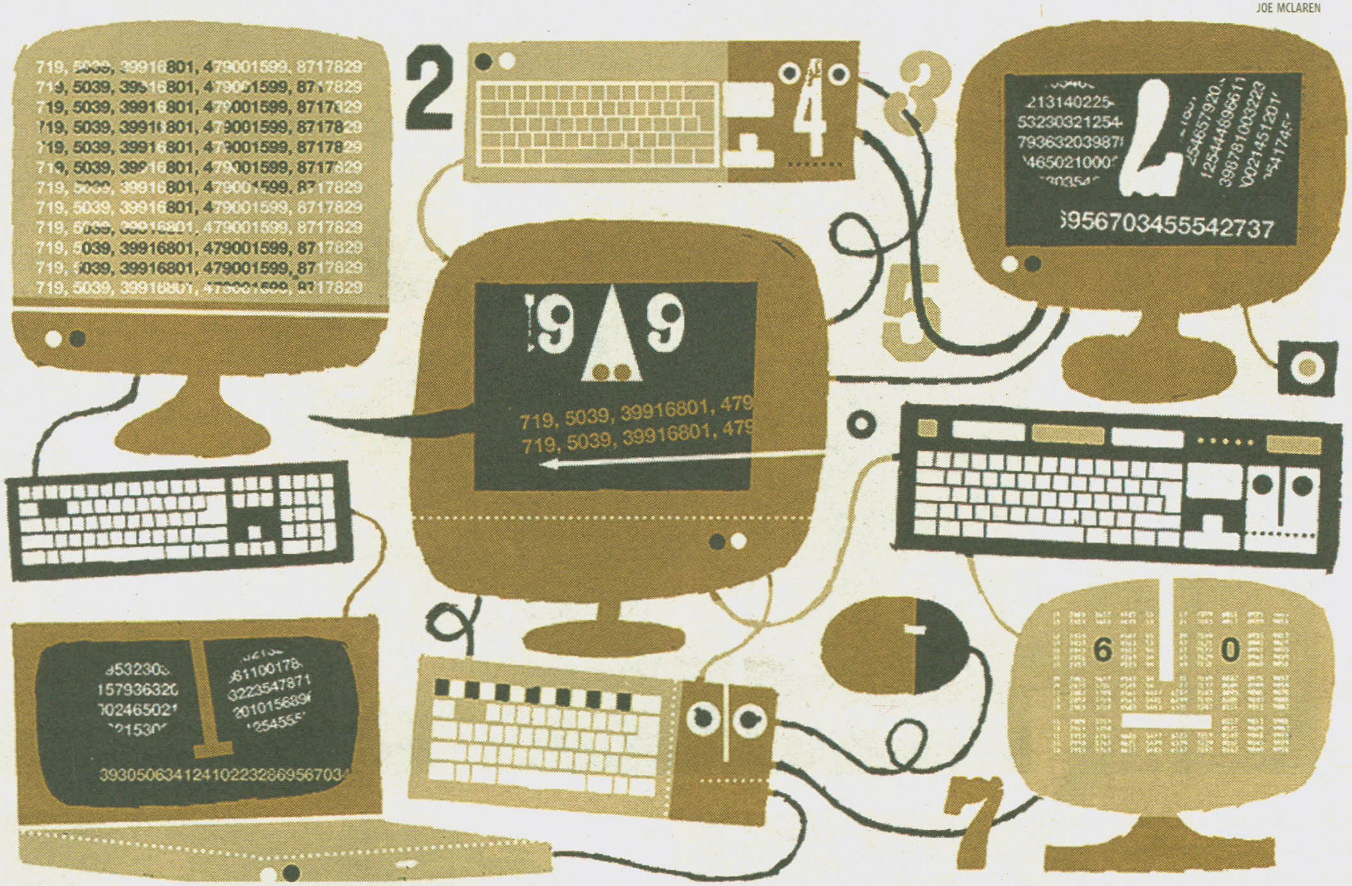
sexy maths

JOE MCLAREN



Primes of passion

Primes of passion

Mathematics is the language of Nature. It helps us to predict climate change, chart the night sky and navigate the inner workings of the atom. The technology we take for granted — mobile phones, the internet, iPods — wouldn't exist without mathematicians.

In this new column I will explore the human side of maths — how it helps you to choose the best partner, to avoid being caught faking your tax return and to smash terrorist cells. Today, though, I start with one of the great enigmas in maths: those numbers that can be divided only by themselves and 1, the primes.

The French composer Messiaen used them to create a sense of timelessness in his music. David Beckham always likes to play in a prime number shirt. Even the security of the internet rests on the mathematics of these indivisible numbers.

Each website has a public code number which is used to encode credit-card numbers. To decode these secret messages a hacker must find the two primes that are multiplied together to give the website's public code number. Unsurprisingly, the websites are using numbers with several hundred digits that are virtually impossible to crack into primes.

Such is our obsession with prime numbers that one organisation, the Electronic Frontier Foundation, offered a prize of $100,000 for whoever discovered the first prime number with more than ten million digits.

I remember as a nerdy kid cutting out newspaper reports of a new big prime discovery. In those days, huge supercomputers claimed the glory but one of the exciting things about the hunt now is that anyone with access to a PC can play the amateur mathematics sleuth simply by downloading a piece of software on to his or her desktop.

I'm not sure if it is my obsession with primes that has rubbed off on my kids, or the prospect of winning thousands of dollars last week, but they were checking a number with 10,853,354 digits to see if it was prime when Edson Smith at UCLA pipped them to the prize with the figure 2 to the power of 43,112,609 -1. Just to give you a sense of how big this number is, it would take you more than two months to read aloud its 12,978,189 digits. It takes a number with only 80 digits to describe the number of atoms in the universe.

To be honest, though, the news of this new prime is not the buzz in mathematics common rooms. The Ancient Greeks proved 2,000 years ago that there are an infinite number of primes, so we know there are primes with as many digits as you want. The trouble is finding them.

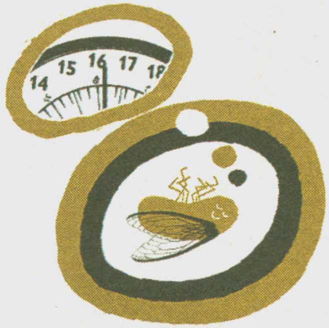
When you look at a list of primes there seems to be no pattern to help you to predict where to find the next big prime. In 1859, the German mathematician Bernhard Riemann came up with a hypothesis about the logic underlying prime numbers that academics have failed to prove for nearly 150 years, despite a $1 million incentive.

While Riemann continues to give me my fair share of sleepless nights, my kids have now set their targets on a different challenge: finding a prime with 100 million digits. There's a $150,000 carrot but I've told them not to get too excited about their prospects. Current estimates are that it will take a desktop computer three years with the current software, and there is only a 1-in-200,000 chance that they'll strike lucky.

But then, who does mathematics for money?

MARCUS DU SAUTOY

The author is Professor of Mathematics at Wadham College, Oxford. His new television series The Story of Maths *starts on BBC Four on Monday at 9pm. To join the search for big primes go to www.gimps.org*



THE CONUNDRUM

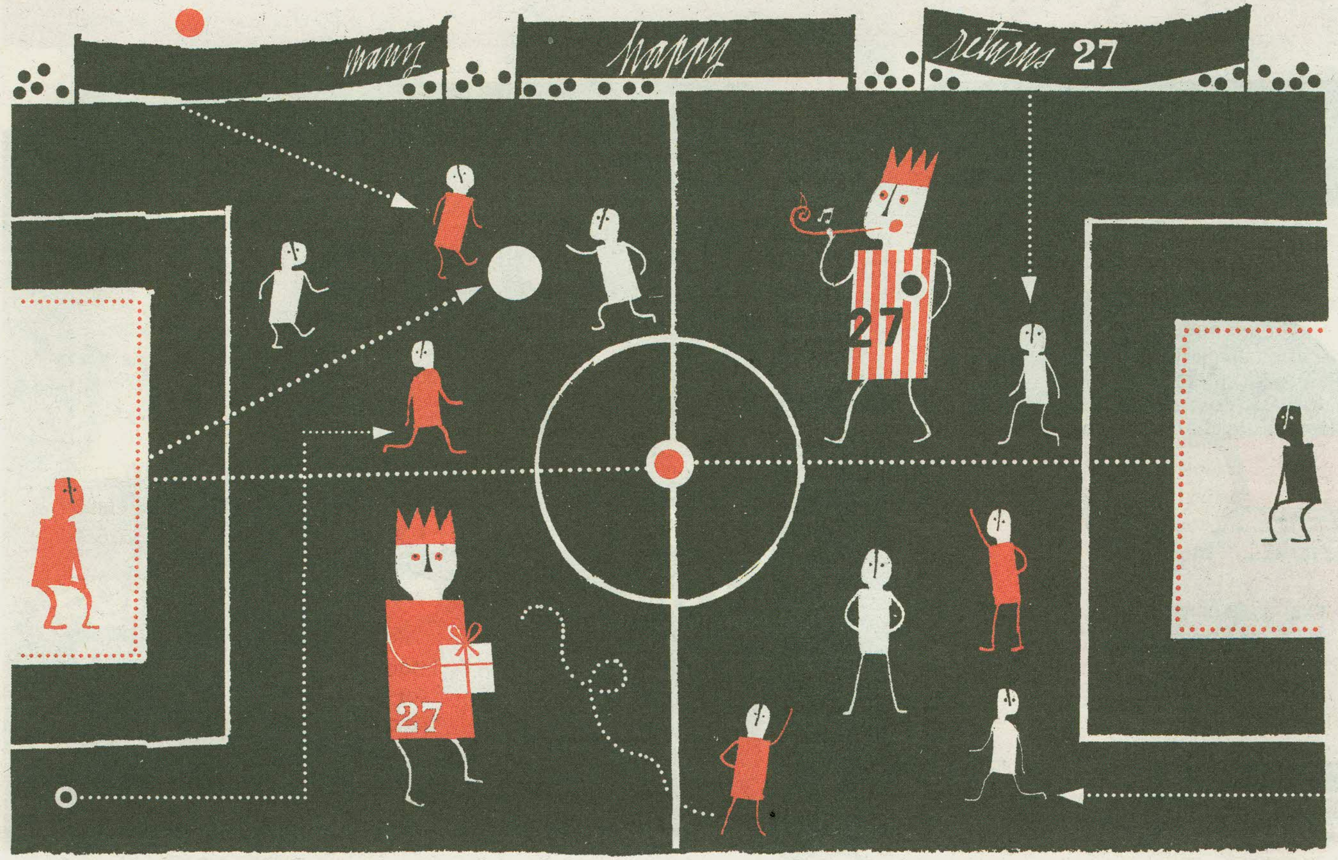
Mathematicians were not the first to search for large prime numbers. Primes seem to have been discovered by a curious cicada that lives in the forests of North America. The cicadas have a strange lifecycle. These insects hide underground doing nothing for 17 years. Then for six weeks they emerge into the forest to mate, lay eggs and then die. The next generation of cicadas waits another 17 years before they emerge. The question is, why?

