DIAMETERS, CENTERS, AND APPROXIMATING TREES OF DELTA-HYPERBOLIC GEODESIC SPACES AND GRAPHS

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δ-Hyperbolic metric spaces have been defined by M. Gromov in 1987 via a simple 4-point condition: for any four points u, v, w, x, the two larger of the distance sums

$$d(u,v)+d(w,x),\ d(u,w)+d(v,x),\ d(u,x)+d(v,w)$$

differ by at most 2δ. They play an important role in geometric group theory, geometry of negatively curved spaces, and have recently become of interest in several domains of computer science. Given a finite set S of points of a δ-hyperbolic space, we present simple and fast methods for approximating the diameter of S with an additive error 2δ and computing an approximate radius and center of a smallest enclosing ball for S with an additive error 3δ: These algorithms run in linear time for classical hyperbolic spaces and for δ-hyperbolic graphs and networks. Furthermore, we show that for δ-hyperbolic graphs $G = (V, E)$ with uniformly bounded degrees of vertices, the exact center of S can be computed in linear time $O(|E|)$. We also provide a simple construction of distance approximating trees of δ-hyperbolic graphs $G = (V, E)$ on n vertices with an additive error $O(\delta \log_2 n)$. This construction has an additive error comparable with that given by M. Gromov for n-point δ-hyperbolic spaces, but can be implemented in linear time $O(|E|)$ (instead of $O(n^2)$). Finally, we establish that several geometrically defined classes of graphs have bounded hyperbolicity.