Universität Hamburg
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Dr. Severin Bunk
Algebra und Zahlentheorie
Fachbereich Mathematik
Universität Hamburg

Algebraic Topology
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# Sheet 7

Solutions are due on 01.06.18.

**Problem 7.1** A continuous map $f\colon \mathbb{S}^n \to \mathbb{S}^n$ is called *even* if $f(-x) = f(x)$ for all $x \in \mathbb{S}^n$. Using Example 1.12.8.4 and that even maps interact nicely with projective spaces (how?), prove the following statements:

(a) in even dimension $n$, every even map has degree zero.

(b) In odd dimension, even maps have even degree.

(c) If $n$ is odd, any even number $2k \in 2\mathbb{Z}$ is the degree of an *even* map.

**Problem 7.2**

(a) Let $F_g$ be the closed, oriented surface of genus $g \in \mathbb{N}_0$. Write down an explicit CW model for $F_g$ and compute the cellular homology of $F_g$.

(b) Give a CW model for the product $\mathbb{S}^n \times \mathbb{S}^m$ for $n, m \in \mathbb{N}$ and compute its cellular homology.

**Problem 7.3** Let $X$ be a CW complex and let $X^n$ denote its $n$-th skeleton.

(a) Can the abelian group $H_n(X^n)$ have torsion?

(b) Show that if $X$ has $k$ many $n$-cells, then $H_n(X)$ is generated by at most $k$ many elements.

**Problem 7.4** Let $G$ be a finitely generated abelian group, and let $n \in \mathbb{N}$ be arbitrary.

(a) Construct a CW complex $M(G, n)$ whose reduced homology groups read as

$$\tilde{H}_k\big(M(G,n)\big) \cong \begin{cases} G\,, & k = n\,,\\ 0\,, & k \neq n\,. \end{cases}$$

Spaces with these reduced homology groups are called *Moore spaces*.

(b) Describe $M(G, n)$ for the case that $G$ is a finite cyclic group. What is $M(\mathbb{Z}_2, n)$?

(c) Let $(A_n)_{n\in\mathbb{N}}$ be a sequence of finitely generated abelian groups. Construct a path-connected space $X$ with

$$\tilde{H}_k(X) \cong A_n \qquad \forall\, k \in \mathbb{N}\,.$$