POTENTIAL THEORY FOR SHOCK REFLECTION BY A LARGE-ANGLE WEDGE

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Abstract

When a plane shock hits a wedge head on, it experiences a reflection and then a self-similar reflected shock moves outward as the original shock moves forward in time. Experimental, computational, and asymptotic analysis has shown that various patterns of reflected shocks may occur, including regular and Mach reflection. However, most fundamental issues for shock reflection phenomena have not been understood, such as the transition among the different patterns of shock reflection; therefore, it is essential to establish a global existence and stability theory for shock reflection. On the other hand, there has been no rigorous mathematical result on the global existence and stability of solutions to shock reflection, especially for potential flow which has widely been used in aerodynamics. The theoretical problems involve several challenging difficulties in the analysis of nonlinear partial differential equations including elliptic-hyperbolic mixed type, free boundary problems, and corner singularity especially when an elliptic degenerate curve meets a free boundary. In this paper we develop a potential theory to overcome these difficulties and to establish the global existence and stability of solutions to shock reflection by a large-angle wedge for potential flow. The techniques and ideas developed will be useful to other nonlinear problems involving similar difficulties.

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One of the most important open problems in mathematical fluid dynamics is the reflection of a shock by a wedge, which arises not only in many important physical situations but also is fundamental in mathematical theory of multidimensional conservation laws since its solutions are building blocks and asymptotic attractors of general solutions to the Euler equations for compressible fluids (cf. [1]–[8]). The complexity of reflection picture was reported by Ernst Mach [9] in 1878, and experimental, computational, and asymptotic analysis has shown that various patterns of reflected shocks may occur, including regular and Mach reflection (cf. [1], [4]–[7], and [10]–[13]). However, most fundamental issues for shock reflection have not been understood, such as the transition among the different patterns of shock reflection. Therefore, it is essential to establish a global existence and stability theory for shock reflection in order to fully understand shock reflection phenomena. On the other hand, there has been no rigorous mathematical result on the global existence and stability of solutions to shock reflection, especially for potential flow which has widely been used in aerodynamics (cf. [4, 5, 7] and [14]–[18]). The theoretical problems involve several challenging difficulties in the analysis of nonlinear partial differential equations including elliptic-hyperbolic mixed type, free boundary problems, and corner singularity especially when an elliptic degenerate curve meets a free boundary.

More precisely, when a plane shock in the $(\mathbf{x}, t)$-coordinates, $\mathbf{x} = (x_1, x_2) \in \mathbf{R}^2$, with left state $(\rho, \nabla_{\mathbf{x}}\Phi) = (\rho_1, u_1, 0)$ and right state $(\rho_0, 0, 0)$, $u_1 > 0, \rho_0 < \rho_1$, hits a symmetric wedge $\Lambda := \{|x_2| < x_1 \tan\theta_w, x_1 > 0\}$ head on, it experiences a reflection. Then the Euler equations for potential flow consist of the conservation law of mass and the Bernoulli law for density ρ and velocity potential Φ:

$$\partial_t\rho + \mathrm{div}_{\mathbf{x}}(\rho\nabla_{\mathbf{x}}\Phi) = 0, \tag{1}$$

$$\partial_t\Phi + \frac{1}{2}|\nabla_{\mathbf{x}}\Phi|^2 + i(\rho) = i(\rho_0), \tag{2}$$

where $i'(\rho) = p'(\rho)/\rho = c^2(\rho)/\rho$ with $c(\rho)$ the speed of sound. For polytropic gas, $p(\rho) = \kappa\rho^\gamma$ and $c^2(\rho) = \kappa\gamma\rho^{\gamma-1}$, $\gamma > 1$. Without loss of generality, we choose $\kappa = (\gamma-1)/\gamma$ so that

$$i(\rho) = \rho^{\gamma-1}, \qquad c^2(\rho) = (\gamma-1)\rho^{\gamma-1},$$

which can be achieved by the scaling: $(\mathbf{x}, t, \rho_0^{\gamma-1}) \to (\alpha\mathbf{x}, \alpha^2 t, \alpha^{-2}\rho_0^{\gamma-1}), \alpha^2 = \kappa\gamma/(\gamma-1)$. Then system (1)–(2) can be written as the following nonlinear equation of second order:

$$\partial_t H(\rho_0^{\gamma-1} - \Phi_t - \frac{1}{2}|\nabla_{\mathbf{x}}\Phi|^2) + \mathrm{div}_{\mathbf{x}}(H(\rho_0^{\gamma-1} - \Phi_t - \frac{1}{2}|\nabla_{\mathbf{x}}\Phi|^2)\nabla_{\mathbf{x}}\Phi) = 0, \tag{3}$$

where $H(s) = s^{1/(\gamma-1)} = i^{-1}(s)$. Then the reflection problem can be formulated as the following mathematical problem.

Problem 1 (Initial-Boundary Value Problem). Seek a global solution of system (1)–(2) with the initial condition at $t = 0$:

$$(\rho, \Phi) = \begin{cases} (\rho_0, 0) & \text{for } |x_2| > x_1 \tan\theta_w, x_1 > 0, \\ (\rho_1, u_1 x_1) & \text{for } x_1 < 0, \end{cases} \tag{4}$$

and the slip boundary condition along the wedge boundary $\partial\Lambda$:

$$\nabla\Phi \cdot \nu = 0, \tag{5}$$

where ν is the unit exterior normal to $\partial\Lambda$ (see Figure 1).

Note that Problem 1 is invariant under the self-similar scaling:

$$(\mathbf{x}, t) \to (\alpha\mathbf{x}, \alpha t), \quad (\rho, \Phi) \to (\rho, \Phi/\alpha) \qquad \text{for } \alpha > 0.$$

Thus, we seek a self-similar solution with the form:

$$\rho(\mathbf{x}, t) = \rho(\xi, \eta), \quad \Phi(\mathbf{x}, t) = t\psi(\xi, \eta) \qquad \text{for } (\xi, \eta) = \mathbf{x}/t,$$

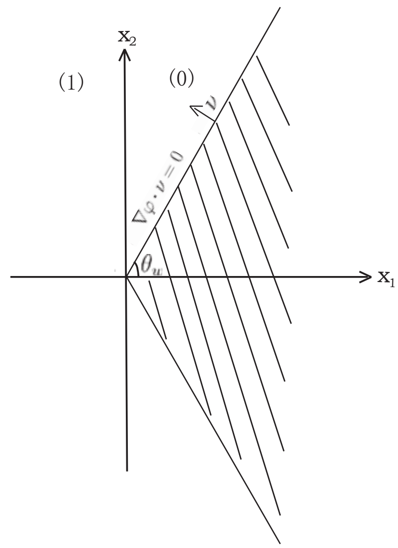

FIGURE 1. Initial-Boundary Value Problem

and set the following quasivelocity potential:

$$\varphi = \psi - \frac{1}{2}(\xi^2 + \eta^2).$$

Then $\varphi(\xi, \eta)$ satisfies the following potential flow equation of second order:

$$\operatorname{div}\left(\rho(|\nabla\varphi|^2, \varphi, \rho_0)\nabla\varphi\right) + 2\rho(|\nabla\varphi|^2, \varphi, \rho_0) = 0 \tag{6}$$

with $\rho(|\nabla\varphi|^2, \varphi, \rho_0) = H(\rho_0^{\gamma-1} - \varphi - \frac{1}{2}|\nabla\varphi|^2)$, where the divergence div and gradient ∇ are respect to (ξ, η). Equation (6) is elliptic-hyperbolic mixed, which is elliptic if and only if

$$|\nabla\varphi| < c(\rho(|\nabla\varphi|^2, \varphi, \rho_0)). \tag{7}$$

Shocks are discontinuities in the quasivelocity $\nabla\varphi$. If Ω^+ and $\Omega^- = \Omega \setminus \overline{\Omega^+}$ are two nonempty open subsets of $\Omega \subset \mathbf{R}^2$ and $S = \partial\Omega^+ \cap \Omega$ is a C^1 curve where $\nabla\varphi$ has a jump, then $\varphi \in W^{1,2}_{loc}(\Omega) \cap C^1(\overline{\Omega^\pm}) \cap C^2(\Omega^\pm)$ is a global weak solution of (6) in Ω if and only if φ is in $W^{1,\infty}_{loc}(\Omega)$ and satisfies equation (6) in $\Omega^\pm$ and the Rankine-Hugoniot condition on S:

$$\left[\rho(|\nabla\varphi|^2, \varphi, \rho_0)\nabla\varphi \cdot \nu\right]_S = 0. \tag{8}$$