Solution, page 23

timesonline.co.uk/news/tech_and_web

sexy maths

JOE MCLAREN



Happy (birthday) coincidences

Happy (birthday) coincidences

Out of the Premier League football fixtures that are played each week, how many matches do you think there are in which two people on the pitch have the same birthday? Rather extraordinarily, the maths predicts that two people will share the same birthday in at least five out of the ten matches (in a group of 23 people — and I'm including the referee).

It is one of the counterintuitive characteristics of randomness that you get such coincidences. I was particularly struck by this when I visited the new exhibition at the Serpentine Gallery in London on Saturday (my team, Arsenal, were playing away at Sunderland, so I had the afternoon off). Currently being exhibited are a series of paintings by the German artist Gerhard Richter that have been created using chance.

Each of the 49 paintings in the exhibition consists of 100 randomly chosen coloured squares arranged in a 10 x 10 grid. The colours are chosen from a palette of 25 colours selected by Richter. Faced with such randomness, the viewer hankers after something that has some structure, some hidden pattern. And so you are drawn to those regions of the paintings where chance has produced blocks of two, or even three, colours the same, one after the other.

The unexpected thing is that throughout the paintings this happens a lot. It looks as though some design is coming through the randomness. Yet when I worked through the maths of these paintings, the calculations predicted exactly the amount of strange coincidences that I was seeing in the canvases. If people were asked to create a painting that they thought was random, the likelihood is that most would not paint three consecutive squares the same colour.

Evidence of the reluctance of people to clump things together can be found in the way people choose their numbers for the national lottery. People rarely choose two consecutive numbers, such as 22 and 23. Yet work through the maths and you will find that half the possible combinations of six numbers that can be chosen will have two consecutive numbers. Look at the winning numbers for last Saturday: 17, 18, 28, 29, 32, 40. Throw in the bonus ball of 39 and you've got three lots of consecutive numbers.

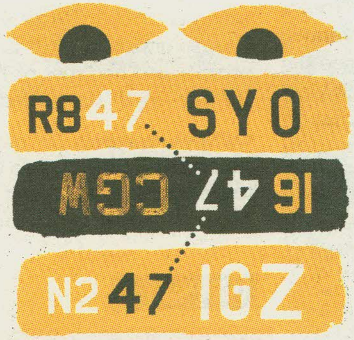
One can use people's desire to spread things out to one's advantage in playing the lottery. This weekend there was just one jackpot winner, who walked off with more than £8 million. Contrast that with what happened on the ninth week of the lottery, when 132 people chose the winning numbers and ended up sharing the jackpot, each getting a measly £122,510. The winning numbers that they all chose were nicely spread out: 7, 17, 23, 32, 38, 42.

Understanding coincidence is important in lots of situations. In legal cases, a person's liberty can depend upon whether something is a rare coincidence, or just an instance of the numbers stacking up (for example, while we may think of DNA evidence as being incontrovertible, this is not necessarily the case). In a poker game, in which only your shirt may be at stake, it still pays to know the odds. For example, in a hand of five-card stud, don't get too excited if you are dealt a pair, as there's nearly an evens chance of being dealt one.

So while I was stunned by Grant Leadbitter's 86th-minute goal for Sunderland against Arsenal on Saturday, I was not surprised to discover that the Gunners' Robin van Persie and Sunderland's Danny Collins did more than share the pitch at the Stadium of Light — they also share a birthday: August 6.

Marcus du Sautoy

The author is Professor of Mathematics at Wadham College, Oxford. His series The Story of Maths *continues on BBC Four at 9pm on October 13.*



THE CONUNDRUM

Here's a game to play on a long car journey with your kids. Look out for vehicles with old-style number plates, such as X847JNK. Take the last two numbers in the registration, in this instance 47. If you were to bet your children that in 20 cars with this style of plate you will see two cars with the same last two numbers, what chance would you have of winning?

Answer on page 28

sexy maths

A calculating approach to love

JOE McLAREN

Remember your first boyfriend or girlfriend? You probably thought that he or she was amazing. Perhaps you even talked romantically of spending your lives together. But then came that nagging feeling that maybe you could do better. There were many more fish in the sea, and perhaps someone else out there was really "the one". The trouble is, if you were to dump your partner, there is generally no going back. So at what point should you just cut your losses and settle for what you've got?

The same dilemma arises in many decisions we have to make. Hunting for a flat is one example. How many times do you see a fantastic one on your first viewing, but then feel that you need to see more before committing yourself — only to risk losing that first flat?

Amazingly, mathematics can help you to maximise your chances of landing the best partner, or the best flat. Even the game show *Deal or No Deal* works on similar principles: once you have opened boxes, you cannot go back to them. So at some point you need to assess when to take the deal and go with what the banker is offering you. And the secret formula for determining this depends on e — not the drug, but the number.

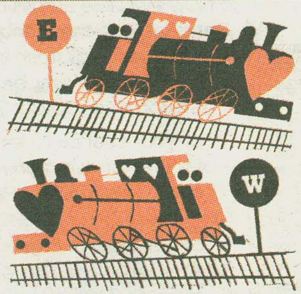
Probably the second most famous number in mathematics — pipped to the post by the enigmatic pi — e has a value of 2.71828... and crops up wherever the concept of growth is important. It is, for example, intimately related to the way in which the interest in your bank account accumulates. Suppose that you are looking at different interest-rate packages being offered by banks to invest £1. One is offering to pay 100 per cent interest after one year. That would increase your investment to £2 at the end of one year, which is not too bad. But another bank is offering to pay 50 per cent interest every half-year. That would give you £1.50 after six months, and after a year would increase to £1.50 + £0.75 = £2.25 — making it a better deal than that offered by the first bank. And the third bank is offering 33.3 per cent, added every third of a year. You do the calculations and find that this bank will give you £$(1.333)^3$ = £2.37. And as you divide the year into smaller and smaller chunks, the compounding of the interest works increasingly to your advantage. The mathematician in you has, we hope, realised that the bank which you really want is Bank Infinity (fortunately unaffected by the credit crunch), which divides the year into infinitely small chunks. The maximum balance that you can possibly achieve from this bank is £2.71828 — the same as the magic number e.

As well as maximising your bank balance, e is also the key to helping you to find the best partner. The process of selecting someone with whom to spend the rest of your life may seem like playing life's lottery but, as we discovered in last week's column, mathematics can help you to maximise your Lotto winnings. As with picking out the box with the biggest prize in *Deal or No Deal*, your decision needn't be made completely randomly: there is a mathematical way to optimise your chances of being lucky in love.

Without overcomplicating matters, mathematical analysis suggests that you should survey the scene for 37 per cent (1/e equals roughly 0.37) of the way through the period that you have set yourself to find a partner. Supposing that you start dating at the age of 16 and aim to find the best partner by the time you reach 60, this would take you to about the age of 32. Then you must choose the next partner who beats all the people you've dated up to that point. It's not going to guarantee you success, but this strategy maximises your chances. Just be sure not to show the formula to your new spouse: it never looks good to be too calculating when it comes to love.

Marcus du Sautoy

The third episode of Marcus du Sautoy's series The Story of Maths *is on BBC Four at 9pm on Monday.*



THE CONUNDRUM

A woman has two lovers, whom she visits by train. One lives at the eastern end of the line, the second to the west. Trains arrive every ten minutes in each direction. She can never decide whom to visit so leaves it to chance, jumping on whichever train arrives first. Explain the circumstances by which she ends up seeing the lover to the east nine times more often than the one to the west.

Answer on page 23

sexy maths

JOE McLAREN

Count, Dracula!

Count, Dracula!

While other kids dressed up as ghosts or zombies on Hallowe'en, I always chose to don false fangs and my grandfather's tails. Recently I discovered that my desire may have been an early sign of my obsession for all things mathematical, as vampires are said to suffer from a condition called arithmomania: a compulsive desire to count things.

Although less well-known than garlic or crosses, one way to ward off the Prince of Darkness is to scatter poppy seeds around his coffin. Theoretically, before Dracula finishes trying to count how many are scattered around his resting place, the sun will have driven him back to his resting place.

Arithmomania is a serious medical condition. The inventor Nikola Tesla, whose studies into electricity gave us the AC current, was obsessed by numbers divisible by three: he insisted on 18 clean towels a day and counted his steps to make sure they were divisible by three. Perhaps the most famous fictional depiction of arithmomania is the Muppets' Count von Count, a vampire who has helped generations of viewers in their first steps along the mathematical path.

Vampires are said to have their own numbers: these are defined as a number that can be written as the product of two smaller numbers of half the length which contain all the digits of the larger number. For example 1395=15x93. The two smaller numbers are called the vampire number's fangs. Although this is little more than a numerical curiosity, mathematicians have proved that there are an infinite number of vampire numbers.

However, research into the science of Hallowe'en has also proved that vampires cannot exist. Vampires need to feed on the blood of a human being at least once a month to survive. The trouble is that once you have feasted on the human, the victim too becomes a vampire. So next month there are twice as many vampires in the search for human blood to feast on.

The world's population is estimated to be 6.7 billion. Each month the population of vampires doubles. Such is the devastating affect of doubling that within 33 months a single vampire would end up transforming the world's population into vampires. Even if one factors in the effect of the birthrate, humans can't reproduce quickly enough to counter the mathematical effect of doubling.

So forget the garlic and the mirrors, it's mathematics that is your best protection against the Prince of Darkness.

Marcus du Sautoy

Marcus du Sautoy will be the new Simonyi Professor for the Public Understanding of Science at Oxford University, succeeding the evolutionist Richard Dawkins.

timesonline.co.uk/news/tech_and_web

sexy maths

JOE McLAREN

Why democracy is an ass

n the two-horse US presidential race ... most points. But if B then drops out, you ... dates drops out you can find that it was

Why democracy is an ass

In the two-horse US presidential race between Barack Obama and John McCain, there should not be too much for the loser to complain about (assuming there is no repeat of the Florida debacle of 2000). A contest involving three candidates, however, would have been a different matter.

