Commun. Math. Anal. Appl. Vol. **3**, No. 2, pp. 121-135
doi: 10.4208/cmaa.2024-V3N2.preface May 2024

Some Contributions of Gui-Qiang G. Chen to Nonlinear Conservation Laws and Partial Differential Equations

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Received 30 April 2024; Accepted 29 May 2024

Special Issue in Honor of Professor Gui-Qiang G. Chen on the Occasion of His 60th Birthday

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Abstract. This special issue and the two follow-up issues are dedicated to Professor Gui-Qiang G. Chen on the occasion of his 60th birthday. Professor Gui-Qiang G. Chen is internationally recognized as a leader in the analysis of partial differential equations (PDEs) and related disciplines in mathematics and science. He has made wide-ranging contributions, both original and significant, to an array of research areas in mathematical analysis, partial differential equations, mathematical physics, nonlinear science, and other disciplines, especially in the areas of nonlinear hyperbolic systems of conservation laws and the mathematical theory of shock waves, free boundary problems in the theory of supersonic and transonic flow, nonlinear degenerate and mixed-type PDEs and their applications, entropy analysis and weak convergence methods, singular limit problems for nonlinear PDEs, measure-theoretical analysis for discontinuous and singular entropy solutions, stability/instability analysis of

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characteristic discontinuities for nonlinear hyperbolic conservation laws, and convergence/stability analysis of shock-capturing methods and related numerical methods, among others.

Key words: Gui-Qiang G. Chen, nonlinear hyperbolic conservation laws, shock waves, free boundary problems, nonlinear degenerate and mixed-type PDEs, discontinuous and singular entropy solutions, characteristic discontinuities, entropy analysis and weak convergence methods, divergence-measure fields.

1 Biographic remarks

Professor Gui-Qiang G. Chen was born in Cixi City (Ningbo), Zhejiang Province, China, in 1963. He received his BSc from Fudan University in 1982 and a PhD from the Chinese Academy of Sciences in 1987 under the supervision of Professor Xiaxi Ding, and was a postdoctoral fellow at the Courant Institute of Mathematical Sciences at New York University from 1987-89 under the direction of Professor Peter D. Lax. He was appointed as Assistant Professor of Mathematics at the University of Chicago in 1989, Associate Professor of Mathematics at Northwestern

University (USA) in 1994, and was promoted to Full Professor of Mathematics in 1996 at Northwestern University (USA). In 2009, he was elected to the Statutory Professor in the Analysis of Partial Differential Equations (PDE Chair) and Professorial Fellow of Keble College, University of Oxford, and has served as the Director of the Oxford Centre for Nonlinear Partial Differential Equations (OxPDE) as successor to Professor Sir John Ball since 2018. He was also a Member or Visiting Professor of many leading universities and institutions around the world, especially including the Institute for Advanced Study (Princeton, USA), Mathematical Sciences Research Institute (Berkeley, USA), the Centre for Advanced Study (Norwegian Academy of Science and Letters, Norway), Mittag-Leffler Institute of Mathematics (Royal Swedish Academy of Sciences, Sweden), CNRS and Ecole Polytechnique (France), Stanford University (USA), University of Cambridge (UK), University of California at Los Angeles (USA), University of Heidelberg (Germany), University of Minnesota-Minneapolis (USA), and University of Nice (France).

Throughout his career, Professor Gui-Qiang G. Chen has also been deeply devoted to advising, helping, and training younger mathematicians. He has trained more than 35 PhD/DPhil students and 25 postdoctoral researchers. Many of them have become leading mathematicians and hold academic positions across the globe. He also served as the Director of the EPSRC Doctoral Training Centre in PDEs (UK): Analysis and Applications, during 2014–24, being responsible for creating the training and research programs and overseeing the training, supervision, and mentoring of 62 DPhil students in PDEs. He has served as the Editor-in-Chief, as well as on the Advisory and Editorial Boards, of more than 15 leading scientific journals. Furthermore, he has served as the Chair, Organizer, or Member of the Scientific/Organizing/Program Committees of more than 200 International Conferences, Workshops, Symposia, Summer/Winter Schools, and Emphasis Years in more than 15 countries around the world. Since 2000, he has delivered more than 300 plenary, keynote, distinguished, and invited talks at international conferences, symposia, workshops, colloquia, seminars, special series, and summer/winter schools around the world.

