1 Itai Benjamini

Take Gromov's density model for random groups. Fix k generators. Pick independently and uniformly at random N relators of length ℓ. Gromov shows that, when ℓ is large, the quotient is infinite if $\log_{2k-1} N < \frac{1}{2}$. Zuk shows that the quotient has property (T) if $\log_{2k-1} N > \frac{1}{3}$.

As Linial explained to us for the Erdös-Renyi model, let us pick relators one after the other. What is the girth of the last infinite quotient ? Growth will collapse, but Girth should behave continuously.

2 Valerio Capraro

With Marco Scarsini, we study relations between game theory and amenable groups.

Recall that a countable group G is amenable iff it admits bi-invariant means (i.e. finitely additive probability measures).

Here is a 2-player game: Fix a subset $W \subset G$. P1 chooses $x_1 \in G$. P2 chooses $x_2 \in G$. P1 and P2 exchange 1 euro wether $x_1 x_2 \in W$ or not.

Does the game admit a Nash equilibrium ?

Our intuition is that, without further knowledge, players play "casually", a notion which makes sens for amenable groups only.

Theorem 1 *If G is amenable and if every left-invariant measure is also right-invariant (and conversely), then for all $W \subset G$, there are Nash equilibria given by bi-invariant means.*

Question. Variational problem in amenable groups. Fix a left-invariant mean λ. Does the following functional on means attain its maximal ?

$$\mu \mapsto \int\int \chi_W(xy)\, d\mu(x)\, d\lambda(y).$$

Beware that Fubini does not hold for means. Also, the functional is not weakly continuous.

Question. Game theory characterization of amenable groups. Is it true that a countable group G is amenable if and only if for all $W \subset G$, there are Nash equilibria ?

3 Ryokichi Tanaka

Let H^2 be hyperbolic plane. Consider a circle packing, and its contact graph. Perform simple random walk on it. When it is transient, what is the hitting distribution on the boundary circle ? Is it singular with respect to Lebesgue measure ?

When the packing is invariant under a Fuchsian group, harmonic measure may be singular.

4 Shalom Eliahou

I start with a hard and ancient problem, the Hadamard conjecture.

Definition 2 *A Hadamard matrix is a square matrix with entries ± 1, which is orthogonal up to factor n.*

In 1893, Jacques Hadamard conjectured that Hadamard matrices exist for all sizes $n \in 4\mathbb{N}$. The least open case is 668.

I am interested in a weaker form, where one merely requires that $HH^\top = nI \mod m$. This was introduced by Marrero and Butson in 1972. They give a positive answer for $m = 6$ and $m = 12$. This is easy.

In 2001, with Michel Kervaire, I gave a positive answer for $m = 32$. It is harder: we provide a list which is a mixture of successful guesses, algebra, number theory. What about $m = 64$? I hope it is

5 Nati Linial

5.1 Continuation

Here is an other relaxation of Hadamard's conjecture.

What is the least $f(n)$ for which it can be shown that there exists $n \times n$ ± 1 matrices which each inner product is between $\pm f(n)$.

5.2 Signings

Let G be a d-regular graph. A signing of G is a symmetric matrix B that is obtained by putting ± 1's on the entries of G's adjacency matrix.

Conjecture. Every d-regular graph has a signing with spectral radius $\leq 2\sqrt{d-1}$.

If it holds, then there exist Ramanujan graphs of arbitrarily large size. Indeed, a signing is a specification for a double cover of G, and the spectrum of B resflects the new eigenvalues occuring for that graph. If G was Ramanujan, so is the cover.

5.3 Improving on the Moore bound

Easy: A d-regular n-vertex graph has girth at most $2\frac{\log n}{\log(d-1)}$.

Conjecture. The factor of 2 is not optimal.