OXFORD CHENGDU

NONLINEAR PDE DAYS

Dec 20 - Dec 22, 2021

Join Zoom Meeting
Meeting ID: 996 8361 2771 Passcode: 212610

This meeting is to bring together experts in Partial Differential Equations related to researchers in Oxford and the greater region of Chengdu, and to boost the connection and collaboration between us in PDEs from natural or social sciences, as well as to strengthen our friendship and support during this difficult time when international collaboration is set online. All talks will be virtual, 45-min long with 40&5 for Q&A, and given from 9 AM to 12:30 PM (Oxford local time) or 5 PM – 8:30 PM (Chengdu local time), and this meeting has a very broad spectrum for the topics of its talks.

Thank you and wish you enjoy the meeting.

Dec 20th MONDAY (DAY 1)		
TIME (40+5 mi)	**THEME**	**CHAIR**
08:55-09:00 (London) 16:55-17:00 (Chengdu)	**OPENING**	
09:00-09:45 (London) 17:00-17:45 (Chengdu)	Jingtang Ma *Finite Difference Methods for the Hamilton-Jacobi-Bellman Equations and Variational Inequalities Arising in Regime Switching Optimal Investment*	Q. WANG
09:45-10:30 (London) 17:45-18:30 (Chengdu)	Yingping PENG *Large Friction-high Force Fields Limit for the Nonlinear Vlasov-Poisson-Fokker-Planck System*	
30 mins	TEA BREAK	
11:00-11:45 (London) 19:00-19:45 (Chengdu)	Gui-Qiang CHEN *Cavitation and Concentration in Entropy Solutions of the Euler/Euler-Poisson Equations and Related Nonlinear PDEs*	Q. WANG
11:45-12:30 (London) 19:45-20:30 (Chengdu)	Yanghong HUANG *Spectral Relations of the Generator of Alpha-stable Processes and Applications to Spectral Methods*	

Dec 21st TUESDAY (DAY 2)		
TIME (40+5 min)	**THEME**	**CHAIR**
09:00-09:45 (London) 17:00-17:45 (Chengdu)	Liwei XU *Some Results on the High Order Methods Solving the Magnetohydrodynamic Equations*	J. CARRILLO
09:45-10:30 (London) 17:45-18:30 (Chengdu)	Ke LIN *The HLS Inequality and Application to the Chemotaxis System*	
30 mins	**TEA BREAK**	
11:00-11:45 (London) 19:00-19:45 (Chengdu)	Jose CANIZO *Improved Bounds for the Fundamental Solution of the Heat Equation in Exterior Domains*	J. CARRILLO
11:45-12:30 (London) 19:45-20:30 (Chengdu)	Ruiwen SHU *Generalized Erdős-Turán Inequalities and Stability of Energy Minimizers*	

Dec 22nd WEDNESDAY (DAY 3)		
TIME (40+5 min)	**THEME**	**CHAIR**
09:00-09:45 (London) 17:00-17:45 (Chengdu)	David GÓMEZ-CASTRO *Concentration Phenomena in Aggregation-Diffusion Equations*	G.-Q. CHEN
09:45-10:30 (London) 17:45-18:30 (Chengdu)	Rishabh GVALANI *Logarithmic Sobolev Inequalities and Equilibrium Fluctuations for Weakly Interacting Diffusions*	
30 mins	**TEA BREAK**	
11:00-11:45 (London) 19:00-19:45 (Chengdu)	Jose CARRILLO *The Landau Equation: Particle Methods & Gradient Flow Structure*	G.-Q. CHEN
11:45-12:30 (London) 19:45-20:30 (Chengdu)	Qi WANG *An Optimal Mass Transport Method for Random Genetic Drift*	
12:30- (London) 20:30- (Chengdu)	CLOSING	

Improved Bounds for the Fundamental Solution of the Heat Equation in Exterior Domains

José A. CAÑIZO
University of Granada
canizo@ugr.es

Abstract. We use entropy methods to show that the heat equation with Dirichlet boundary conditions on the complement of a compact set in R^d shows a self-similar behaviour much like the usual heat equation on R^d, once we account for the loss of mass due to the boundary. Giving good lower bounds for the fundamental solution on these sets is surprisingly a relatively recent result, and we find some improvements using some advances in logarithmic Sobolev inequalities.