I should know. Hours of debate have been spent in Oxford colleges trying to come up with fair voting systems for college elections. Is there a system that will give a result that people feel reflects the college's wishes however the fellows decide to vote? Lewis Carroll, the author of *Alice in Wonderland* and a fellow of Christ Church, spent many hours trying to devise a system to elect fellows fairly to his college. But as he wrote in his diary in 1873, it "proved to be much more complicated than I had expected".

The problem is illustrated by the following hypothetical vote for the new Master of Jordan College. Three candidates have been shortlisted: Dr Abel, Professor Bernoulli and Professor Calculus, or A, B and C for short. The 54 fellows of Jordan College were asked to rank the candidates in order of preference.

21 preferred A to B and B to C.
18 preferred B to C and C to A
15 preferred C to A and A to B.

At first sight you might say that A got the most first votes, so should be elected. But then the other 18+15 fellows object. A total of 33 fellows would have preferred C over A. But try to elect C and there is uproar again; 21+18 fellows would have preferred B to C. So perhaps it looks like B should get the post. Once again a majority, 21+15, would rather see A elected over B.

Perhaps you can come up with a system. For example, suppose yours gives A the most points. But if B then drops out, you realise that C should really win because he's clearly got most votes in a straight run-off. If you change the system and somehow devise a score that results in C coming top and then A drops out, again you look silly because B has the most votes. The trouble is that, however you try to score this election, if one of the candidates drops out you can find that it was the other candidate who should have won.

Time and again voting systems around the world have thrown up anomalous results. In 1998 a former pro-wrestler became Governor of Minnesota, beating the Democrat and Republican candidates thanks to the voting system employed. The International Ice Skating Union was rather shocked in 1995 at the World Figure Skating Championship when a skater who came fourth had the effect of swapping the two skaters who were in the silver and bronze positions. How could someone coming below these skaters have an effect on their ranking?

In 1950 a 28-year-old mathematician called Kenneth Arrow proved that no voting system can be devised that can satisfy a set of innocent-looking criteria that you would hope for in a fair voting system. Arrow's Theorem has the extraordinary consequence that, whatever system you tried to devise, there will always be one voter who, by changing his or her preferences, can change the outcome of the election. This voter is given the status of dictator by the voting system. Fortunately, it is impossible in any practical setting to identify the dictator in any society voting. Arrow was awarded the Nobel Prize for Economics in 1971 for his discovery.

Mathematics may have revealed that democracy is fundamentally flawed but, as Winston Churchill once said, it's still better than all the other forms of government that we have tried. ●

Marcus du Sautoy

The author will be in conversation with Mark Haddon, author of The Curious Incident of the Dog in the Night-time, *at the Royal Society at 6.30pm on Monday*

THE CONUNDRUM

Ten pirates have dug up 100 gold pieces. Each pirate can propose a way to divide the treasure and a vote will be taken whether to accept the proposal. Pirate One, the Cabin Boy, goes first. The proposal is accepted if half the pirates or more vote for his proposal (he's allowed to vote too). Otherwise Pirate One walks the plank, dies and it's Pirate Two's chance. If Pirate Two loses the vote, he dies and it moves to Pirate Three, and so on until a pirate wins the vote. What is the best strategy for Pirate One? He wants to survive but he also wants as much of the treasure as he can get. Remember these pirates are very clever, very greedy and none of them wants to die.

Answer on page 21

sexy maths

JOE McLA

How to get the upper hand at poker

How to get the upper hand at poker

Yesterday, Peter Eastgate left Las Vegas with nearly $10 million after winning the 2008 World Series of Poker. There were more than 6,000 entrants to poker's premier event but the 22-year-old Dane outlasted the game's greatest exponents to become its youngest winner.

Card games are often considered to be games of pure luck but poker also requires psychology — the ability to read the temperaments of your opponents — and a good handle on mathematics, which is why time and again you see the same faces finishing in the money in major poker tournaments.

Professionals constantly calculate the probability of winning hands with the cards they hold. So, in the final hand of the tournament, Eastgate had to disguise his excitement. The cards he was holding, the ace of diamonds and five of spades, combined with three cards on the table, gave him a straight (ascending numerical cards 1-2-3-4-5). The only way his opponent, the Russian Ivan Demidov, could beat him was with a higher straight. But the maths told him that of the 990 hands that Demidov could hold, only 12 would win, about a 1 per cent chance. Sure enough, Demidov had only two pairs and Eastgate's maths had won him the 2008 World Series of Poker bracelet.

Even before you start dealing the cards there is a lot of mathematics that it's worth being wise to. Hustlers and magicians spend years perfecting something called the perfect shuffle, which allows them to dictate where in the deck cards appear. The deck is split exactly into two equal piles and then the cards are perfectly interweaved like a zipper, alternating one at a time from the right and left hand. It is difficult to perform this trick but, once mastered, it can be put to devastating effect. This is because the person holding the cards knows exactly how the cards are arranged.

So, for example, suppose you and an accomplice want to sting two players in a round of poker. Put four aces on top of the pack. After one perfect shuffle the aces are two cards apart. After another perfect shuffle the aces are four cards apart, perfectly placed for you as the dealer to deal your accomplice all four "bullets".

Magicians exploit an even more amazing mathematical property of the perfect shuffle. If you do it eight times in a row, although the audience is convinced that the pack must be totally random, the magician knows that the deck has returned to its original arrangement. The perfect shuffle is a bit like rotating an eight-sided coin. Each shuffle is like moving the coin round an eighth of a turn. After eight shuffles, just as the coin has returned to its original position, the deck is just as it was before you started shuffling.

But what if you are shuffling cards for a round of poker at home tonight with your friends. How many times should you shuffle the deck to make sure that the cards are properly scrambled?

Mathematicians have analysed the way most of us shuffle. If you are doing a riffle shuffle (also called a dovetail shuffle, the one favoured by dealers in casinos) and there are L cards in your left hand and R cards in your right then a sensible model is to say that there is an L/(L+R) probability that the card is going to fall from your left hand. After analysing the mathematics of this shuffle, it transpires that you need to shuffle the pack seven times for it to become random. Any less than this and the pack retains information from the previous game.

So, if you have aspirations to be sitting there with the finalists at the 2009 World Series of Poker, just remember, it's the maths that will make you your millions. Lucky players don't last. ●

Marcus du Sautoy

THE CONUNDRUM

In Texas Hold 'em you start by being dealt two cards face down, called your pocket cards. The best starting hand you can be dealt is a pair of aces, but any high pair is good. For example, if you are dealt a pair of sixes, there is only a 25 per cent chance of the other eight players on a table having a higher pair. So, what is the probability that the first two cards you are dealt are a pair?

Answer on page 21

timesonline.co.uk/news/tech_and_web

sexy maths

JOE McLAREN

To infinity — and beyond

To infinity — and beyond

The credit crunch has caused some big numbers to be cited over the past months: millions lost in the financial meltdown in Iceland, billions promised to save British banks, trillions wiped off the markets in one day of trading. But, as all children know, it's easy to beat any of those numbers. You lost a trillion dollars? Well, I lost a trillion plus one. But the playground clever-clogs may venture a number to trump all others: infinity. It can't be so easily beaten by adding one.

Suppose an hotel has an infinite number of rooms, and each one is occupied. If a new guest arrives, Hotel Infinity's proprietor can still accommodate the new guest. By shifting each guest one room along, room 1 is freed up for the new guest. So no one is left without a room, because there is always another one.

You might try to beat it by adding infinity to infinity. But Hotel Infinity can still soak up infinitely many new guests. Ask the existing guests to move to the room number double that of their present one, ie, the person in room 5 moves to room 10. Now all the odd-number rooms are empty and can take the new guests.

So perhaps infinity is the biggest number? One exciting moment in mathematical history was the realisation at the end of the 19th century that infinity isn't the biggest number: there are infinite infinities, each bigger than the previous one. It was Georg Cantor, a German mathematician, who came up with a beautiful argument for why there is more than one. So hold on to your mathematical hats as I take you to infinity — and beyond.

Hotel Infinity's rooms are numbered by whole numbers: 1, 2, 3 etc. Hotel Uncountable has rooms numbered using all the infinite decimal numbers — those that continue for ever after the decimal point — such as $\Pi = 3.14159\ldots$ and $\sqrt{2} = 1.41421\ldots$. Both have infinitely many rooms but Cantor showed why Hotel Uncountable's infinity is bigger than Hotel Infinity's. If two hotels have a finite number of rooms, the way to tell which has the most is to pair up the rooms; the hotel with rooms left over is the bigger one. This is how Cantor realised that you should compare infinities.

So, imagine that Hotel Infinity's owner thinks that he's found a way to pair up all his rooms so that they match all the rooms in Hotel Uncountable; for example (to choose some infinite decimal numbers at random), suppose that room 1 is paired with room 0.342565..., room 2 with room 0.578027466..., room 3 with room 0.55472882... and so on. Hotel Uncountable's owner can always come up with a room that has been missed. To do this, she cooks up an infinite decimal number so that the first decimal differs from the first decimal of the room paired with room 1; in this case change the 3 to a 4. The second decimal is chosen to differ from the second one of room 2; for example, change the 7 to an 8. Keep going, arranging each time that, for example, the 100th decimal is different from the 100th decimal of the room paired with room 100. So, from these examples, the new number starts 0.485....

Why has the room with this door number not been counted? Suppose that Hotel Infinity's owner claims that this room was paired with a later number, room 412, say. Hotel Uncountable's owner can show that it wasn't. Look at the 412th decimal place of this new number: because of the way we constructed the number, it must be different from the 412th decimal place of the room you paired with room 412. So it's not the number paired with room 412.

