John Forbes Nash, Jr (1928–2015)

The lives of John Forbes Nash, Jr and his wife Alicia came to a tragic end on 23 May 2015. John Nash was 86 years old. The Nashes were on their way home to Princeton, New Jersey from Oslo, Norway, where Nash had received the Abel Prize in Mathematics [1], when the driver of their taxi lost control of the vehicle and crashed into a guard rail on the New Jersey Turnpike.

Nash's co-recipient of the Abel Prize, Louis Nirenberg of New York University's Courant Institute of Mathematical Sciences recalled [2] that it had been a 'dream week' in Norway for both men who were hailed in the Abel Prize citation as 'towering figures' and 'mathematical giants'. Nirenberg added: 'Just the horror of it after this wonderful week, it's unimaginable.'

John Forbes Nash, Jr was born in Bluefield, West Virginia, on 13 June 1928 and attended the Carnegie Institute of Technology receiving both a BS and MS in Mathematics in 1948. Ironically, he was originally a chemical engineering major, but then switched to chemistry and eventually on the advice of his teacher J.L. Synge to mathematics. A well known quote from his Carnegie teacher and adviser R.L. Duffin in the one sentence letter of recommendation to Princeton University on Nash's behalf simply stated: 'This man is a genius'. Nash received his PhD from Princeton under the direction of A.W. Tucker in 1950. In his thesis, he defined what is now well known as the Nash equilibrium in non-cooperative games. His work in game theory was awarded the Nobel Prize in Economics in 1994. In the 1950s, Nash's behaviour was affected by schizophrenia. His battle against the disease along with his epic life in mathematics has become familiar to the public via the popular book *A Beautiful Mind* by Sylvia Nasar and the movie of the same name. The movie won the Oscar for best picture in 2001.

While many distinguished mathematicians and scientists have commented on Nash's life and death, one remark by R. Dijkgraaf, Director of the Institute for Advanced Study in Princeton, seems particularly striking in summing up Nash's approach to mathematics:

> One consistent element of Nash's work was that he was always going in directions that were thought to be impossible, or actively discouraged. It's amazing the problems he was thinking of. They were really the biggest problems in mathematics. People think there are these really big problems that everyone is working on, but people simply cannot find the internal courage to address the bigger issues. Nash suffered for that; he was really a mathematician that pushed his mind to go far, far beyond where other peoples' would have dared to go. [3]

Nash's work exemplified the word 'power' in mathematics. He exhibited it in his seminal papers in both game theory and partial differential equations. While the work in game theory is perhaps more widely known to the public because of the 1994 Nobel Economics Prize, applied mathematicians and analysts have long had a deep appreciation of his distinctive and creative contributions in analysis. In particular, we cite his work on the isometric embedding problem in Riemannian geometry, the nonlinear implicit function theorem for solving systems of nonlinear partial differential equations, Hölder estimates for the solution of linear partial differential equations with bounded measurable coefficients (independently with E. De Giorgi), and the local existence (in time) and uniqueness of solutions to the initial value problem for the Navier-Stokes equations of a compressible, viscous, heat conducting fluid.

In the paper (written in French) on the Navier-Stokes equations, Nash set up an iteration scheme for resolving the Cauchy problem via a solution to a parabolic system at each step in the iteration. The role of an iteration also appears in Nash's theorems on the nonlinear implicit function theorem, where a Newtonian iteration scheme must be smoothed at each iteration due to a possible loss of derivatives in the scheme. This approach was further developed in the work of J. Moser and L. Hörmander.

Nash also employed his iteration scheme to answer a fundamental problem in differential geometry. Recall that, in his 1854 Habilitationsschrift at the request of C.F. Gauss, B. Riemann had established a generalisation of Gauss's theory of surfaces from our usual two-dimensional surfaces to n-dimensional abstract objects which we nowadays call Riemannian manifolds. Riemannian manifolds like the surfaces are intrinsic, in the sense that the basic properties of distance, curvature, and angle depend only on the manifold itself and are independent of its embedding into an ambient space. Then the fundamental question is whether the Riemannian manifold could be thought of as a classical surface in some possibly high dimensional Euclidean space. Locally, the answer was quickly resolved in the analytic and later C^∞ cases, but a global result escaped the techniques of mathematicians until Nash's epic 1956 paper, in which Nash settled the issue in the affirmative if the target ambient space has sufficiently large dimension. In contrast to this case where the embedding is smooth, Nash established a remarkable result on nonsmooth embeddings. In that paper, Nash showed that any short embedding, i.e., a subsolution of the embedding system of partial differential equations, can be uniformly approximated by a C^1–isometric embedding. This means for example that the standard 2-sphere S^2 has C^1–embedding into an arbitrarily small ball in R^3, where the small ball itself provides the relevant subsolution. Of course, the C^1–embedding is very rough in differential geometry: it is not even smooth enough to define the curvatures. In recent years, Nash's fundamental idea behind the nonsmooth embedding has given rise to the h-principle of Gromov and the discovery of nonsmooth solutions to quasilinear elliptic systems (S. Müller & V. Šverák) and nonunique wild entropy solutions of the incompressible and compressible Euler equations (C. De Lellis & L. Székelyhidi).

Dr Nash is survived by his sons, John David Stier and John Charles Martin Nash, and a sister, Martha Nash Legg.

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