The Landau Equation: Particle Methods & Gradient Flow Structure

José A. CARRILLO
University of Oxford
carrillo@maths.ox.ac.uk

Abstract. The Landau equation introduced by Landau in the 1930's is an important partial differential equation in kinetic theory. It gives a description of colliding particles in plasma physics, and it can be formally derived as a limit of the Boltzmann equation where grazing collisions are dominant. The purpose of this talk is to propose a new perspective inspired from gradient flows for weak solutions of the Landau equation, which is in analogy with the relationship of the heat equation and the 2-Wasserstein metric gradient flow of the Boltzmann entropy. Moreover, we aim at using this interpretation to derive a deterministic particle method to solve efficiently the Landau equation. Our deterministic particle scheme preserves all the conserved quantities at the semidiscrete level for the regularized Landau equation and that is entropy decreasing. We will illustrate the performance of these schemes with efficient computations using treecode approaches borrowed from multipole expansion methods for the 3D relevant Coulomb case. From the theoretical viewpoint, we use the theory of metric measure spaces for the Landau equation by introducing a bespoke Landau distance. Moreover, we show for a regularized version of the Landau equation that we can construct gradient flow solutions, curves of maximal slope, via the corresponding variational scheme. The main result obtained for the Landau equation shows that the chain rule can be rigorously proved for the grazing continuity equation, this implies that H-solutions with certain apriori estimates on moments and entropy dissipation are equivalent to gradient flow solutions of the Landau equation. We crucially make use of estimates on Fisher information-like quantities in terms of the Landau entropy dissipation developed by Desvillettes.

Cavitation and Concentration in Entropy Solutions of the Euler/Euler-Poisson Equations and Related Nonlinear PDEs

Gui-Qiang CHEN
University of Oxford
gui-qiang.chen@maths.ox.ac.uk

Abstract. In this talk, we will discuss the intrinsic phenomena of cavitation/decavitation and concentration/deconcentration in entropy solutions of the compressible Euler/Euler-Poisson equations and related nonlinear PDEs, which are fundamental to understand the well-posedness and solution behaviour of nonlinear PDEs. We will start to discuss the formation process of cavitation and concentration in the entropy solutions of the isentropic Euler equations with respect to the initial data and the vanishing pressure limit. Then we will analyse a longstanding fundamental problem in fluid dynamics: Does the concentration occur generically so that the density develops into a Dirac measure at the origin generically in spherically symmetric entropy solutions of the multi-dimensional compressible Euler/Euler-Poisson equations? We will report our recent results and approaches developed for solving this longstanding open problem for the Euler/Euler-Poisson equations and related nonlinear PDEs and discuss its close connections with entropy methods and theory of divergence-measure fields. Further perspectives and open problems in this direction will also be addressed.

Concentration Phenomena in Aggregation-Diffusion Equations

David GÓMEZ-CASTRO
University of Oxford
gomezcastro@maths.ox.ac.uk

Abstract. Over the last two decades, intense work has been devoted to the Aggregation-Diffusion equation

$$\partial_t \rho = \operatorname{div}\Big(\rho \nabla (U'(\rho) + V + W * \rho)\Big).$$

This family of problem model, amongst other things, the mean-field limit of systems with a large number of interacting particles arising in biology. Their mathematical structure is rather interesting, since they can be seen as the gradient flow in a suitable metric of a free energy functional. Thus, we can expect the system to asymptotically approximate minimisers as $t \to \infty$. This is known to be true when the energy is suitably convex. In the case of Fast Diffusion ($U(\rho) = \rho^m/(m-1)$ with $0 < m < 1$), this can fail, so the gradient flow techniques do not guarantee convergence to the minimiser.