Professor Gui-Qiang G. Chen has received numerous prestigious honors, fellowships, and awards for his distinguished scientific contributions including: The 2024 LMS Pólya Prize (London Mathematical Society, UK), Member of Academia Europaea (elected, 2022), Fellow of the European Academy of Sciences (elected, 2020), Fellow of the American Mathematical Society (elected, 2017), Fellow of the Institute of Mathematics and its Applications (elected, 2014), Turner-Kirk Fellow of the Isaac Newton Institute for Mathematical Sciences (Cambridge, UK, 2013–14), Fellow of the Society of Industrial and Applied Mathematics (elected, 2012), the 2011 SIAG/Analysis of Partial Differential Equations Prize (SIAM 2011), the Royal Society Wolfson Research Merit Award (UK, 2009), Alexander von Humboldt Foundation Fellow (Germany, 2003), Alfred P. Sloan Foundation Fellow (USA, 1991), Second Prize of the 1991 State Natural Science Award by the State Council of China, First Prize of the 1989 Advances in Science and Technology Award by the Chinese Academy of Sciences, as well as his recently awarded higher doctorate – Doctor of Science (DSc) by the University of Oxford (UK, 2024).

This article aims to present some of Professor Gui-Qiang G. Chen's significant work to illustrate his contributions to nonlinear conservation laws and partial differential equations. It is not intended as, nor does it come close to, a comprehensive review of all his work.

2 Existence, compactness, and convergence of exact and approximate solutions of nonlinear PDEs via the development of entropy analysis and weak convergence methods

Fundamental to nonlinear hyperbolic systems of conservation laws are the problems of existence, compactness, and convergence of exact and approximate solutions, which require novel ideas involving entropy analysis, weak convergence, compensated compactness, *a priori* estimates, and *ad hoc* structural analysis of the PDEs themselves. In all these areas, Professor Gui-Qiang G. Chen has made profound contributions. For example:

The Lax-Friedrichs scheme is a numerical method for solving nonlinear hyperbolic conservation laws, including the isentropic Euler equations for gas dynamics studied first by Riemann in 1858. Chen (with Ding and Luo) in [6,73–75] was the first to prove rigorously the convergence of the Lax-Friedrichs scheme for the Euler equations with arbitrarily large initial data for the whole physically relevant range of exponents $\gamma \in (1,5/3]$. As a result, the existence and compactness of bounded entropy solutions was established for all $\gamma \in (1,5/3]$, and the convergence of the Godunov scheme was a corollary. Before that, the existence of global solutions of bounded variation (BV) had been obtained, based on a constructive scheme due to Glimm (1965), with the restriction being either to small initial data or to exponents γ close to 1. The existence of global bounded solutions was proven by DiPerna (1983) for the discrete exponent $\gamma = (N+2)/N$, where $N \geq 5$ is odd. The convergence of the Lax-Friedrichs scheme had been established only in the scalar case analyzed by Oleinik (1957) and others. Chen succeeded in establishing sophisticated and original estimates to handle the general case $\gamma \in (1,5/3]$ by combining compensated compactness arguments with his novel ideas of adapting fractional derivatives, the Hilbert transform, and related techniques into entropy analysis. This represented a major advance in the existence, compactness, and convergence theory of hyperbolic conservation laws, and was further extended to the other ranges of exponents: $\gamma \geq 3$ by Lions, Perthame and Tadmor (1994) and $\gamma \in (5/3,3)$ by Lions, Perthame and Souganidis (1996), and to a class of general pressure laws in [42] (also see [35]). Chen's work was recognized through receiving the First Prize of the 1989 Advances in Science and Technology Award from the Chinese Academy of Sciences and the Second Prize of the 1991 State Natural Science Award from the State Council of China.

In [14,36], Chen with his collaborators established the first compensated compactness framework for sonic-subsonic approximate solutions to the two-dimensional Euler equations, for both steady irrotational and rotational flows, which may contain stagnation points. Following this framework, by carefully constructing the approximate solutions, the first global existence result of two-dimensional steady characteristic discontinuities (vortex sheets and contact discontinuities) with large vorticity and no background solution was established in [37].

In [52], Chen (with Perepelitsa) proved that, as the viscosity vanishes, the solutions of the barotropic Navier-Stokes equations converge to a solution of the Euler equations of one-dimensional gas dynamics, thereby successfully settling an issue that had been open since the work of Stokes (1848). This question was difficult as there exists no natural invariant region for the Navier-Stokes solutions, and convex entropy pairs may not produce signed entropy dissipation measures. To overcome these difficulties, uniform energy-type estimates of the Navier-Stokes solutions were first established. Then it was shown how special features of the Euler equations can lead to the conclusion that a class of weak entropy dissipation measures of the Navier-Stokes solutions is confined in a compact set in $H^{-1}_{\rm loc}$. This in turn leads to the existence of a measure-valued solution that satisfies a commutator relation. A careful analysis and characterization of the unbounded support of the measure-valued solution then yields its reduction to a Dirac mass, which in turn leads to the convergence of the Navier-Stokes solutions to an entropy solution of the Euler equations with large initial data as the viscosity vanishes.

