-1-

DEF. A HOMOMORPHISM BETWEEN SPHERICAL BUILDINGS IS A MAP $f : \Sigma \to \Sigma'$ S.T. $\forall$ Δ CHAMBER OF Σ, $f|_\Delta$ IS AN ISOMETRY.

EXAMPLE COMING LATER WHEN X IS AN EUCL. BUILDING, $\cdot\,(\partial_\infty X, \angle_{Tits})$ SPH. BUILDING

$\cdot\, \forall v \in X, (\Sigma_v X, \angle_v)$ — " —

$$f : \partial_\infty X \to \Sigma_v X, \quad \alpha \in X \mapsto [r_\alpha] \in \Sigma_v X,$$

WHERE r_α IS THE UNIQUE RAY, $r_\alpha(0) = v$, $r_\alpha(\infty) = \alpha$

=3=

EUCLIDEAN BUILDINGS

EUCLIDEAN COXETER COMPLEXES

DEF. AN EUCLIDEAN COX. COMPLEX = A PAIR:

- $(\mathbb{R}^n, d_{\|\ \|_2}) =: E$
- A GROUP OF ISOMETRIES W_{aff} S.T.
- GIVEN $\rho : \mathrm{ISOM}(E) \longrightarrow \mathrm{ISOM}(\partial_\infty E)$, $\rho(W_{aff})$ IS FINITE
- W_{aff} GENERATED BY REFLECTIONS WRTO HYPERPLANES.

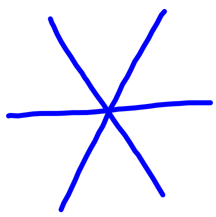
-3-

$\Delta_{mod} = S/W$ SPACE OF SLOPES / DIR.

EVERY GEOD. RAY, ORIENTED SEGMENT HAS A Δ_{mod}-DIR. ATTACHED TO IT.

W_{aff} CALLED AFFINE WEYL GROUP.

$\forall$ x, CONSIDER THE HYPERPLANES GENERATING W THROUGH x. IT DIVIDES E INTO CONES, CALLED WEYL CHAMBERS.

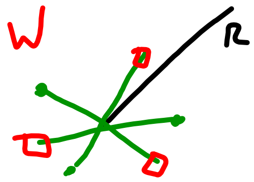


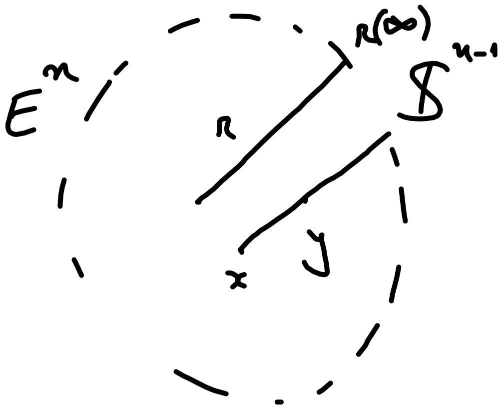
WALL = HYPERPLANE OF REFLECTION

SINGULAR FLAT = AN INTERSECTION OF WALLS

=6=

$$\theta(r) = \theta_{\partial_\infty E}(r(\infty))$$





$$W_{aff} = W \ltimes \Lambda$$

$$\rho(W_{aff}) = W \subset \mathrm{ISOM}(\partial_\infty E)$$

$$\theta : S \to \Delta_{mod} = S/W$$

$\theta(xy) = \theta($GEOD. RAY EXTENDING $[x,y]$ BEYOND $y)$.

=8=

DEF. AN EUCLIDEAN BUILDING MODELLED ON AN EUCLIDEAN COX. CPLX. (E, W_{aff}) IS A HADAMARD SPACE X AND A COLL. OF MAPS $i : E \to X$, ISOMETRIC EMBEDDING S.T. $\forall\, w \in W_{aff}$, $i \circ w$ IS ALSO IN THE COLLECTION

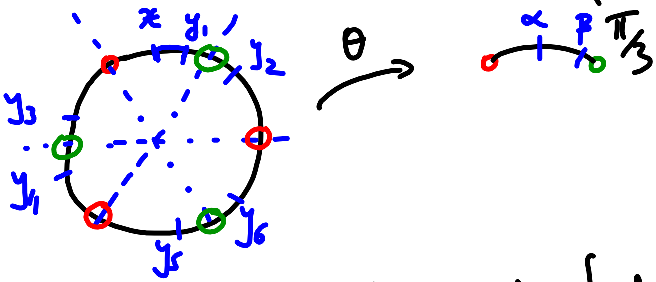
DENOTE COLLECT. OF MAPS $\mathcal{A}$.