There is a long literature characterising the minimisers of the energy functional and, in particular, discussing the existence or not of delta Deltas. The presence of a delta is usually described as a concentration phenomena.

In this talk, we will discuss some of the previous literature, and present one of the first results in the literature proving the asymptotic formation of a Dirac, in the case $W = 0$ and V suitably regular. This result has been accepted for publication in Journal des Mathématiques Pures et Appliquées. The talk presents joint work with J.A. Carrillo and J.L. Vázquez

---

Logarithmic Sobolev Inequalities and Equilibrium Fluctuations for Weakly Interacting Diffusions

Rishabh GVALANI
Max Planck Institute, Leipzig
rishabh.gvalani@mis.mpg.de

Abstract. We study the mean field limit of interacting diffusions for confining and interaction potentials that are non-convex. The limiting behaviour is described by the nonlocal McKean–Vlasov PDE. We explore the relationship between the limit $N \to \infty$ of the constant in the logarithmic Sobolev inequality (LSI) for the N-particle system and the presence or absence phase transitions for the mean field limit, conjecturing a limiting form of the LSI constant. We also explore the consequences of the non-degeneracy of the LSI constant as they relate to uniform-in-time propagation of chaos and equilibrium fluctuations. Our results extend previous results on unbounded spin systems as well as recent results on (uniform-in-time) propagation of chaos using novel coupling arguments. Joint work with Matías Delgadino, Greg Pavliotis, and Scott Smith.

Spectral Relations of the Generator of Alpha-stable Processes and Applications to Spectral Methods

Yanghong HUANG
University of Manchester
yanghong.huang@manchester.ac.uk

Abstract. In this talk, Jacobi polynomials are shown to establish spectral relations of the generator of alpha-stable processes, generalising existing ones for the (symmetric) fractional Laplacian to non-symmetric case. The resulting spectral relations will be used to characterise the singularity of the solution near the boundary and the regularity to the Dirichlet problem. The results can also be used to develop higher order spectral methods, even for problems posed on the joint of multiple intervals.

The HLS Inequality and Application to the Chemotaxis System

Ke LIN
Southwestern University of Finance and Economics, Chengdu
linke@swufe.edu.cn

Abstract. In this talk, starting from the well-known Hardy-Littlewood-Sobolev inequality in the whole space, we will firstly introduce several critical mass results for the classical single-species Patlak-Keller-Segel chemotaxis system. Then, exploiting the sharp constants for some variants to the HLS inequality for system, we get the threshold values which classify the behavior of solutions to the multi-species PKS system.

Finite Difference Methods for the Hamilton-Jacobi-Bellman Equations and Variational Inequalities Arising in Regime Switching Optimal Investment

Jingtang MA
Southwestern University of Finance and Economics, Chengdu
mjt@swufe.edu.cn

Abstract. The value functions of the regime switching optimal investment satisfy the coupled Hamilton-Jacobi-Bellman (HJB) equations or variational inequalities. In this talk I will present the iterative finite difference methods (FDMs) with iteration policy to solve the problems. The convergence to the viscosity solutions of the HJB equations and variational inequalities is proved and numerical examples are carried out to confirm the results. This is joint work Jianjun Ma.

Large Friction-high Force Fields Limit for the Nonlinear Vlasov-Poisson-Fokker-Planck System

Yingping PENG
Southwest Jiaotong University, Chengdu
yingping_peng@163.com

Abstract. We provide a quantitative asymptotic analysis for the nonlinear Vlasov–Poisson–Fokker–Planck system with a large linear friction force and high force-fields. The limiting system is a diffusive model with nonlocal velocity fields often referred to as aggregation-diffusion equations. We show that a weak solution to the Vlasov–Poisson–Fokker–Planck system strongly converges to a strong solution to the diffusive model. Our proof relies on the modulated macroscopic kinetic energy estimate based on the weak-strong uniqueness principle together with a careful analysis of the Poisson equation.