Chen and his collaborators also established general compactness frameworks for various important and difficult problems such as the construction of the two-dimensional steady irrotational transonic flows in [61], hyperbolic conservation laws with umbilic degeneracy in [38,39], and the Kolmogorov's theory of turbulence and inviscid limit of the Navier-Stokes equations in [32]. Recently, Chen and his collaborators further developed the ideas and approaches to construct global solutions to the compressible Euler equations and Euler-Poisson equations with spherical symmetry for general pressure law in [33, 35, 53, 57, 68]. The central feature here is the possibility that waves grow as they move radially inward; indeed there had been a long debate involving Guderley (1942) and Courant and Friedrichs (1948) as to whether spherically symmetric solutions might concentrate at the origin. This problem was first solved in [53, 57] where the initial data have finite energy. In his more recent work [68], Chen (with Wang) developed a new approach by adapting a class of degenerate density-dependent viscosity terms including those for shallow water (Saint Venant) flows, and succeeded in employing this approach to prove that the inviscid limit of global weak solutions of the Navier-Stokes equations with the density-dependent viscosity terms is a solution with relative finite energy of the Euler equations with spherical symmetry and large initial data of positive far-field density (with unbounded total initial-energy). Chen's most recent vanishing-viscosity results with related estimates have led to the conclusion that there is no concentration at the origin in the limit, even with unbounded total initial-energy, which provides an affirmative answer to this longstanding open problem.

These new results, ideas, and approaches have also been successfully applied to solving problems and related questions for the compressible Euler-Poisson equations with gravitational potential and general pressure law (esp. including the constitutive equation of white dwarf stars) in [33, 35], entropy flux-splittings for hyperbolic conservation laws (e.g. [42]), the relativistic Euler equations (e.g. [58]), and the Euler equations in nonlinear elastodynamics and the Gauss-Codazzi-Ricci equations (e.g. [1, 46, 47, 62, 63]).

3 The first complete solution to von Neumann's conjectures and new approaches/techniques for multi-dimensional transonic shocks and related free boundary problems for nonlinear conservation laws of mixed type

Multi-dimensional shock problems are important in many applications and fundamental to the mathematical multi-dimensional theory of nonlinear hyperbolic conservation laws[†‡]. In a series of papers, beginning with [19] in 2003, Chen (with Feldman) introduced new ideas and developed approaches/techniques for solving fundamental open problems involving transonic shocks and related free boundary problems for nonlinear PDEs of mixed hyperbolic-elliptic type. In particular, in the Annals paper [22], a new approach was introduced and corresponding techniques were developed to solve the global problem of shock reflection-diffraction by large-angle wedges. This paper was awarded the 2011 Analysis of Partial Differential Equations Prize by the Society for Industrial and Applied Mathematics (SIAM). The SIAM Prize citation said:

> "For their paper 'Global Solutions of Shock Reflection by Large-Angle Wedges for Potential Flow', Annals of Mathematics, Volume 171, Issue 2, 2010, in which they prove the existence and stability of a solution for the equations of two-dimensional compressible gas dynamics, for the case of a shock reflection from a wedge. This problem, originating with the work of Ernst Mach, has long defied careful mathematical analysis."

On account of this significant breakthrough, the global multi-dimensional transonic shock problems can now be solved rigorously in [23, 26, 27]. In particular, the first complete proof of the existence of solutions to both von Neumann's sonic conjecture and detachment conjecture (both proposed in 1943) was provided by Chen and Feldman in [23]. Further results on the existence/uniqueness/stability/optimal regularity of global solutions of the von Neumann problem for shock reflection-diffraction, up to the detachment angle, was established through rigorous analysis of the two-dimensional Euler equations for potential flow in [2, 24, 26, 27]).

The approaches/techniques developed by Chen and his collaborators have been widely applied to solving other transonic shock problems and related problems involving similar difficulties for nonlinear PDEs of mixed type, which arise in fields such as fluid dynamics, amongst others (e.g. [2,3,12,15,24–26] and the references cited therein). These especially include the first complete solution to the other three longstanding open problems – the Prandtl conjecture for the Prandtl-Meyer configuration for supersonic flow onto a solid ramp up to the detachment angle in [3], the Lighthill problem for shock diffraction in [15, 25], and the two-dimensional Riemann problem with four-shock interactions [12].

[†]Multidimensional Hyperbolic Problems and Computations, James Glimm & Andrew Majda (Eds), Springer, 1991.

[‡]Supersonic Flow and Shock Waves, Richard Courant & Kurt Otto Friedrichs, Springer, 1976.