The continuity of φ is followed by the continuity of the tangential derivative of φ across S, which is a direct corollary of irrotationality of the velocity. The discontinuity S of $\nabla\varphi$ is called a shock if φ further satisfies the physical entropy condition that the corresponding density function $\rho(|\nabla\varphi|^2, \varphi, \rho_0)$ increases across S in the quasiflow direction. We remark that the Rankine-Hugoniot condition (8) with the continuity of φ across a shock for (6) is also fairly good approximation to the corresponding Rankine-Hugoniot conditions even for the full Euler equations for the shock with small strength since the errors are third-order in strength of the shock.

The plane incident shock solution in the $(\mathbf{x}, t)$-coordinates with two states (0) and (1) corresponds to a continuous weak solution φ of (6) in the self-similar coordinates (ξ, η) with the following form for states (0) and (1):

$$\varphi_0(\xi, \eta) = -\frac{1}{2}(\xi^2 + \eta^2) \qquad \text{for} \quad \xi > \xi_0, \tag{9}$$

$$\varphi_1(\xi, \eta) = -\frac{1}{2}(\xi^2 + \eta^2) + u_1(\xi - \xi_0) \qquad \text{for} \quad \xi < \xi_0, \tag{10}$$

respectively, where $\xi = \xi_0$ is the location of the incident shock with

$$\xi_0 = \rho_1 \sqrt{\frac{2(\rho_1^{\gamma-1} - \rho_0^{\gamma-1})}{\rho_1^2 - \rho_0^2}} = \frac{\rho_1 u_1}{\rho_1 - \rho_0} > 0.$$

Since the problem is symmetric with respect to the ξ-axis, it suffices to consider the problem in the half-plane $\eta > 0$ outside the half wedge

$$D := \{\xi < 0, \eta > 0\} \cup \{\eta > \xi \tan\theta_w, \xi > 0\}.$$

Then Problem 1 in the $(\mathbf{x}, t)$-coordinates can be formulated as the following boundary value problem in the unbounded domain in the self-similar coordinates (ξ, η).

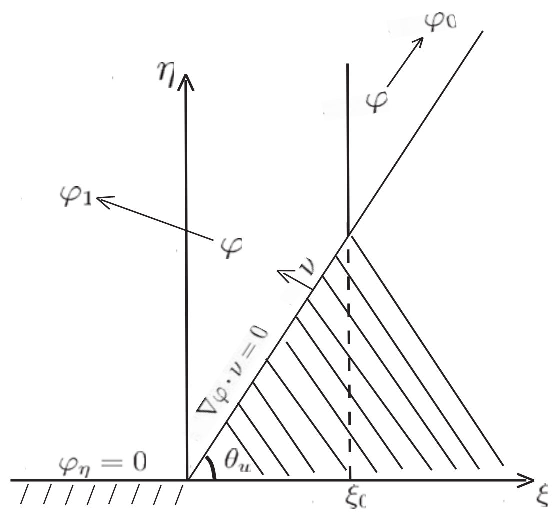

FIGURE 2. Boundary Value Problem in the Unbounded Domain

Problem 2 (Boundary Value Problem) (see Figure 2). Seek a solution φ of equation (6) in the self-similar domain D with the slip boundary condition on the wedge boundary:

$$\nabla\varphi \cdot \nu = 0 \qquad \text{on} \;\; \partial D \tag{11}$$

and the asymptotic boundary condition at infinity:

$$\varphi \to \overline{\varphi} := \begin{cases} \varphi_0 & \text{for } \; \xi > \xi_0, \eta > \xi\tan\theta_w, \\ \varphi_1 & \text{for } \; \xi < \xi_0, \eta > 0, \end{cases} \qquad \text{in } C^1 \text{ when } \xi^2 + \eta^2 \to \infty, \tag{12}$$

where (12) holds in the sense $\lim_{R\to\infty} \|\varphi - \overline{\varphi}\|_{C^1(D\setminus B_R(0))} = 0$.