Generalized Erdős-Turán Inequalities and Stability of Energy Minimizers

Ruiwen SHU
University of Oxford
shu@maths.ox.ac.uk

Abstract. The classical Erdős-Turán inequality on the distribution of roots for complex polynomials can be equivalently stated in a potential theoretic formulation, that is, if the logarithmic potential generated by a probability measure on the unit circle is close to 0, then this probability measure is close to the uniform distribution. We generalize this classical inequality from $d = 1$ to higher dimensions $d > 1$ with the class of Riesz potentials which includes the logarithmic potential as a special case. In order to quantify how close a probability measure is to the uniform distribution in a general space, we use Wasserstein-infinity distance as a canonical extension of the concept of discrepancy. Then we give a compact description of this distance. Then for every dimension d, we prove inequalities bounding the Wasserstein-infinity distance between a probability measure ρ and the uniform distribution by the L^p-norm of the Riesz potentials generated by ρ. Our inequalities are proven to be sharp up to the constants for singular Riesz potentials. Our results indicate that the phenomenon discovered by Erdős and Turán about polynomials is much more universal than it seems. Finally we apply these inequalities to prove stability theorems for energy minimizers, which provides a complementary perspective on the recent construction of energy minimizers with clustering behavior.

An Optimal Mass Transport Method for Random Genetic Drift

Qi WANG
Southwestern University of Finance and Economics, Chengdu
qwang@swufe.edu.cn

Abstract. In population genetics, genetic drift describes random fluctuations in the numbers of gene variants (alleles) over time. Mathematical modeling of genetic drift such as the Wright–Fisher model employs a discrete stochastic process to model dynamics of finite populations at the individual level such that each copy of the gene of the new generation is selected independently and randomly from the whole gene pool of the previous generation. In the limit of a large population, these processes can be approximated by the diffusion that describes the probability of fixation of a mutant with frequency-independent fitness. While the continuum framework makes a systematic qualitative and quantitative analysis of the discrete model possible thanks to tools from modern analysis, it also inherits degenerate diffusion from the discrete stochastic process that conveys to the blow-up into Dirac-delta singularities hence bringing great challenges to the analytical and numerical studies.

In this talk, we describe and analyze an optimal mass transport method for a random genetic drift problem driven by a Moran process formulated as a degenerate reaction-advection-diffusion equation. The proposed numerical method can quantitatively capture to the fullest possible extent the development of Dirac-delta singularities for genetic segregation on one hand, and preserves several sets of biologically relevant and computationally favored properties of the random genetic drift on the other. Moreover, the numerical scheme exponentially converges to the unique numerical stationary state in time at a rate independent of the mesh size up to a mesh error. Numerical evidence is given to illustrate and support these properties, and to demonstrate the spatio-temporal dynamics of random generic drift. This talk is based on a joint work with J. A. Carrillo and L. Chen.

Some Results on the High Order Methods Solving the Magnetohydrodynamic Equations

Liwei XU
University of Electronic Science and Technology of China, Chengdu
xul@uestc.edu.cn

Abstract. The magnetohydrodynamic (MHD) equation and its variants, including ideal MHD equation, incompressible MHD equations, two-phase MHD equations, resistive MHD equations, Drude models, have many applications in sciences and engineering. In this talk, we first quickly review some of our results on high order methods solving these MHD equations. Then, we present a temporally second-order accurate, finite element method for the incompressible magnetohydrodynamic equations. A modified Crank–Nicolson method is used for the temporal discretization, and appropriate semi-implicit treatments are adopted for the approximation of the fluid convection term and two coupled terms. Then a linear system with variable coefficients is presented and its unique solvability can be theoretically proved. The energy stability analysis and optimal error estimates are provided for the scheme. We also apply the scheme to the two-phase MHD equations, and present essential theoretical results. Several numerical examples are presented to demonstrate the robustness and accuracy of the proposed scheme.