4 Relaxation theory via entropy for nonlinear hyperbolic systems of conservation laws

Relaxation is important in many physical situations, including gas flow near the thermo-equilibrium, kinetic theory with a small mean-free path, and viscoelasticity with vanishing memory. A relaxation theory via entropy was developed by Chen (with Levermore and Liu) in [45, 49] for the study of stability and the singular limits of zero-relaxation time of nonlinear hyperbolic systems of conservation laws with relaxation. In particular, the local equilibrium approximation and its first correction for general hyperbolic systems of conservation laws with appropriate relaxation terms were constructed. A general notion of entropy for such systems was introduced to ensure the hyperbolicity of the local equilibrium approximation and the dissipativity of its first correction. For general 2×2 strictly hyperbolic systems, the existence of dissipative entropies is implied from the strict stability criterion that the equilibrium characteristic lies between the frozen characteristics. The validity of the local equilibrium limit for such 2×2 systems was then established through entropy analysis, which proved that oscillations did not develop in the limit. The analysis includes the construction of suitable entropy pairs to derive energy estimates. Such energy estimates are relevant for not only the compactness properties but also the deviation from the equilibrium of solutions of relaxation systems. Moreover, the weakly nonlinear limit for 2×2 systems was derived and justified through energy estimates. This relaxation theory via entropy has recently motivated successful numerical methods, including relaxation schemes, equilibrium schemes, and kinetic schemes, for the computation of physically relevant solutions of nonlinear hyperbolic systems of conservation laws. Thus, Chen's work has stimulated a whole range of new ideas, paving the way for a wide variety of research in the highly active fields of theoretical and numerical analysis of nonlinear PDEs.

5 Theory of divergence-measure fields and related analysis and applications

The systematic analysis by Chen and his collaborators (e.g. [13, 29, 30, 64–66]) for divergence-measure fields (whose distributional divergences are measures) has led to the development of a representation of the interior (respectively exterior) normal trace of the field on the boundary of any given open set, as the limit of classical normal traces over the boundaries of the interior (respectively exterior) smooth approximations of the open set, as well as to the new understanding of other fundamental properties of these vector fields, resulting in the establishment of the Gauss-Green formula for divergence-measure fields over any open set. An intrinsic connection between weak entropy solutions for hyperbolic conservation laws and divergence-measure fields was first observed by Chen and Frid in [29, 30]. This theory has wide applications, including to nonlinear conservation laws, continuum mechanics, measure-theoretical analysis, and the calculus of variations. Included amongst these applications is the provision of the solution to many longstanding open problems for non-BV entropy solutions, such as the trace of the solution on a surface of co-dimension one, the decay and structure of the solution, free boundary problems, and initial-boundary value problems for nonlinear hyperbolic systems of conservation laws, as well as the first rigorous equivalence proof between the bounded entropy solutions (non-BV) of the multi-dimensional nonlinear PDE systems of balance laws and of the mathematical formulation of physical balance laws via the Cauchy flux through the constitutive relations in the axiomatic foundation of continuum physics (see [66]); see also [13, 29, 64, 65] and the references cited therein.

6 Kinetic theory for anisotropic degenerate parabolic-hyperbolic PDEs of second order and related nonlinear problems

Well-posedness theories for initial value problems are important for both the theoretical and the numerical analysis of all classes of time-dependent PDEs. For nonlinear degenerate parabolic-hyperbolic PDEs, well-posedness issues are understood relatively well if the diffusion term is absent, on account of the work of Lax (1957), Oleinik (1957), Volpert (1967), Kruzhkov (1970), and Lions, Perthame and Tadmor (1994). They are also well-understood when the strict parabolic degeneracy is located at isolated points (as in the porous medium equation). In [54, 55], Chen (with Perthame) developed a unified approach for dealing with both the parabolic and the hyperbolic phases for general anisotropic degenerate parabolic-hyperbolic PDEs (see also [17, 56]). A new notion of kinetic solutions and a corresponding unified kinetic approach was developed so that the well-posedness theory was established. Its advantage is that the resulting kinetic PDEs are well-defined even when the macroscopic fluxes are not locally integrable so that L^1 is a natural space on which the kinetic solutions are posed. Based on this notion, a new, simpler, and more effective approach has been developed to prove the contraction property of kinetic solutions in L^1. A novel ingredient here is a chain-rule-type condition – an essential difference from the isotropic case. Moreover, this new approach was successfully employed in establishing the decay of periodic solutions by recognizing the role of the nonlinearity-diffusivity of the PDE, then exploiting time translations and the monotonicity-in-time property of the solutions. These ideas and approaches have also been applied to solving other nonlinear deterministic and stochastic problems, including the recent work on the well-posedness, the continuous dependence, and the fractional BV regularity for stochastic kinetic solutions in [50], as well as the existence of invariant measures for nonlinear stochastic degenerate parabolic-hyperbolic equations in [51].