Hotel Infinity's owner missed this room number. But even if he tried to shift all the rooms along one and add this new room to the count, Hotel Uncountable's proprietor can play the same trick, producing another missing room number. Hence the infinity of rooms in Hotel Uncountable is bigger than the infinity of Hotel Infinity. ●

Marcus du Sautoy

The author appears in Zero to Infinity *at the Dana Centre on November 20 at 7pm.*

THE CONUNDRUM

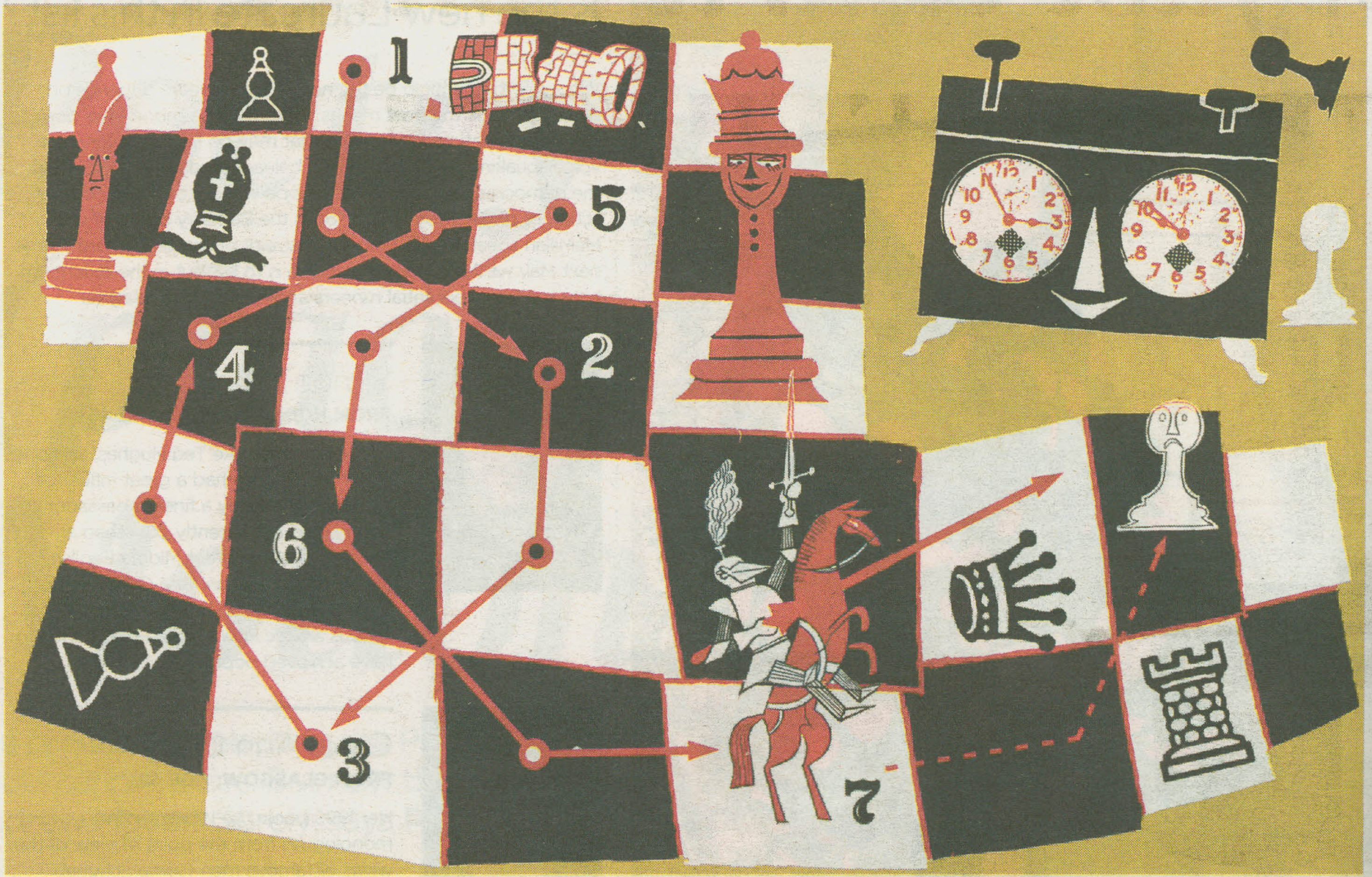
If infinitely many buses arrive at Hotel Infinity with infinitely many guests in each bus, can the proprietor fit them all in?

Answer on page 21

timesonline.co.uk/news/tech_and_web

sexy maths

JOE McLAREN



The skills of a Grandmaster

For a while, the chess Olympiad this year looked like producing a surprise winner but closer inspection of Israel's team sheet revealed that it was pretty much business as usual: half the players were named Boris!

Other than a brief blip in the 1970s, the biennial event has produced remarkably consistent results. From 1952 to 1990, the Soviet Union ruled the contest, and after the superstate's fragmentation either Russia or one of its former union satellites struck gold every time. As it turned out this year, the Soviet diaspora's turn in the spotlight was short-lived and Armenia triumphed for its second successive Olympiad.

Despite being connected by being born under the red flag, those that dominate the game are better categorised by their membership of a different club: the mathematical mafia. Legend has it that the game was invented by a mathematician in India who elicited a huge reward for its creation. The King of India was so impressed with the game that he asked the mathematician to name a prize as reward. Not wishing to appear greedy, the mathematician asked for one grain of rice to be placed on the first square of the chess board, two grains on the second, four on the third and so on. The number of grains of rice should be doubled each time.

The King thought that he'd got away lightly, but little did he realise the power of doubling to make things big very quickly. By the sixteenth square there was already a kilo of rice on the chess board. By the twentieth square his servant needed to bring in a wheelbarrow of rice. He never reached the 64th and last square on the board. By that point the rice on the board would have totalled a staggering 18,446,744,073,709,551,615 grains.

Playing chess has strong resonances with doing mathematics. There are simple rules for the way each chess piece moves but beyond these basic constraints, the pieces can roam freely across the board. Mathematics also proceeds by taking self-evident truths (called axioms) about properties of numbers and geometry and then by applying basic rules of logic you proceed to move mathematics from its starting point to deduce new statements about numbers and geometry. For example, using the moves allowed by mathematics the 18th-century mathematician Lagrange reached an endgame that showed that every number can be written as the sum of four square numbers, a far from obvious fact. For example, $310 = 17^2 + 4^2 + 2^2 + 1^2$.

Some mathematicians have turned their analytic skills on the game of chess itself. A classic problem called the Knight's Tour asks whether it is possible to use a knight to jump around the chess board visiting each square once only. The first examples were documented in a 9th-century Arabic manuscript. It is only within the past decade that mathematical techniques have been developed to count exactly how many such tours are possible.

It isn't just mathematicians and chess players who have been fascinated by the Knight's Tour. The highly styled Sanskrit poem *Kavyalankara* presents the Knight's Tour in verse form. And in the 20th century, the French author Georges Perec's novel *Life: A User's Manual* describes an apartment with 100 rooms arranged in a 10x10 grid. In the novel the order that the author visits the rooms is determined by a Knight's Tour on a 10x10 chessboard.

Mathematicians have also analysed just how many games of chess are possible. If you were to line up chessboards side by side, the number of them you would need to reach from one side of the observable universe to the other would require only 28 digits. Yet Claude Shannon, the mathematician credited as the father of the digital age, estimated that the number of unique games you could play was of the order of 10^{120} (a 1 followed by 120 0s). It's this level of complexity that makes chess such an attractive game and ensures that at the Olympiad in Russia in 2010, local spectators will witness games of chess never before seen by the human eye, even if the winning team turns out to have familiar names. ●

Marcus du Sautoy

THE CONUNDRUM

Take a chess board and cut off the two opposite corners so that there are only 62 squares on the board. Can you lay 31 dominos on the board so that all the squares are covered? (A domino is assumed to be big enough to cover two adjacent squares. You can't lay dominos diagonally.)

Answer on page 21

timesonline.co.uk/news/tech_and_web

sexy maths

JOE McLAREN

The symmetry of sneezing

The symmetry of sneezing

At this time of year more than any other, the air is full of symmetrical objects. Not just the five-pointed stars that adorn Christmas trees, or the six-armed snowflakes that accompany the increasingly Arctic weather. It's the explosion of symmetrical shapes shot through the air every time someone sneezes that accounts for the surge of seasonal symmetry. The viruses that carry the coughs and influenza that lay us low each winter are invariably constructed with beautiful but sometimes deadly symmetry.

In 1918, the Spanish flu pandemic killed 50 million people, more than the casualties of the First World War. Such devastation concentrated scientists' minds on determining the mechanism of this dangerous disease. They soon realised that bacteria was not the cause, but something that could not be seen under a conventional microscope. They called these new agents viruses, after the Latin for poison.

The discovery of the true nature of these viruses had to wait for the development of a new piece of equipment called the electron microscope, which gave scientists a way of penetrating the underlying molecular structure of the organisms that were reaping such havoc. A molecule is a bit like a collection of ping-pong balls connected together with toothpicks. The pictures that you get from the electron microscope are a bit like shining a light on one of these structures and looking at the shadow created by the arrangement of ping-pong balls.

This is where mathematics can become a powerful ally in unravelling the information contained in these shadows. The game is to identify what three-dimensional shapes could give rise to the two-dimensional shadows from the electron microscope. Progress often depends on getting the right angle to reveal the molecule's true character.

When scientists studied the 2-D pictures that these new techniques were revealing they found, rather than a tangled mess of molecules, shapes full of symmetry. The first views revealed dots arranged in triangles, which implied that the three-dimensional shape could be spun by a third of a turn and the shape would realign. When the biologists looked in the mathematicians' cabinet of shapes, the Platonic solids seemed to be the best candidates.

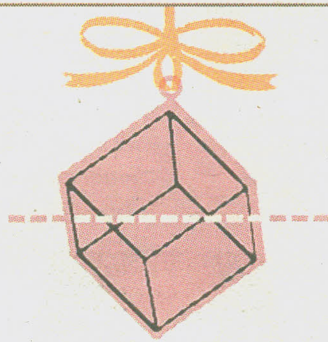
The Platonic solids are the symmetrical shapes that make good dice, such as the cube made up of six square faces. As any *Dungeons & Dragons* aficionado will know, we can construct four other dice in addition to the cube. The tetrahedron, for example, comprises four equilateral triangles; the first die used in history, it featured in a forerunner of backgammon, played in 2,500BC by the Babylonians.

The trouble was that all five of Plato's dice had axes through which you could spin the shape by a third of a turn such that all the faces would realign. It was when they looked at the shadow from another perspective that they were able to pin down the shapes of these viruses more precisely. Suddenly dots arranged in pentagons appeared. This allowed the scientists to home in on one of the more interesting of Plato's dice, the icosahedron, a shape made up of 20 triangles with five triangles meeting at each point.

Subsequent analysis has revealed that the underlying structure of some of the most deadly viruses are constructed using the shape of an icosahedron. From influenza to herpes, from polio to the Aids virus, symmetry is the key to the mechanism by which these viruses can so easily replicate themselves. The symmetry of the shape provides a simple formula for constructing multiple copies of the virus, which is what makes it so virulent.

