Universität Hamburg
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Algebraic Topology
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Sheet 9

Solutions are due on 15.06.18.

Problem 9.1

Let A be a finitely generated abelian torsion group. Explain how the abelian groups $\mathrm{Tor}(A, \mathbb{Q}/\mathbb{Z})$ and $\mathrm{Hom}(A, \mathbb{Q}/\mathbb{Z})$ are related to A.

Problem 9.2

Let M be an abelian group and let

$$0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0 \tag{1}$$

be a short exact sequence of abelian groups. Make explicit statements on the exactness of the sequences

$$0 \longrightarrow \mathrm{Hom}(M, A) \xrightarrow{\alpha_*} \mathrm{Hom}(M, B) \xrightarrow{\beta_*} \mathrm{Hom}(M, C) \longrightarrow 0$$

and

$$0 \longrightarrow \mathrm{Hom}(C, M) \xrightarrow{\beta^*} \mathrm{Hom}(B, M) \xrightarrow{\alpha^*} \mathrm{Hom}(A, M) \longrightarrow 0$$

at each of the non-trivial groups by giving proofs or counterexamples. What can you say, if the sequence (1) is split?

Problem 9.3

Recall the definition of a Moore space $M(A, n)$ from Problem 7.4, or you can find it in, for instance, [Example 2.40, Hatcher]. Let $M(\mathbb{Z}_p, n)$ and $M(\mathbb{Z}_q, m)$ be two Moore spaces with p, q prime and $n, m \geq 1$. Compute the homology groups of $M(\mathbb{Z}_p, n) \times M(\mathbb{Z}_q, m)$ in all cases.

Problem 9.4

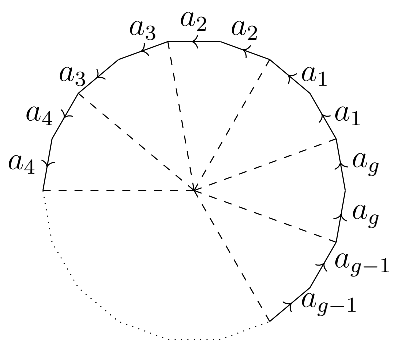
Let $f \colon X \to Y$ be a continuous map of topological spaces.

(a) Show that whenever f induces an isomorphism $H_*(f; \mathbb{Z}) \colon H_*(X; \mathbb{Z}) \to H_*(Y; \mathbb{Z})$ in integer homology, it also induces an isomorphism $H_*(f; G) \colon H_*(X; G) \to H_*(Y; G)$ in homology with coefficients in any abelian group G.

(b) Show that the converse does not hold true: give an example of a continuous map $f\colon X \to Y$ and an abelian group G such that $H_*(f;G)$ is an isomorphism, but $H_*(f;\mathbb{Z})$ is not.

Problem 9.5

Let N_g denote the connected, closed, *non-orientable* surface of genus $g \in \mathbb{N}$, obtained by identifying edges with the same label according to the orientations indicated in the $2g$-gon

$N_g :=$ 

What is N_1? Can you see that N_2 is the Klein bottle?
Compute $H_*(N_g;G)$ for all $g \in \mathbb{N}$ and an arbitrary abelian group G.