Further examples of important areas and directions of research to which Chen has contributed include: stability of multidimensional shock waves past a wedge/cone or in a nozzle (e.g. [8,9,18–21,24,41,71,72] and the references cited therein), stability and instability of characteristic discontinuities (vortex sheets and contact discontinuities) and related free boundary problems for nonlinear hyperbolic systems of conservation laws in magnetohydrodynamics, relativistic fluid dynamics, and thermoelasticity (e.g. [37, 59, 60, 67] and the references cited therein), weak continuity theory for the Gauss-Codazzi-Ricci equations, isometric immersions and related curvatures for connections (e.g. [11, 46–48, 62, 63] and followup papers), stability and uniqueness of Riemann solutions with large oscillation (e.g. [5,7,28,31]), comparison of entropy solutions of nonlinear hyperbolic systems of conservation laws in the BV space (e.g. [10, 69]), nonlinear stochastic hyperbolic PDEs of balance laws (e.g. [16, 50, 51]), global solutions with combustion phenomena (e.g. [34, 70]), vanishing viscosity limit with degenerate viscosities and far-field vacuum (e.g. [4]), amongst other research areas in the analysis of nonlinear PDEs and related applications.

Based on the contributions illustrated above, Chen has published more than 200 original research papers in peer-reviewed premier SCI journals, and more than 10 research books/volumes via leading international publishers.

Professor Gui-Qiang G. Chen's distinguished scientific achievements, inspirational mentoring, exemplary humanity, along with his profound contributions to promoting mathematics across the sciences, have profoundly influenced and inspired us all. The papers contributed by his former students, postdoctoral fellows, collaborators, and friends within the issues of this journal and other journals stand as a modest testimony and demonstration of their high appreciation for Professor Chen's scientific eminence and unfattering friendship. On the special occasion of his 60th birthday, we wish Professor Gui-Qiang G. Chen continued success and fulfillment, in all his endeavors, for many decades to come.

References

[1] A. Acharya, G.-Q. Chen, S. Li, M. Slemrod, and D. Wang, *Fluids, elasticity, geometry, and the existence of wrinkled solutions*, Arch. Ration. Mech. Anal. 226 (2017), 1009–1060.

[2] M. Bae, G.-Q. Chen, and M. Feldman, *Regularity of solutions to regular shock reflection for potential flow*, Invent. Math. 175 (2009), 505–543.

[3] M. Bae, G.-Q. Chen, and M. Feldman, *Prandtl-Meyer reflection configurations, transonic shocks, and free boundary problems*, arXiv:1901.05916, 2024.

[4] G. Chen, G.-Q. Chen, and S. Zhu, *Vanishing viscosity limit of the three-dimensional barotropic compressible Navier-Stokes equations with degenerate viscosities and far-field vacuum*, Ann. Inst. H. Poincaré Anal. Non Linéeaire 39 (2022), 121–170.

[5] G.-Q. Chen, *Vacuum states and global stability of rarefaction waves for compressible flow*, Methods Appl. Anal. 7 (2000), 337–361.

[6] G.-Q. Chen, *Convergence of the Lax-Friedrichs scheme for isentropic gas dynamics (III)*, Acta Mathematica Scientia, 6 (1986), 75–120, Acta Mathematica Scientia (Chinese Ed.) 8 (1988), 243–276.

[7] G.-Q. Chen and J. Chen, *Stability of rarefaction waves and vacuum states for the multidimensional Euler equations*, J. Hyperbolic Differ. Equ. 4 (2007), 105–122.

[8] G.-Q. Chen, J. Chen, and M. Feldman, *Transonic shocks and free boundary problems for the full Euler equations in infinite nozzles*, J. Math. Pures Appl. 88(2) (2007), 191–218.

[9] G.-Q. Chen, J. Chen, and W. Xiang, *Stability of attached transonic shocks in steady potential flow past three-dimensional wedges*, Commun. Math. Phys. 387 (2021), 111–138.

[10] G.-Q. Chen, C. Christoforou, and Y. Zhang, *Continuous dependence of entropy solutions to the Euler equations on the adiabatic exponent and Mach number*, Arch. Ration. Mech. Anal. 189 (2008), 97–130.

[11] G.-Q. Chen, J. Clelland, M. Slemrod, D. Wang, and D. Yang, *Isometric embedding via strongly symmetric positive systems*, Asian J. Math. 22 (2018), 1–40.