Since φ_1 does not satisfy the slip boundary condition (11), the solution must differ from φ_1 in $\{\xi < \xi_0\} \cap D$ and thus a shock diffraction occurs. In this paper, we develop a potential theory for shock reflection to solve Problem 2 (i.e., Problem 1) and establish the global existence and stability of solutions to shock reflection by a large-angle wedge for potential flow.

Some efforts have been made mathematically for the reflection problem first for simplified models. One of these models, the transonic small-disturbance equation, was derived and used in [5, 11, 12, 13] and the references cited therein for asymptotic analysis of shock reflection; and some steps of this analysis have been justified in [19] and further extended to the nonlinear wave system. Also see [20] for the pressure-gradient system. On the other hand, in order to deal with the reflection problem, some asymptotic methods have also been developed in [21, 22] under the assumption that the wedge angle θ_w is either very small or close to $\pi/2$, and [12, 13, 23] under the assumption that the shock is so weak that its motion can be approximated by an acoustic wave. For a weak incident shock and a wedge with small angle in the context of potential flow, the nature of the shock reflection pattern was further explored in Morawetz [5] by a number of different scalings, a study of mixed equations, and matching asymptotics for the different scalings.

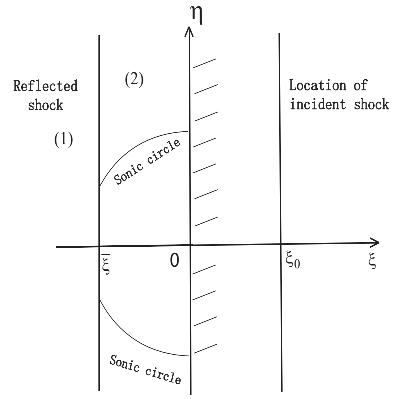

FIGURE 3. Normal Reflection

Normal Reflection. When the wedge becomes flat, i.e., $\theta_w = \pi/2$, the reflection becomes the normal reflection, the simplest case (see Figure 3). In this case, the incident shock normally reflects, the reflected shock is also a plane at $\xi = \bar{\xi}$, and $(u_2, v_2, \rho_2) = (0, 0, \bar{\rho}_2)$, where

$$\bar{\xi} = -\frac{\rho_1 u_1}{\bar{\rho}_2 - \rho_1} < 0, \tag{13}$$

and $\bar{\rho}_2 > \rho_1$ is the unique solution of the equation

$$\bar{\rho}_2^{\gamma-1} = \rho_1^{\gamma-1} + \frac{1}{2}u_1^2 + \frac{\rho_1 u_1^2}{\bar{\rho}_2 - \rho_1} \tag{14}$$

from the Bernoulli law. Then state (1) has form (10) for $\xi < \bar{\xi}$, state (2) has the form:

$$\varphi_2(\xi, \eta) = -\frac{1}{2}(\xi^2 + \eta^2) + u_1(\bar{\xi} - \xi_0) \qquad \text{for } \xi \in (\bar{\xi}, 0), \tag{15}$$

and the reflected shock $\xi = \bar{\xi}$ indeed satisfies the entropy condition: $\bar{\rho}_2 > \rho_1$.

Moreover, it can be shown that $|\bar{\xi}| < \bar{c}_2 := c(\bar{\rho}_2)$. Then $\sqrt{\xi^2 + \eta^2} = \bar{c}_2$ is the sonic circle, and the subsonic region of state (2) is $B_{\bar{c}_2}(0) \cap \{\bar{\xi} < \xi < 0\}$.