Marcus du Sautoy

The author will be Kirsty Young's castaway on *Desert Island Discs* on BBC Radio 4 on Friday at 9am



THE CONUNDRUM

Imagine hanging a cube-shaped decoration on your Christmas tree. Now cut the cube along the middle, horizontally. You have two identical pieces, each with a new face. What is the shape of that new face?

Answer on page 21

timesonline.co.uk/news/tech_and_web

sexy maths

JOE McLAREN

Warm up with a few festive candles

On the first day of Christmas my true love sent to me a partridge in a pear tree. So how many presents did I get from my true love by the twelfth day of Christmas? This is the sort of problem my family gets tortured with at this time of year by their mathematical father. But it also illustrates some classic traits of the mathematical mind.

There is, of course, the tedious method: counting through the days of Christmas, keeping a tally of all the presents. "OK, on the second day you've got two turtle doves and another pear tree. So that's three presents, plus the first pear tree . . . ." But soon there will be so many lords a-leaping and species of birds flying round that it will be hard to keep track of everything.

Mathematicians are lazy beasts at heart, so we like a solution that avoids too much computation. Surprisingly, most mathematicians are not that hot on mental arithmetic, preferring a clever bit of lateral thinking to lengthy sums. One classic trick is to limber up by solving a simpler version of the problem. Conveniently, my family also celebrates Chanukkah (although, as Richard Dawkins's successor as the Simonyi Professor for the Public Understanding of Science, I probably should not be admitting that). But this Jewish festival does provide a slimmed-down version of the Christmas problem and makes a good mathematical warm-up.

On Sunday evening, the first day of Chanukkah, my children will light two candles. The next evening they will light three — and so on until on the eighth day, when they will light all nine candles on the candelabra. So how many candles are needed for the whole festival?

To make things slightly easier, let's add an extra day at the beginning, with one candle. So we want to calculate $1 + 2 + 3 + \ldots + 9$ candles. The slow way is to add up the numbers one by one; the mathematical way is to change the sum into something else. Arrange the candles in a triangle, with one at the top and nine along the bottom. Make a copy of this triangle, but lay it upside-down, with nine candles in the top row and one in the bottom. If we join these triangles together, every row now has ten candles. There are nine rows, making a total of $9 \times 10 = 90$ candles. But we took two triangles, so we need to halve the number to get 45 candles. Remember that we added an extra day to Chanukkah to make the calculation easier. So we need 44 candles to celebrate Chanukkah.

This trick of turning numbers into geometry is a powerful weapon in attacking such problems, and the 12 days of Christmas poser requires three-dimensional geometry to solve it. As the presents arrive each day, you want to stack them to make a pyramid of presents rather than a triangle. The first layer contains one pear tree; the second a pear tree and two turtle doves; the last a pear tree in one corner and 12 drummers drumming down the long side of the triangle.

If we take six of these pyramids, they can be put together to make a rectangular box 12 presents high, 13 presents long and 14 presents deep. So the total number in each pyramid is $12 \times 13 \times 14/6 = 364$, one present for every day of the year, bar one.

Having solved the two-dimensional Chanukkah problem and three-dimensional Christmas one, the mathematician in me is on the search for another religious festival my family could celebrate that might have a four-dimensional version of our counting problem: the Hindu festival of Diwali, or African festival of Kwanzaa? Or maybe I'll have to start my own four-dimensional cult in hyperspace.

Marcus du Sautoy

Stuck for a Christmas present? Why not get a symmetrical shape in hyperspace named after a loved one in exchange for a donation to Common Hope, a charity that provides education, housing and healthcare to Guatemalan street kids; visit firstgiving.com/findingmoonshine

THE CONUNDRUM

One hundred guests attended the Mathematical Institute Christmas party. How many handshakes must take place for all 100 guests to shake hands with each other?

Answer on page 23

sexy maths

JOE McLAREN

How to be a perfect timekeeper

How to be a perfect timekeeper

Every year millions of people around the world do not mark the beginning of their new year when the clock strikes 12 on December 31. For the Chinese, the Year of the Ox will begin on January 26. For Jews the next new year starts on the evening of September 18, while Muslims have just celebrated the beginning of 1430, which arrived on the evening of December 28. As it happens, tonight nobody around the globe should be popping their corks at midnight, because the Gregorian calendar is having a blip and the new year will officially start when the second hand reaches minus 1.

We are used to adding extra days for leap years but every now and again it is necessary to add a leap second to ensure that our calendar keeps in step with the passage of the Earth around the Sun. One of the reasons that we periodically have to adjust our timekeeping is because the Earth, Sun and Moon play a crazy, syncopated rhythm as they dance through the sky.

One of the things unifying the world's calendars is the role of mathematics in trying to make sense of the passage of time; the other is that the day is a basic unit of measurement common to all of them. This is not the time it takes the Earth to spin once on its axis. That actually takes 23 hours 56 minutes and 4 seconds. Use this as the length of a day and over time day and night would flip over. Instead, a day consists of the time it takes for the Earth to spin so that the Sun returns to its original position in the sky. Because the Earth is moving around the Sun, this adds the four minutes needed to bring a day to a full 24 hours. The leap second being added at the end of 2008 is due to the gradual slowing of the Earth spinning on its axis.

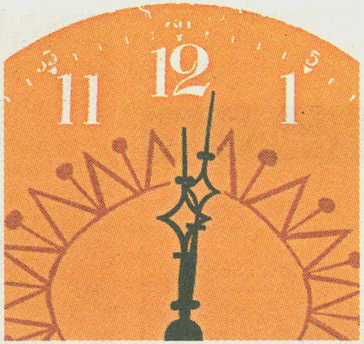
Ask most people how long it takes for the Earth to travel around the Sun and they'll probably say 365 days; in reality, it takes an average of 365.2422. Again, what you mean by this is complicated but what is important is to match up seasons. So this is the number of days it takes to match up the equinoxes. The Gregorian calendar, which tells us that tomorrow is the beginning of the new year, uses an approximation to this cycle to record the passage of time. Because 0.2422 is almost a quarter, by adding an extra day into the calendar every four years, one can keep the Gregorian calendar in synch with the passage of the Earth around the Sun. Little tweaks are required because 0.2422 isn't quite 0.25. So every 100 years you miss a leap year, but every 1,000 you skip the skipping and retain the leap year.

The Islamic calendar prefers to use the cycle of the Moon instead. Here the basic unit is a lunar month, and 12 of these make up a year. The beginning of a lunar month is determined by the sighting of the new moon at Mecca. This averages out at 29.53 days, which makes a lunar year 11 days shorter than a solar year. This is why Ramadan slips each year. Because 365 divided by 11 is approximately 33, it takes 33 years for Ramadan to cycle through the year.

The Jewish and Chinese calendars try to mix and match. They attempt to unify the cycle of the Earth around the Sun with the passage of the Moon around the Earth. They do this by adding a leap month a little bit more than every third year. And the key to their calculations is the number 19, because 19 solar years (= 19 x 365.2422 days) almost exactly matches 235 lunar months (= 235 x 29.53 days). So the Chinese calendar has seven leap years in every 19-year cycle to keep the lunar and solar calendars in synch.

So if you wake up with a cracking headache tomorrow morning, just say that it was because you spent the night trying to sort out the maths to determine exactly when you were meant to start celebrating.

MARCUS DU SAUTOY



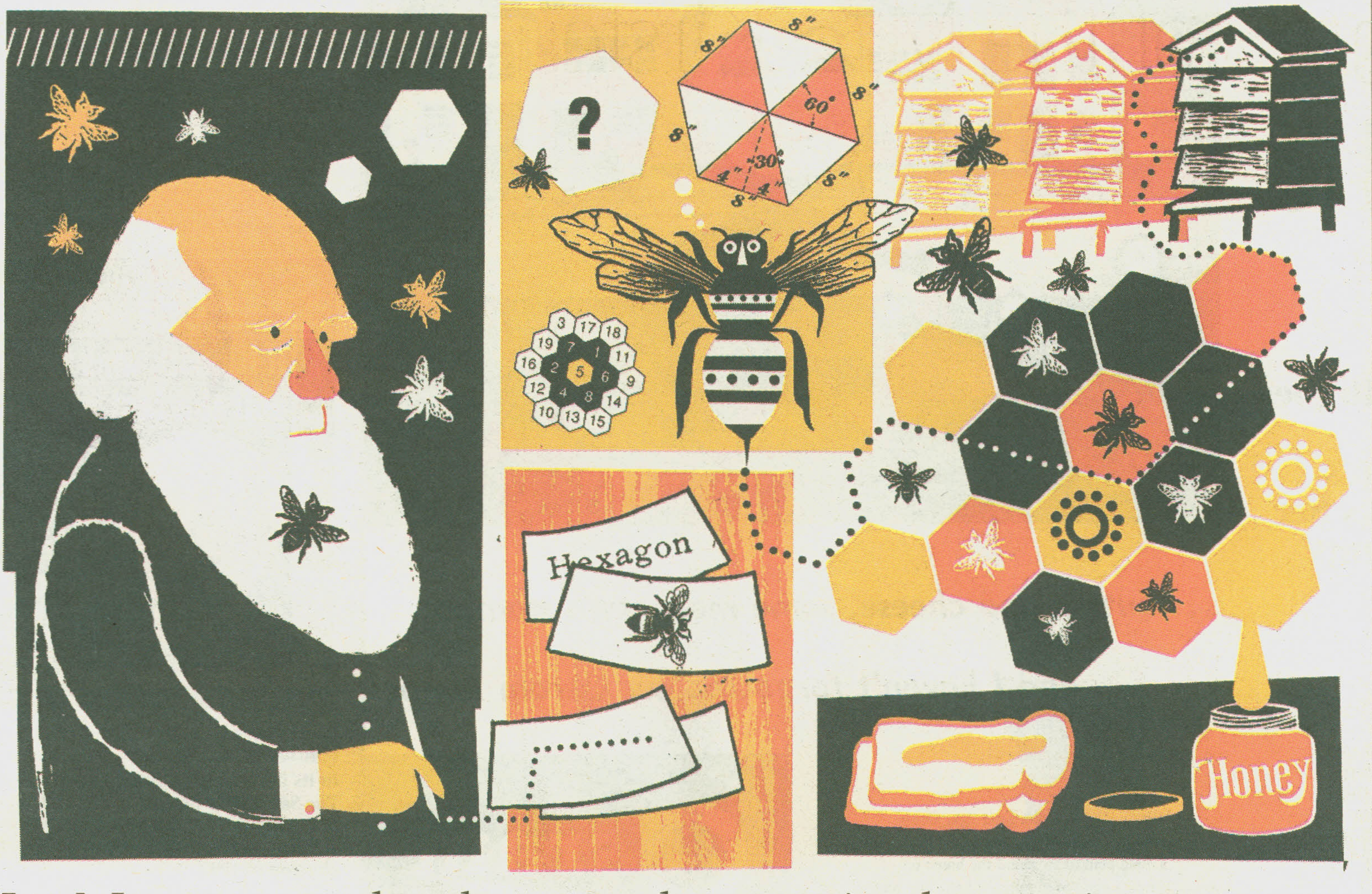
THE CONUNDRUM

From midnight yesterday to midnight tonight, how many times does the minute hand pass over the hour hand?