[12] G.-Q. Chen, A. Cliffe, F. Huang, S. Liu, and Q. Wang, *Global solutions of the two-dimensional Riemann problem with four-shock interactions for the Euler equations for potential flow*, J. Eur. Math. Soc. 2024. To appear. arXiv:2305.15224, 2023.

[13] G.-Q. Chen, G. E. Comi, and M. Torres, *Cauchy fluxes and Gauss-Green formulas for divergence-measure fields over general open sets*, Arch. Ration. Mech. Anal. 233 (2019), 87–166.

[14] G.-Q. Chen, C. Dafermos, M. Slemrod, and D. Wang, *On two-dimensional sonic-subsonic flow*, Commun. Math. Phys. 271 (2007), 635–647.

[15] G.-Q. Chen, X. Deng, and W. Xiang, *Shock diffraction by convex cornered wedges for the nonlinear wave system*, Arch. Ration. Mech. Anal. 211 (2014), 61–112.

[16] G.-Q. Chen, Q. Ding, and K. Karlsen, *On nonlinear stochastic balance laws*, Arch. Ration. Mech. Anal. 204 (2012), 707–743.

[17] G.-Q. Chen and E. DiBenedetto, *Stability of entropy solutions to the Cauchy problem for a class of nonlinear hyperbolic-parabolic equations*, SIAM J. Math. Anal. 33 (2001), 751–762.

[18] G.-Q. Chen and B. Fang, *Stability of transonic shocks in steady supersonic flow past multidimensional wedges*, Adv. Math. 314 (2017), 493–539.

[19] G.-Q. Chen and M. Feldman, *Multidimensional transonic shocks and free boundary problems for nonlinear equations of mixed type*, J. Amer. Math. Soc. 16 (2003), 461–494.

[20] G.-Q. Chen and M. Feldman, *Steady transonic shocks and free boundary problems in infinite cylinders for the Euler equations*, Comm. Pure Appl. Math. 57 (2004), 310–356.

[21] G.-Q. Chen and M. Feldman, *Existence and stability of multidimensional transonic flows through an infinite nozzle of arbitrary cross-sections*, Arch. Ration. Mech. Anal. 184 (2007), 185–242.

[22] G.-Q. Chen and M. Feldman, *Global solutions to shock reflection by large-angle wedges for potential flow*, Ann. of Math. (2) 171 (2010), 1067–1182.

[23] G.-Q. Chen and M. Feldman, *The Mathematics of Shock Reflection-Diffraction and von Neumann's Conjectures*, in: Princeton Math Series in Annals of Mathematical Studies, Vol. 197, Princeton University Press, 2018.

[24] G.-Q. Chen and M. Feldman, *Multidimensional transonic shock waves and free boundary problems*, Bull. Math. Sci. 12(1) (2022), 2230002.

[25] G.-Q. Chen, M. Feldman, J. Hu, and W. Xiang, *Loss of regularity of solutions of the Lighthill problem for shock diffraction for potential flow*, SIAM J. Math. Anal. 52 (2020), 1096–1114.

[26] G.-Q. Chen, M. Feldman, and W. Xiang, *Convexity of self-similar transonic shocks and free boundaries for the Euler equations for potential flow*, Arch. Ration. Mech. Anal. 238 (2020), 47–124.

[27] G.-Q. Chen, M. Feldman, and W. Xiang, *Uniqueness and stability for the shock reflection-diffraction problem for potential flow*, AIMS Series on Applied Mathematics 10 (2020), 2–24.

[28] G.-Q. Chen and H. Frid, *Decay of entropy solutions of nonlinear conservation laws*, Arch. Ration. Mech. Anal. 146 (1999), 95–127.

[29] G.-Q. Chen and H. Frid, *Divergence measure fields and hyperbolic conservation laws*, Arch. Ration. Mech. Anal. 147 (1999), 89–118.

[30] G.-Q. Chen and H. Frid, *Extended divergence-measure fields and the Euler equations for gas dynamics*, Commun. Math. Phys. 236 (2003), 251–280.

[31] G.-Q. Chen, H. Frid, and Y. Li, *Uniqueness and stability of Riemann solutions with large oscillation in gas dynamics*, Commun. Math. Phys. 228 (2002), 201–217.

[32] G.-Q. Chen and J. Glimm, *Kolmogorov's theory of turbulence and inviscid limit of the Navier-Stokes equations in R^3*, Commun. Math. Phys. 310 (2012), 267–283.

[33] G.-Q. Chen, L. He, Y. Wang, and D. Yuan, *Global solutions of the compressible Euler-Poisson equations with large initial data of spherical symmetry*, Comm. Pure Appl. Math.

77 (2024), 2861–3140.