The von Neumann Criterion and Local Theory for Regular Shock Reflection. For a possible two-shock configuration at the reflected point $A = (\xi_0, \xi_0 \tan\theta_w)$ satisfying the corresponding boundary condition on the wedge $\eta = \xi \tan\theta_w$, the three state functions $\varphi_j, j = 0, 1, 2$, must be of form (9), (10), and $\varphi_2(\xi, \eta) = -\frac{1}{2}(\xi^2 + \eta^2) + u_2(\xi + \tan\theta_w \eta) + C$ for some constant C. Assume that the line that coincides with the reflected shock in state (2) will intersect with the ξ-axis at point $P = (\hat{\xi}, 0)$ with angle θ_s between the line and the ξ-axis. Then φ_2 can be written as

$$\varphi_2(\xi, \eta) = -\frac{1}{2}(\xi^2 + \eta^2) + u_2(\xi + \eta\,\tan\theta_w) + \Big(u_1(\hat{\xi} - \xi_0) - u_2\hat{\xi}\Big) \tag{16}$$

by the continuity of φ at A and the relation between ξ_0 and $\hat{\xi}$, which is equivalent to the continuity of φ at P.

Also we have $\hat{\xi} = \xi_0 - \xi_0 \frac{\tan\theta_w}{\tan\theta_s}$, that is,

$$(\hat{\xi} - \xi_0)\cos\theta_w + \xi_0 \sin\theta_w \cot\theta_s = 0. \tag{17}$$

The continuity of φ on the shock implies that $\nabla(\varphi_2 - \varphi_1)$ is orthogonal to the tangent direction of the reflected shock, which yields $u_2 = u_1 \frac{\cos\theta_w \cos\theta_s}{\cos(\theta_w - \theta_s)}$. Then the Bernoulli law becomes

$$\rho_2^{\gamma-1} + \frac{\cos^2\theta_s}{2\cos^2(\theta_w - \theta_s)} u_1^2 + \frac{\sin\theta_w \sin\theta_s}{\cos(\theta_w - \theta_s)} u_1 \hat{\xi} - u_1 \xi_0 - \rho_0^{\gamma-1} = 0. \tag{18}$$

Moreover, the Rankine-Hugoniot condition along the reflected shock yields

$$\rho_2 \Big(u_1 \cos\theta_s \tan(\theta_s - \theta_w) - \hat{\xi} \sin\theta_s \Big) - \rho_1 (u_1 - \hat{\xi}) \sin\theta_s = 0. \tag{19}$$

Thus, we obtain system (17)–(19) for $(\rho_2, \theta_s, \hat{\xi})$. The condition for the solvability of this system is the necessary condition for the existence of regular shock reflection.

Now we compute the Jacobian J in terms of $(\rho_2, \theta_s, \hat{\xi})$ for system (17)–(19) at the normal reflection solution state $(\bar{\rho}_2, \frac{\pi}{2}, \bar{\xi})$ for state (2) when $\theta_w = \pi/2$ to obtain

$$J = -\xi_0 \Big((\gamma - 1)\bar{\rho}_2^{\gamma-2}(\bar{\rho}_2 - \rho_1) - u_1 \bar{\xi} \Big) < 0,$$

since $\bar{\rho}_2 > \rho_1$ and $\bar{\xi} < 0$. Then, by the Implicit Function Theorem, when θ_w is near $\pi/2$, there exists a unique solution $(\rho_2, \theta_s, \hat{\xi})$ of system (17)–(19). This implies that the two-shock configuration exists locally near A and, behind the straight reflected shock emanating from A, state (2) is supersonic up to the sonic circle of state (2). Furthermore, this local structure is stable in the limit $\theta_w \to \pi/2$ to the normal reflection solution.