CALL i CHARTS, $i(E)$ APARTMENTS.

=10=

REMINDER GIVEN A SPH. COX. COMPLEX (S, W)

AND $\theta : S \to \Delta_{mod}$



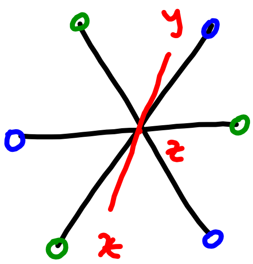
$\alpha, \beta \in \Delta_{mod}$, $D(\alpha, \beta) = \{ d_S(x, y) : \theta(x) = \alpha, \theta(y) = \beta \}$

=12=

(EB3) (PLENTY OF APARTMENTS): $\forall$ GEOD. SEGMENT, GEOD RAY OR BI-INFINITE GEOD. ($\simeq \mathbb{R}$) IS CONTAINED IN AT LEAST ONE APART.

(EB4) (COMPATIBILITY): $\forall\ i_1 : E \to X$, $i_2 : E \to X$ S.T. $C = i_1(E) \cap i_2(E) \neq \phi$ THEN FOR $i_j^{-1}(C) = C_j$, $j = 1,2$, $i_2^{-1} \circ i_1 |_{C_1} : C_1 \to C_2$ IS A RESTRICTION OF SOM $w \in W_{aff}$ TO C_1.

=13=



$\theta(\overline{zy})$ 

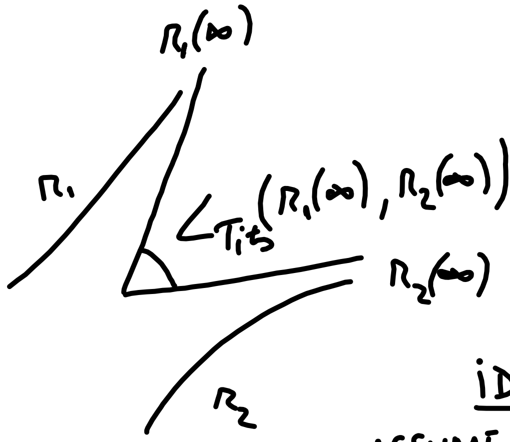
$\theta(\overline{zx})$ 

② TWO ASYMPTOTIC GEODESIC RAYS r_1, r_2

(HAUSDORFF DIST. $(r_1, r_2) < +\infty \iff d(r_1(t), r_2(t))$ BOUNDED)

$\theta(r_1) = \theta(r_2)$. (EX.)

=15=



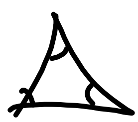
X HADAMARD

$\Rightarrow (\partial_\infty X, \angle_{Tits})$

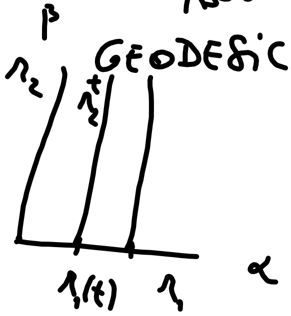
IDEA OF PROOF FOR BOTH

ASSUME $[x,y],[x,z]$ EXTEND TO 2 GEODESIC RAYS:

CAT(0)



$\leq \pi$



$\theta(r_1) = a \in \Delta_{mod}$

$\theta(r_2) = \theta(r_2^t) = b \in \Delta_{mod}$

$\angle_t(r_1^t, r_2) \in D(a,b)$

THEN $\angle_t(\alpha,\beta)$ WILL BE CONST. ON $[0,t_0], [t_1,+\infty)$ AND INCREASES WITH t.

=17=

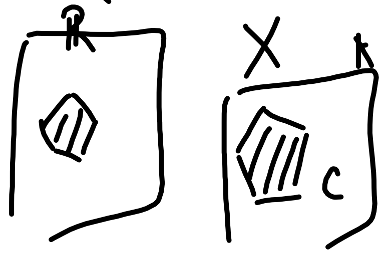
(3) THE MAP $\partial_\infty X \to \Sigma_v X$ IS A SPH. B. HOMOM

$$\alpha \longmapsto \overline{v\alpha}$$

PROOF (1), (2) USE THE PROPOSITION ABOUT "MINIMAL DATA" REQUIRED FOR A SPH. BUILDING.

PROP. $\forall$ CONVEX SUBSET $C \subseteq X$, C ISOMETRIC TO $C' \subseteq E$, C IS CONTAINED IN AN APARTMENT.

IN PART. $C \simeq \mathbb{R}^k$, $k \leq \dim(E)$



= 19 =

IN A TREE:

$J_1 \times J_2$