[34] G.-Q. Chen, D. Hoff, and K. Trivisa, *Global solutions to a model for exothermically reacting, compressible flows with large discontinuous initial data*, Arch. Ration. Mech. Anal. 166 (2003), 321–358.

[35] G.-Q. Chen, F. Huang, T. Li, W. Wang, and Y. Wang, *Global finite-energy solutions of the compressible Euler-Poisson equations for general pressure laws with large initial data of spherical symmetry*, Commun. Math. Phys. 405(3) (2024), 1–85.

[36] G.-Q. Chen, F. Huang, and T.-Y. Wang, *Subsonic-sonic limit of approximate solutions to multidimensional steady Euler equations*, Arch. Ration. Mech. Anal. 219 (2016), 719–740.

[37] G.-Q. Chen, F. Huang, T.-Y. Wang, and W. Xiang, *Steady Euler flows with large vorticity and characteristic discontinuities in arbitrary infinitely long nozzles*, Adv. Math. 346 (2019), 946–1008.

[38] G.-Q. Chen and P. Kan, *Hyperbolic conservation laws with umbilic degeneracy, I*, Arch. Ration. Mech. Anal. 130 (1995), 231–276.

[39] G.-Q. Chen and P. Kan, *Hyperbolic conservation laws with umbilic degeneracy*, Arch. Ration. Mech. Anal. 160 (2001), 325–354.

[40] G.-Q. Chen, J. Kuang, W. Xiang, and Y. Zhang, *Hypersonic similarity for steady compressible full Euler flows over two-dimensional Lipschitz wedges*, Adv. Math. 451 (2024), 109782.

[41] G.-Q. Chen, J. Kuang, and Y. Zhang, *Two-dimensional steady supersonic exothermically reacting Euler flow past Lipschitz bending walls*, SIAM J. Math. Anal. 49 (2017), 818–873.

[42] G.-Q. Chen and P. LeFloch, *Entropy flux-splittings for hyperbolic conservation laws part I: General framework*, Comm. Pure Appl. Math. 48 (1995), 691–729.

[43] G.-Q. Chen and P. LeFloch, *Compressible Euler equations with general pressure law*, Arch. Ration. Mech. Anal. 153 (2000), 221–259.

[44] G.-Q. Chen and P. LeFloch, *Existence theory for the isentropic Euler equations*, Arch. Ration. Mech. Anal. 166 (2003), 81–98.

[45] G.-Q. Chen, D. Levermore, and T.-P. Liu, *Hyperbolic conservation laws with stiff relaxation terms and entropy*, Comm. Pure Appl. Math. 47 (1994), 787–830.

[46] G.-Q. Chen and S. Li, *Global weak rigidity of the Gauss-Codazzi-Ricci equations and isometric immersions of Riemannian manifolds with lower regularity*, J. Geom. Anal. 28 (2018), 1957–2007.

[47] G.-Q. Chen and S. Li, *Weak continuity of the Cartan structural system and compensated compactness on semi-Riemannian manifolds with lower regularity*, Arch. Ration. Mech. Anal. 241 (2021), 579–641.

[48] G.-Q. Chen, S. Li, and M. Slemrod, *On asymptotic rigidity and continuity problems in nonlinear elasticity on manifolds and hypersurfaces*, J. Math. Pures Appl. 160 (2022), 29–53.

[49] G.-Q. Chen and T.-P. Liu, *Zero relaxation and dissipation limits for hyperbolic conserva-*

tion laws, Comm. Pure Appl. Math. 46 (1993), 755–781.

[50] G.-Q. Chen and P. H. C. Pang, *Nonlinear anisotropic degenerate parabolic-hyperbolic equations with stochastic forcing*, J. Funct. Anal. 281(12) (2021), 109222.

[51] G.-Q. Chen and P. H. C. Pang, *Invariant measures for nonlinear conservation laws driven by stochastic forcing*, Chinese Ann. Math. Ser. B 40 (2019), 967–1004.

[52] G.-Q. Chen and M. Perepelitsa, *Vanishing viscosity limit of the Navier-Stokes equations to the Euler equations for compressible fluid flow*, Comm. Pure Appl. Math. 63 (2010), 1469–1504.

[53] G.-Q. Chen and M. Perepelitsa, *Vanishing viscosity solutions of the compressible Euler equations with spherical symmetry and large initial data*, Commun. Math. Phys. 338 (2015), 771–800.

[54] G.-Q. Chen and B. Perthame, *Well-posedness for anisotropic degenerate parabolic-hyperbolic equations*, Ann. Inst. H. Poincaré Anal. Non Linéaire 20 (2003), 645–668.