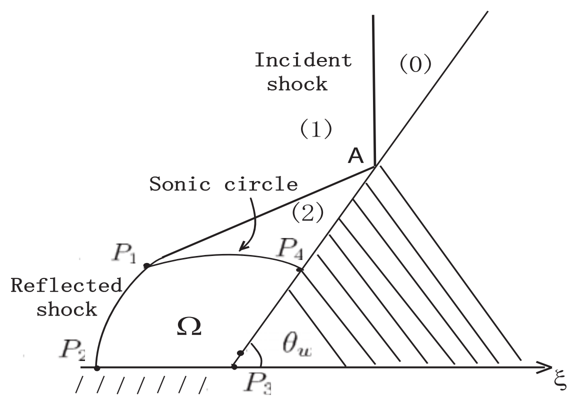

Figure 4. Regular Reflection

Global Theory for Shock Reflection by a Large-Angle Wedge. We now develop a rigorous mathematical approach to extend the local theory to a global theory for solutions to regular reflection, which converge to the unique solution of the normal reflection when θ_w tends to $\pi/2$. The solution φ_2 is quasi-subsonic within the sonic circle for state (2) with center $(u_2, u_2 \tan\theta_w)$ and radius $c_2 := c(\rho_2) > 0$ and is quasi-supersonic outside this circle containing arc P_1P_4 in Figure 4, so that φ_2 is the unique solution in the domain AP_1P_4 as argued in [24, 25]. In the domain Ω, the solution is proved to be quasi-subsonic, smooth, and $C^{1,1}$-smoothly matching with state (2) across P_1P_4 and to satisfy $\varphi_\eta = 0$ on P_2P_3; the transonic shock P_1P_2 matches up to second-order with AP_1 and is orthogonal to the ξ-axis at P_2 so that the standard reflection about the ξ-axis yields a global solution in the whole plane. Then the solution of Problem 2 can be shown to be the solution of Problem 1.

Theorem. There exist $\theta_c = \theta_c(\rho_0, \rho_1, \gamma) \in (0, \frac{\pi}{2})$ and $\alpha = \alpha(\rho_0, \rho_1, \gamma) \in (0, 1)$ such that, when $\theta_w \in (\theta_c, \frac{\pi}{2})$, there exists a global self-similar solution

$$\Phi(\mathbf{x}, t) = t\,\varphi(\frac{\mathbf{x}}{t}) + \frac{|\mathbf{x}|^{\mathbf{2}}}{2t} \qquad \text{for } \mathbf{x/t} \in D,\ t > 0$$

with $\rho(\mathbf{x}, t) = (\rho_0^{\gamma-1} - \Phi_t - \frac{1}{2}|\nabla_{\mathbf{x}}\Phi|^2)^{\frac{1}{\gamma-1}}$ of Problem 1 (equivalently, Problem 2) for shock reflection by the wedge, which satisfies that, for $(\xi, \eta) = \mathbf{x}/t$,

$$\varphi \in C^\infty(\Omega) \cap C^{1,\alpha}(\bar{\Omega}),$$

$$\varphi = \begin{cases} \varphi_0 & \text{for } \xi > \xi_0 \text{ and } \eta > \xi \tan\theta_w, \\ \varphi_1 & \text{for } \xi < \xi_0 \text{ and above the reflection shock } AP_1P_2, \\ \varphi_2 & \text{in } AP_1P_4, \end{cases}$$

φ is quasi-subsonic in Ω and $C^{1,1}$ across the part P_1P_4 of the sonic circle including the endpoints P_1 and P_4, and the reflected shock AP_1P_2 is C^2 at P_1 and C^∞ except P_1. Moreover, the solution φ is stable with respect to changes in the angle in $W^{1,1}_{loc}(\overline{D})$ and converges in $W^{1,1}_{loc}(\overline{D})$ to the solution of normal reflection as $\theta_w \to \pi/2$.

Difficulties and Methods. One of the main difficulties for the global existence is that the ellipticity condition (7) for (6) is more difficult to control, even only for handling the transonic shock than that for steady potential flow as in [26, 27]. The second difficulty is that the ellipticity degenerates at the sonic circle P_1P_4 that is the boundary of the subsonic flow. The third difficulty is that, on P_1P_4, it requires to match the solution in Ω with φ_2 at least in C^1, that is, the two conditions on the fixed boundary P_1P_4: the Dirichlet and conormal conditions, which are generically overdetermined for an elliptic equation since the conditions on the rest boundary have been prescribed. Thus we have to prove that, if φ satisfies (6) in Ω, the Dirichlet continuity condition on the sonic circle, and the appropriate conditions on the other parts of $\partial\Omega$ derived from Problem 2, then the normal derivative $\nabla\varphi\cdot\nu$ automatically matches with $\nabla\varphi_2 \cdot \nu$ along P_1P_4. In fact, this follows from the structure of elliptic degeneracy of (6) on P_1P_4 for the solution φ. Indeed, equation (6), written in terms of function $u = \varphi - \varphi_2$ in the coordinates (x, y) defined near P_1P_4 such that P_1P_4 becomes a segment on $\{x = 0\}$, has the form:

