration of zeroes of Gaussian stationary functions

Abstract: The mean number of zeroes of a real Gaussian stationary function (GSF) was computed already in the 1940's by Kac and Rice. However, it is far more complicated to estimate the probability of a significant deficiency or abundance in the number of zeroes in a long interval (compared to the expectation). We do so for a specific family of GSF's with additional smoothness and absolutely summable correlations, using tools from real and complex analysis. Joint work with R. Basu, A. Dembo and O. Zeitouni, arXiv:1709.06760.

Maxime Ingremeau.

Title: Local weak limits of eigenfunctions and Berry's conjecture

Abstract: We will recall Berry's conjecture, some of its mathematical interpretations, and propose a new one, based on new limiting objets for Laplace eigenfunctions.

Michael McAuley.

Title: The Nazarov-Sodin constant and critical points of Gaussian fields

Abstract: The Nazarov-Sodin constant, which describes the average number of nodal set components of Gaussian fields over large areas, can be generalised to a functional describing the corresponding number of level set components for arbitrary levels. Using results related to those of Morse theory, this functional can be expressed as an integral over the level densities of different types of critical points. I will discuss some consequences of this integral representation, including upper and lower bounds on the functional for isotropic fields, which show that the functional is at least bimodal for the random plane wave.

Alejandro Rivera.

Title: Decoupling distant local events for Gaussian fields

Abstract: Consider a stationary Gaussian field f on $\mathbb{R}^n$ (e.g. a random monochromatic wave). You might be interested in counting connected components of its zero sets, detecting large connected components or connected components of a specific topology etc. Assuming that the covariance of f decays fast enough, it might seem intuitive that, given two disjoint balls far enough from each other, the topology of the zero set inside the first ball should be almost independent from its topology in the second ball. This type of information is useful to study large scale events or space-wise averages of semi-local events, such as component counting. However, if your field is analytic, say, measuring it on the first ball immediately determines it on the whole plane. In particular, the restrictions of the field to the two balls cannot be asymptotically independent. I will present different strategies to overcome this difficulty.

Hugo Vanneuville.

Title: The critical threshold for Bargmann-Fock percolation

Abstract: Let f be the planar Bargmann-Fock field, i.e. the analytic Gaussian field with covariance kernel $\exp(-|x-y|^2/2)$. We compute the critical point for the percolation model induced by the level sets of f. More precisely, we prove that there exists a.s. an unbounded component in $\{f > p\}$ if and only if $p < 0$. Such a percolation model has been studied recently by Beffara-Gayet and Beliaev-Muirhead. One important aspect of our work is the derivation of a KKL-type inequality for correlated Gaussian variables. The idea to use a KKL-type result to compute a critical point goes back to Bollobs-Riordan. This is joint work with Alejandro Rivera.

Nadav Yesha.

Title: CLT for small scale mass distribution of toral Laplace eigenfunctions

Abstract: In this talk we discuss the fine scale L^2-mass distribution of toral Laplace eigenfunctions with respect to random position. For the 2-dimensional torus, under certain flatness assumptions on the Fourier coefficients of the eigenfunctions and generic restrictions on energy levels, both the asymptotic shape of the variance and the limiting Gaussian law are established, in the optimal Planck-scale regime. We also discuss the 3-dimensional case, where the asymptotic behaviour of the variance is analysed in a more restrictive scenario. This is joint work with Igor Wigman.