Answer on page 19

Marcus du Sautoy's Sexy maths

JOE McLAREN



In Nature, only the mathematical survive

In Nature, only the mathematical survive

The new year is a great excuse to celebrate some anniversaries. With the arrival of 2009 you should expect to hear a good deal of Samuel Johnson quoted as we celebrate his 300th birthday. Prepare to be scared by the macabre stories of Edgar Allan Poe, who was born 200 years ago. The airwaves and concert halls will reverberate to the strains of Mendelssohn, born in the same year as Poe, and to the baroque refrains of Handel, who died 250 years ago this year. And you may even hear the tune of *Happy Birthday* a little more often than you expect, as its composer, the American Mildred Hill, was born 150 years ago.

But undoubtedly the biggest celebrations will be among the scientific community, which is celebrating the 200th anniversary of the birth of the scientist who rocked the world with his thesis that we are descended from apes: Charles Darwin. In fact, it will be a double celebration as 2009 is also the 150th anniversary of the publication of his *On The Origin of Species*, or, if you want its full title, *On the Origin of Species by Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life*.

You may think that, as a mathematician, I will be feeling a little on the fringes of this party. After all, evolutionary biology is generally perceived as being at the other end of the scientific spectrum from the hard, abstract world of mathematics. But not at all. Darwin is one of my heroes because he provides the perfect retort to all those people who, on hearing at a party that I am a mathematician, defensively counter: "I was terrible at maths . . . just didn't have the brain for it."

Darwin's theory of natural selection suggests that the opposite is true: our brains are hard-wired to do mathematics. A concept of numbers is an essential skill in being able to negotiate the world. If your group is being attacked by a rival group, knowing whether there are more of them than of you will inform your decision on whether to fight or fly. Those who can count survive. This applies as much to animals as to human beings — and there is lots of evidence to suggest that animals, too, can count.

People and animals have evolved brains that are also good at geometry. To navigate your surroundings successfully requires an ability to judge space and distances, and some animals take their geometry very seriously. Darwin himself was deeply impressed by the engineering feat of the bee's honeycomb. He marvelled at how natural selection had led bees to achieve such a perfect balance of economy of labour and productivity of honey.

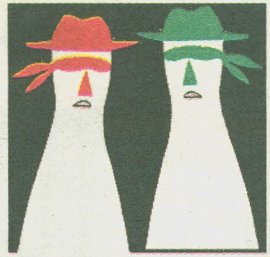
> “Those creatures that can spot a symmetrical pattern in the jungle are more likely to survive”

Although bees have long known that hexagons are the most efficient shape for building their hives, it is only recently that mathematicians have fully explained what they call the Honeycomb Conjecture: why hexagons use the least wax to create the most cells. There is an infinite choice of different layouts that the bee could have built with its wax. Only in 1999 did the mathematician Thomas Hales finally come up with a watertight proof that nothing would beat the hexagonal symmetry of the beehive.

A sensitivity to symmetry is something with which all animal brains seem to be programmed, and the explanation for this comes again from the struggle for life. Those creatures that can spot a symmetrical pattern in the chaotic tangle of the jungle are more likely to survive. Symmetry in the undergrowth is either something about to eat you, or something that you could eat.

Of course, translating this intuitive ability to do mathematics into the sophisticated subject that we study today is what marks out humans as a special species in the animal kingdom. The modern world that we take for granted is built on our ability to capitalise on the fact that we are innately mathematical. In the Preservation of Favoured Races in the Struggle for Life, it was those with the best mathematical brains that came out top. Which is why I shall be raising a glass on February 12 to celebrate Darwin's 200th anniversary.

Marcus du Sautoy will be appearing tonight on Richard Bacon's show on Radio 5 Live, starting at 11.00

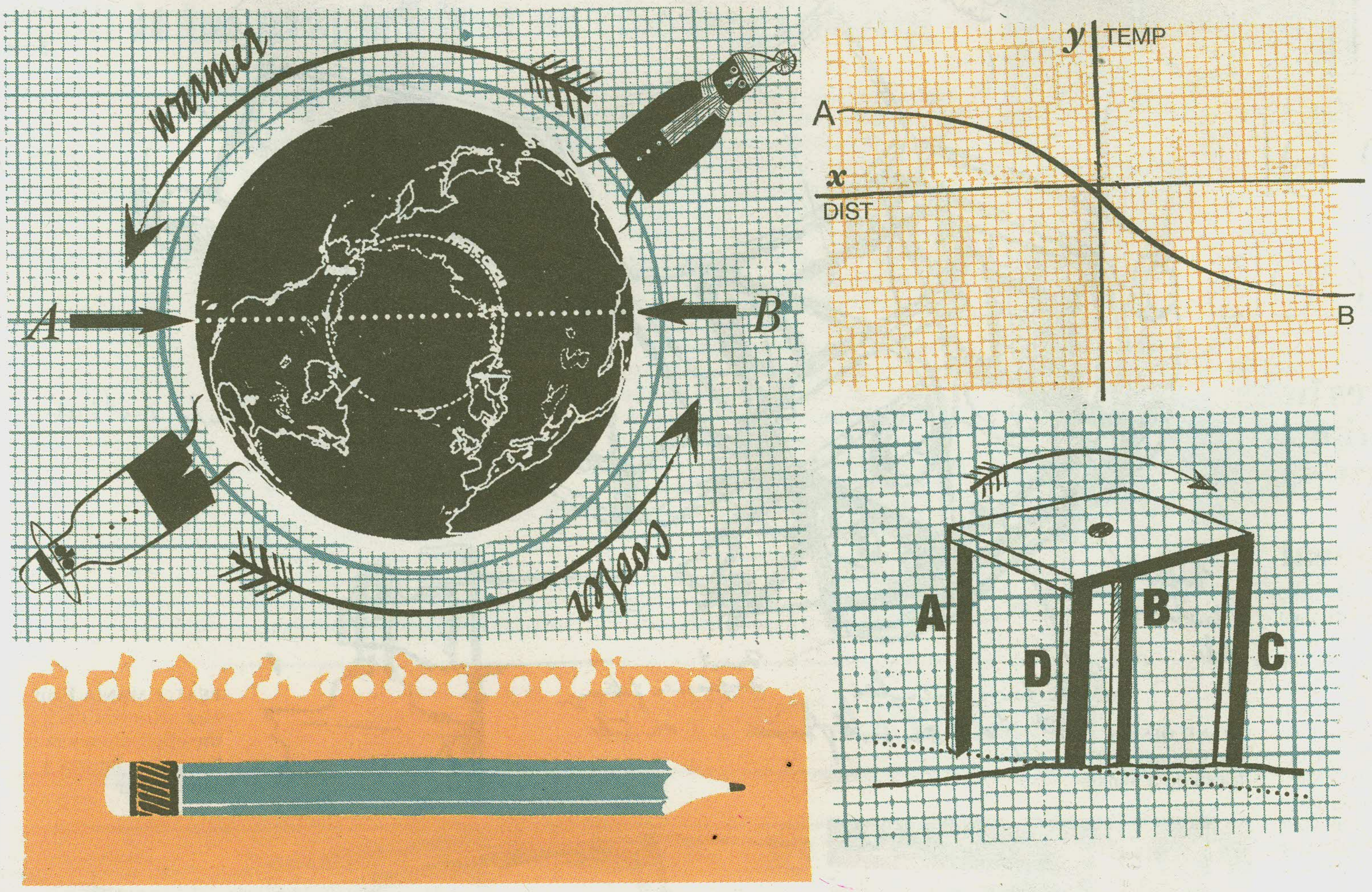


Conundrum

Two prisoners are told that they are going to be blindfolded and taken into a room where hats will be put on their heads. They are told that the hats are either red or green. The blindfolds will be removed and then each will have to write down on a piece of paper (without talking to the other) the colour of his own hat. If a prisoner gets his colour correct, he survives; otherwise he is executed. Can you propose a strategy for them that will guarantee that at least one prisoner survives?
(Answer on page 21)

Marcus du Sautoy's Sexy maths

JOE McLAREN



Eureka! No more wobbly restaurant tables

Eureka! No more wobbly restaurant tables

How many times have you sat down in a restaurant or pub and found that the table is wobbly? Wedging beer mats and bits of paper under the legs never quite seems to do the job, but mathematicians have come to the rescue with a geometric solution to this table conundrum.

I had better warn you that this week's problem requires a little more mental dexterity than usual — sexy maths isn't always easy maths. That said, the solution does use a seemingly obvious but very powerful result in mathematics called the intermediate value theorem. This states that if you have two points on either side of an infinite line then any continuous path joining the two points must cross the line at some point.

To illustrate this, suppose I pick a location on the equator and measure the temperature. Let's call the temperature A. You are stationed exactly on the other side of the globe. You measure the temperature at your location, which we will call B. Unless we were very lucky, these temperatures will be different. Calculate A minus B.

Now plot a graph of how this difference between the temperatures changes as we move round the equator. Along the horizontal axis of the graph is the distance round the equator and the height of the graph is the difference in temperatures. Let's suppose that A is hotter than B so the graph starts above the horizontal axis. By the time I go halfway round the equator I'm at the position at which you started. Similarly, you will have arrived at my starting location. So my thermometer gives a temperature of B and yours has temperature A. The difference in temperatures is B minus A, which is a negative number because it was hotter where I started. By the time we've walked halfway round the equator the graph that records the difference in temperatures is below the horizontal axis because the height of the graph is negative.

But the difference in temperatures will be a continuous graph because the temperatures vary as we walk. The graph starts above the horizontal axis and finishes below it. Now apply the intermediate value theorem to say that at some point on our journey round the globe the graph must cross the horizontal axis as it goes from positive to negative. But this means that, at that point, the difference between our temperatures is exactly 0. In other words, there is a place on our journey where our thermometers must show the same temperatures.