$$(2x - (\gamma+1)u_x)\,u_{xx} + \frac{1}{c_2^2}u_{yy} - u_x = 0 \qquad \text{in } x > 0 \text{ and near } x = 0, \tag{20}$$

plus "small" terms which are controlled by $\pi/2 - \theta_w$ in appropriate norms. Equations (20) is elliptic if $u_x < 2x/(\gamma+1)$. Thus we need to obtain the $C^{1,1}$ estimates near P_1P_4 to ensure $|u_x| < 2x/(\gamma+1)$ that implies both the ellipticity of the equation in Ω and the match of normal derivatives $\nabla\varphi\cdot\nu = \nabla\varphi_2\cdot\nu$ along P_1P_4. Taking into account the "small" terms to be added to equation (20), we need to make the stronger estimate $|u_x| \le 4x/[3(\gamma+1)]$ and assume that $\pi/2 - \theta_w$ is appropriately small in order to control these additional terms. A further issue is the non-variational structure and nonlinearity of our problem which makes it difficult to apply the approaches of [28]–[32]. Moreover, the elliptic degeneracy and geometry of our problem makes it difficult to apply the hodograph transform [33, 34] to fix the free boundary.

For these reasons, our approach is to apply the iteration scheme in [26, 27] to a partially modified equation. We modify equation (6) in Ω by a proper cutoff that depends on the distance to the sonic circle, so that the original and modified equations coincide for φ satisfying $|u_x| \le 4x/[3(\gamma+1)]$, and the modified equation $\mathcal{N}\varphi = 0$ is elliptic in Ω with elliptic degeneracy on P_1P_4. Then we solve a free boundary problem for this modified equation: The free boundary is the curve P_1P_2 and the conditions on P_1P_2 are $\varphi = \varphi_1$ and the Rankine-Hugoniot condition (8), via an iteration scheme developed in [26, 27].

On each step, an "iteration free boundary" curve P_1P_2 is given, and a solution of the modified equation $\mathcal{N}\varphi = 0$ is constructed in Ω with the boundary condition (8) on P_1P_2, the Dirichlet condition $\varphi = \varphi_2$ on the degenerate circle P_1P_4, and $\nabla\varphi \cdot \nu = 0$ on P_2P_3 and P_3P_4. The solvability issue is settled by constructing the solutions φ_ε of the uniformly elliptic equations $\mathcal{N}\varphi + \varepsilon\Delta\varphi = 0$, $\varepsilon > 0$, in Ω with the boundary conditions indicated above, and by making careful estimates for φ_ε to obtain a solution φ of the boundary value problem for the degenerate elliptic equation $\mathcal{N}\varphi = 0$ in the limit as $\varepsilon \to 0$. Then it is proved that φ is in fact $C^{1,1}$ up to the boundary part P_1P_4, especially $|\nabla(\varphi - \varphi_2)| \le Cx$, by using the nonlinear structure of the elliptic degeneracy near P_1P_4 which is modeled by equation (20) and by using a scaling technique similar to [35, 36]. Furthermore, we modify the "iteration free boundary" curve P_1P_2 by using the Dirichlet condition $\varphi = \varphi_1$ on P_1P_2. A fixed point φ of this iteration procedure is a solution of the free boundary problem for the modified equation. Moreover, we prove the precise gradient estimate $|u_x| \le 4x/[3(\gamma + 1)]$, which implies that φ satisfies the original equation (6). Note that, with this approach, the solution is proved to be smooth in Ω, $C^{1,1}$ across P_1P_4, and uniformly quasi-subsonic in Ω away from the degenerate circle P_1P_4.

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