[55] G.-Q. Chen and B. Perthame, *Large-time behavior of periodic entropy solutions to anisotropic degenerate parabolic-hyperbolic equations*, Proc. Amer. Math. Soc. 137 (2009), 3003–3011.

[56] G.-Q. Chen and B. Perthame, *What is a kinetic solution for degenerate parabolic-hyperbolic equations?* Notices Amer. Math. Soc. 57(6) (2010), 737–739.

[57] G.-Q. Chen and M. Schrecker, *Vanishing viscosity approach to the compressible Euler equations for transonic nozzle and spherically symmetric flows*, Arch. Ration. Mech. Anal. 229 (2018), 1239–1279.

[58] G.-Q. Chen and M. Schrecker, *Global entropy solutions and Newtonian limit for the relativistic Euler equations*, Ann. PDE 8 (2022), 10.

[59] G.-Q. Chen, P. Secchi, T. Wang, *Nonlinear stability of relativistic vortex sheets in three-dimensional Minkowski spacetime*, Arch. Ration. Mech. Anal. 232(2) (2019), 591–695.

[60] G.-Q. Chen, P. Secchi, and T. Wang, *Stability of multidimensional thermoelastic contact discontinuities*, Arch. Ration. Mech. Anal. 237 (2020), 1271–1323.

[61] G.-Q. Chen, M. Slemrod, and D. Wang, *Vanishing viscosity method for transonic flow*, Arch. Ration. Mech. Anal. 189 (2008), 159–188.

[62] G.-Q. Chen, M. Slemrod, and D. Wang, *Isometric immersion and compensated compactness*, Commun. Math. Phys. 294 (2010), 411–437.

[63] G.-Q. Chen, M. Slemrod, and D. Wang, *Weak continuity of the Gauss-Codazzi-Ricci system for isometric embeddings*, Proc. Amer. Math. Soc. 138 (2010), 1843–1852.

[64] G.-Q. Chen and M. Torres, *Divergence-measure fields, sets of finite perimeter, and conservation laws*, Arch. Ration. Mech. Anal. 175 (2005), 245–267.

[65] G.-Q. Chen and M. Torres, *Divergence-measure fields: Gauss-Green formulas and normal traces*, Notices Amer. Math. Soc. 68 (2021), 1282–1290.

[66] G.-Q. Chen, M. Torres, and W. P. Ziemer, *Gauss-Green theorem for weakly differentiable fields, sets of finite perimeter, and balance laws*, Comm. Pure Appl. Math. 62 (2009), 242–304.

[67] G.-Q. Chen and Y.-G. Wang, *Existence and stability of compressible current-vortex sheets in three-dimensional magnetohydrodynamics*, Arch. Ration. Mech. Anal. 187(3) (2008), 369–408.

[68] G.-Q. Chen and Y. Wang, *Global solutions of the compressible Euler equations with large initial data of spherical symmetry and positive far-field density*, Arch. Ration. Mech. Anal. 243 (2022), 1699–1771.

[69] G.-Q. Chen, W. Xiang, and Y. Zhang, *Weakly nonlinear geometric optics for hyperbolic systems of conservation laws*, Comm. Partial Differential Equations 38 (2013), 1936–1970.

[70] G.-Q. Chen, C. Xiao, and Y. Zhang, *Existence of entropy solutions to two-dimensional steady exothermically reacting Euler equations*, Acta Math. Sci. Ser. B 34 (2014), 1–38.

[71] G.-Q. Chen and H. Yuan, *Uniqueness of transonic shock solutions in a duct for steady potential flow*, J. Differential Equations 247 (2009), 564–573.

[72] G.-Q. Chen, Y. Zhang, and D. Zhu, *Existence and stability of supersonic Euler flows past Lipschitz wedges*, Arch. Ration. Mech. Anal. 181 (2006), 261–310.

[73] X. Ding, G.-Q. Chen, and P. Luo, *Convergence of the Lax-Friedrichs scheme for isentropic gas dynamics (I)*, Acta Math. Sci. Ser. B (Engl. Ed.) 5 (1985), 415–432. Acta Math. Sci. Ser. A (Chinese Ed.) 7 (1987), 467–480.

[74] X. Ding, G.-Q. Chen, and P. Luo, *Convergence of the Lax-Friedrichs scheme for isentropic gas dynamics (II)*, Acta Math. Sci. Ser. B (Engl. Ed.) 5 (1985), 433–472. Acta Math. Sci. Ser. A (Chinese Ed.) 8 (1988), 61–94.

[75] X. Ding, G.-Q. Chen, and P. Luo, *Convergence of the fractional step Lax-Friedrichs scheme and Godunov scheme for isentropic gas dynamics*, Commun. Math. Phys. 121 (1989), 63–84.