> "Wedging beer mats or bits of paper under the legs never seems to work, but maths can come to the rescue"

How does this help us with our wobbly table? Let's suppose it's the pub floor that is uneven and the legs are all the same length. We'll also suppose that the table is a perfect square. Start by placing the table so that three legs are on the ground. This is always possible. Two must always touch and then just lean the table over to one side until the third leg touches. Let's call the leg in the air A and, going clockwise, label the others B, C and D.

Now, forget leg A for a moment. Rotate the table clockwise around the axis through the middle of the table (especially easy if it's a table with an umbrella going through the middle). If leg A wasn't there you can do this while still keeping the three legs B, C and D in contact with the floor. But after a quarter of a turn because of the symmetry of the square, the table looks like it did when it started, except that leg D, which has moved to the position of leg A, is on the ground. So if we tried to put leg A back, it would be digging into the floor in the position where leg B started. In fact the table would be in the same position as if, instead of rotating the table, we'd simply pushed the table down on one side pushing leg B into the ground until leg A touched the floor.

But now comes the clever bit. At some point as you spin the table with leg A now in place, there must be one point where it changes from hovering in the air to digging into the ground. It's at this point that all four legs will be perfectly aligned to give you a stable table.

So, the next time you go to the pub, take a mathematician. Sometimes it's useful to have a friend who knows their tables. (Incidentally, a stable table isn't necessarily a horizontal one; times2 accepts no liability for spilt beer!)

Conundrum

A traveller set out on a journey and stopped only when he had returned to his starting point. During that journey, his head travelled 11 metres farther than his feet, yet his feet remained attached to his body. What is the explanation for this?

Answer: The journey was round the globe. The man's head was 1.75m, say, from the ground, so the radius of the circle travelled by his head was 1.75m greater than the circle travelled by his feet. As we all know, the circumference of a circle is $2\pi r$ (r being the radius). The difference in the circumferences of the circles is $2\pi \times 1.75$m, which is approximately 11m.

Marcus du Sautoy's Sexy maths

JOE McLAREN

It's true, young Muggles. Maths can be magic

It's true, young Muggles. Maths can be magic

Parents across the world are on the lookout for a secret formula to unlock their children's mathematical skills. For some, Kumon is the answer. More than four million children in 43 countries are signed up to the method developed by the Japanese educator Toru Kumon, who believed that repetition and speed are the basis of making a good mathematics student.

Others think that the solution lies elsewhere. As reported in the education pages of *The Times* last week, parents in France have been flocking to Stella Baruk — the "J. K. Rowling of figures" — in the hope that she can perform the same magic for mathematics that the Harry Potter stories did for children's literacy.

Kumon's philosophy is in tune with that of the music teacher who insists that a pupil must master scales and arpeggios before moving on to anything more interesting. Yet although it is essential to have the technical skills at your fingertips, if pupils are left with the impression that this is what maths is about, I fear that Kumon will only accentuate a student's boredom with mathematics and antagonism to the subject. Many have criticised Kumon as "drill and kill".

Baruk believes instead that language and understanding are the magic ingredients. Born in Iran, home of the great poet and mathematician Omar Khayyám, Baruk focuses on teaching maths as a living language with meaning. She bemoans the response that eight and nine-year-olds in France gave to this question: "On a boat there are 26 sheep and ten goats. How old is the captain?"

Of the 97 children who were asked, 76 responded by using the numbers contained in the statement — giving the captain's age as 26 or 10 or maybe 36.

Maybe it's all those posters pinned on young children's bedroom doors that cause the problem. Seeing pictures of two apples and three dogs followed by five medicine bottles can be confusing for a child trying to make sense of how you manage to get cough mixture by combining fruit with pets.

Baruk is keen to get children looking beyond the objects, to understand the abstract nature of numbers. But she is not adverse to using a little sorcery in the shape of such mathematical curiosities as magic squares — and it is bringing alive the magic and playfulness of mathematics that, for me, is key for anyone hoping to spark a child's interest in the subject.

I witnessed the power of a simple bit of algebra to capture people's imagination last year at the Barbican in Complicité's hit play *A Disappearing Number*. At the start of the show, an actor asks members of the audience to think of a number. "Now double your number. Add 14 to the new number. Divide this number by 2. Finally, subtract the first number you thought of." I was staggered at the gasp of surprise as the actor revealed that everyone was thinking of the number seven.

> "If I taught at Hogwarts, I'd be keen to impress Hermione Granger with maths spells: 'Primenumeros incantato!'"

Although the maths behind this trick is trivial, at its heart is the essence of what makes the subject so magical. It is the power of algebra that reveals why this trick works whatever the numbers.

There are similar tricks which are far more surprising — and which produced a gasp from me when I first encountered them. For example, the French mathematician Fermat discovered this bit of magic:

Think of a number and take a prime number bigger than the number you first thought of. Now raise your first number to the power of that prime number, then divide the resulting number by the prime. The remainder is the number you started with. For example, if you were thinking of 2 and you raise it to the power of the prime number 5, the answer is $2^5=32$. Divide by the prime 5 and the remainder is 2. This is far from just a mathematical curiosity. It is actually the basis of the mathematics at the heart of internet cryptography.

If I were asked to teach arithmancy at Hogwarts, I'd be keen to impress the likes of Hermione Granger with maths spells: "*Primenumeros incantato!*"

When you see a magician do a trick, the magic often vanishes if he tells you how he did it. For me, this is where maths and magic differ. Understanding why Fermat's trick always works only enhances the magic — and it's such magic that will capture children's imagination.

Conundrum

Think of a three-digit number where the first and last digit differ by 2 or more. Reverse the digits to make a new number, then subtract the smaller of the two numbers from the larger one. Take this new number. Reverse the digits of this new number and add these two new numbers together. Using my magic mathematical power, I can reveal that your answer is 1,089. But how did I know? See page 21 for the explanation.

Marcus du Sautoy's Sexy maths

JOE McLAREN

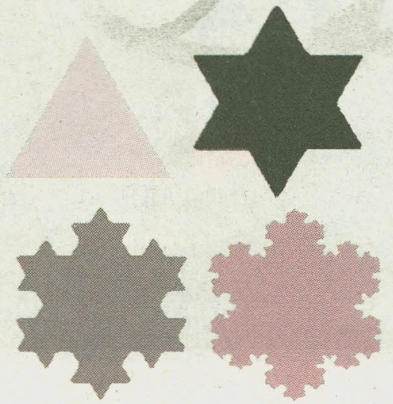
Searching for the snowflake solution

Searching for the snowflake solution

While my kids have been throwing snowballs at each other and sliding down hills in plastic bags, I've been trying to make up for their missing school by getting them interested in a question that has vexed generations of scientists. Why do snowflakes have six arms?

The great 17th-century astronomer Johannes Kepler wrote a whole treatise on the question. He dedicated *On the Six-Cornered Snowflake* to the Imperial Treasurer as a new year's present, a canny move to secure funding for future scientific ventures. To find an explanation for why snowflakes don't have seven or five arms Kepler looked to other structures in nature that like sixfold symmetry. The beehive was an obvious candidate. Hexagons are an efficient way of packing shapes together. Kepler proposed that maybe this was the reason that snow liked the number six. He believed that snowflakes packed together like a beehive in the clouds and then would break apart as they fell to earth. But why didn't they like packing in squares or triangles, two equally good alternatives?

Ultimately Kepler's theory was incomplete. The true explanation for the snowflake's choice of six had to wait for the development of X-ray crystallography. Only then could scientists stare deep inside the snowflake and unmask its secrets. Everyone knows that water is made up of one oxygen atom and two hydrogen atoms. When the water molecules bind together to form crystals, the oxygen atom shares its hydrogen atoms with neighbouring oxygen atoms and in turn borrows two other hydrogen atoms from other water molecules. The crystal, therefore, is constructed so that each oxygen atom is connected to four hydrogen atoms.



Conundrum

To make your own fractal snowflake, start with a triangle, each of whose sides is 1cm long. Add three smaller triangles a third the size of the original triangle on to all the straight edges of the first triangle. Repeat this process adding ever-smaller triangles (a third the size of the previous triangles added) on to all the straight edges of the previous shape. If you repeat this an infinite number of times you create a fractal called the Koch snowflake. The perimeter of the snowflake will be infinite, as length increases by a factor of 4/3 each time you add a new set of triangles. So why is the area finite?

Chemists like to build models of atoms using coloured ping-pong balls joined by toothpicks. The four ping-pong balls representing hydrogen atoms arrange themselves around the central oxygen atom in the shape that ensures that each hydrogen atom is as far away from the three other hydrogen atoms as possible.

The mathematical solution to such a desire is to position each hydrogen atom at the corner of a tetrahedron, the platonic shape made up of four equilateral triangles, with the oxygen atom at its centre joined by toothpicks to the hydrogen atoms.

The crystal structure that emerges has something in common with oranges being stacked at the grocer where three oranges on one layer have a fourth orange placed on top to make a tetrahedron. But look instead at each layer of oranges and you see hexagons everywhere. It's these hexagons that appear in the crystals of ice that are key to the snowflake's shape. That this molecular structure based on the hexagon translates itself to large-scale flakes with six arms is still quite surprising. It seems that as the snowflake grows, the water molecules attach themselves to the six points of the hexagon.

It is this passage from the molecular to the large snowflake where the individuality of each snowflake asserts itself. And while symmetry is at the heart of the birth of the water crystal, it is another important mathematical shape that controls the evolution of each flake: the fractal. This geometry is characterised by having infinite complexity at all scales. Zoom in on a section of a fractal and it is impossible to judge what scale you are looking at the object. It never seems to smooth out.

The shape of the fractal is found all over Nature. Ferns, the human lung, lightning and the coastline of Britain all show this complex structure. The snowflake, like the fern, has the property that as one magnifies the shape it looks like the shape before one zoomed in on the arms.

So although your kids may be off school, don't despair: there is still some great maths you can pack into those snowballs flying through the air.

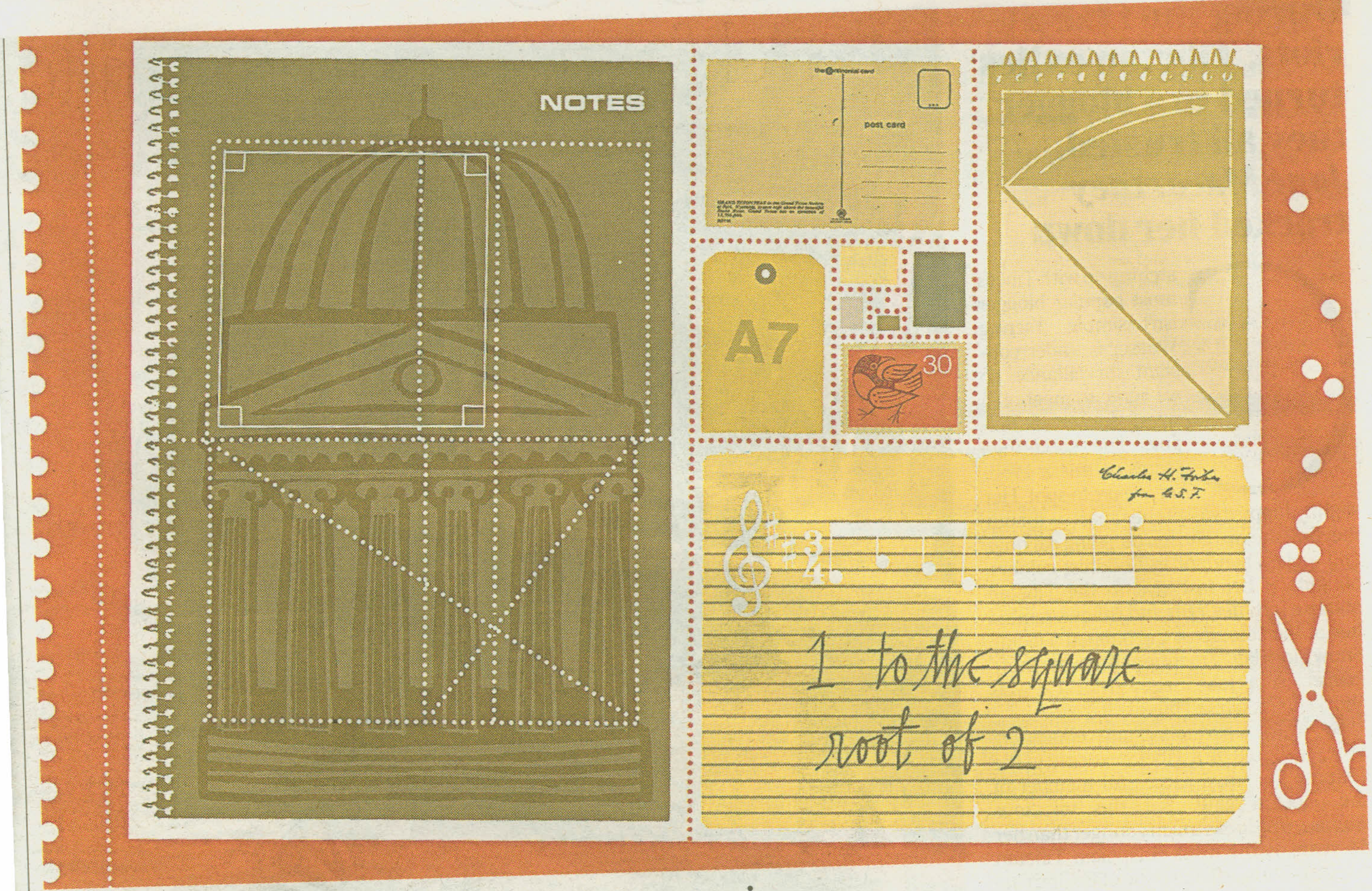
> "The crystal structure that emerges has something in common with oranges being stacked at the grocer"

Answer:
Although each stage adds more area, the snowflake will never extend beyond a triangle whose sides are 2cm long. So it is finite.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN



Why Palladio's proportions are pleasing on the eye – and the ears

Why Palladio's proportions are pleasing on the eye – and the ears

What's so special about the dimensions of an A4 piece of paper? A lot, according to the 16th-century architect Palladio. He believed that the proportions were one of the fundamental ratios that every architect should employ in his or her designs. So influential was his style that architects have copied his blueprints ever since. London is full of buildings that capture Palladio's perfect proportions, from those designed by Inigo Jones, such as the Banqueting House in Whitehall, through to the villa in Regent's Park built in 2004 by Quinlan Terry.

The proportions of an A4 piece of paper are so appealing because, if you halve the paper, going from A4 to A5, the resulting proportions are precisely the same. No other proportion has this property. Palladio exploited this in his villas. He knew that if he built a room in the proportions of an A4 piece of paper and then cut the room in half, a second room built to these new dimensions would be in perfect harmony with the first.

In addition to the A4 ratio, Palladio's buildings are full of squares, cubes and rectangles, where the sides are in perfect, whole-number ratios to each other. Many refer to his style as frozen music because it captures precisely the proportions that Pythagoras recognised as producing combinations of notes that sounded harmonious to the ear.

> Legend has it that the Pythagoreans drowned the person who had made the geometric discovery

For example, take a rectangle where the long side is twice the length of the short side. If you cut pieces of string to the same lengths as the two dimensions of the shape — and hold them under the same tension — then the notes made when they are plucked will be a perfect octave apart. Do the same on a rectangle whose sides have a 2:3 ratio and you will hear a perfect fifth.

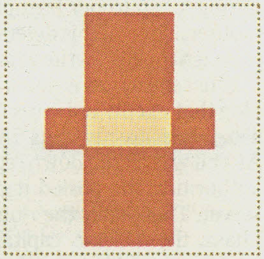
The architect Zaha Hadid, who trained as a mathematician in Iraq, captured this connection in the piece she built for the 2008 Venice Biennale. She used the shapes of the waves produced by these harmonic notes as the inspiration for the beautiful, curvaceous, 3-D structure that she built inside the rooms of Palladio's Villa Foscari last autumn.

But the A4 proportions that Palladio was so taken with were not so popular with the Pythagoreans and gave them a lot of grief. One way to build a room in these proportions is to start with a perfect square. Then draw in the diagonal line from one corner of the square to the opposite corner. Now move this line round to make the side of a new wall. A room constructed with this as the long wall and the side of the original square as the short one will be in the proportions of an A4 sheet of paper. Using Pythagoras's theorem, you can calculate that the proportions of this room are 1 to the square root of 2.

The thing that so shocked the Pythagoreans was the discovery that these proportions cannot be expressed as whole-number ratios. The square root of 2 is a number that cannot be written as a fraction. Its infinite decimal expansion never repeats itself. In music, two notes corresponding to these lengths produce a rather challenging interval called an augmented fourth. Yet in architecture, because of its close relationship to the perfect square, it worked.

Legend has it that the Pythagoreans were so startled by the revelation that simple geometry gave rise to such disharmony that they tried to suppress the news and even drowned the person who had made the discovery. But today every respected architecture firm is full of mathematically literate architects who, as well as knowing their Ps from their Qs, know their threes from their square root of twos.

So what would Palladio or Pythagoras have made of the proportions of *The Times*? The compact format is based on a rectangle whose proportions correspond to musical notes that are positively discordant to the ear. I'm afraid that it wouldn't have made the cut either in ancient Greece or in 16th-century Venice. But who said newspapers had to be pretty?



Conundrum

In a Palladian villa the owner is having a party and has placed a rectangular dance floor in the main room. He wants to put square carpets down adjacent to each side of the dance floor, making four squares in all. To determine the area of carpet he would need in total, what is the smallest number of measurements he could take?

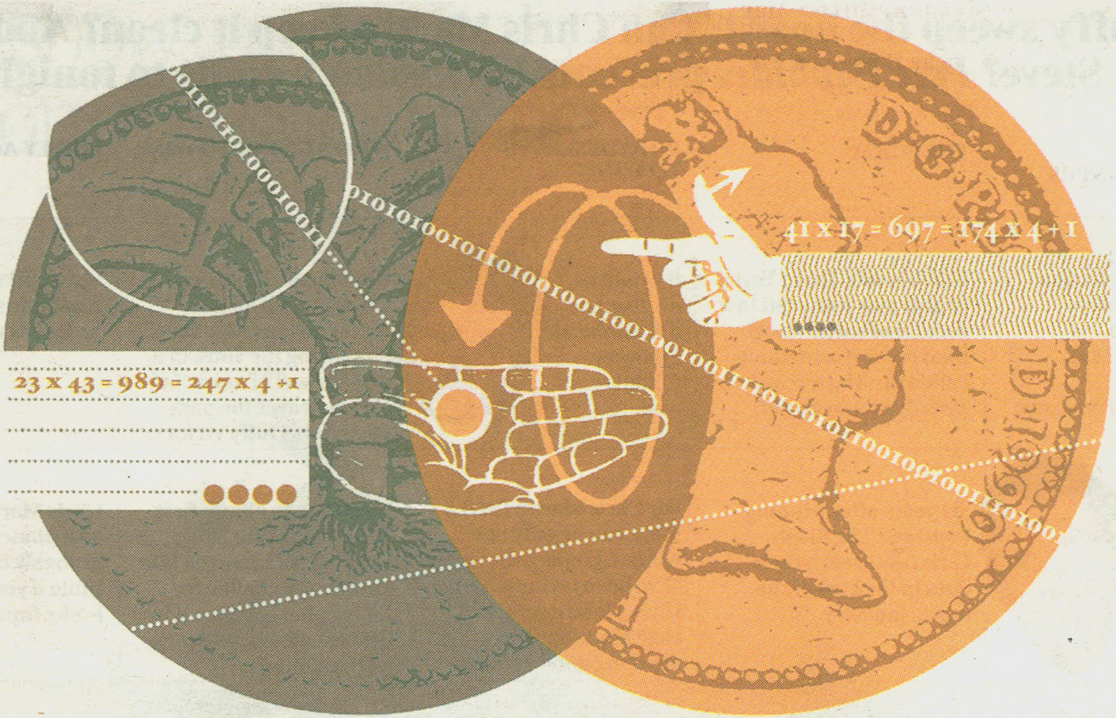
Answer:

One. The distance between the opposite corners of the rectangular dance floor. Pythagoras's theorem states that the sum of the area of the two squares adjacent to the short sides of a right-angled triangle is equal to the area of the square adjoining the hypotenuse. So the area of carpet the owner needs is double the area of the square on the diagonal line from one corner of the rectangle to the opposite corner.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN



Winning at heads and tails

Winning at heads and tails

Heads or tails? If you were standing next to me it would be easy for you to check that I wasn't cheating and to see how the coin I've just tossed landed. But you're sitting somewhere miles away from me, eating your toast over breakfast or travelling to work on the train. You've never met me. How can you trust me if, after you call heads, I say: "Sorry, it landed tails"?

Is there any way to toss a coin when we are miles apart and we've never met each other and avoid the possibility of my cheating once you've made your call? Such issues of trust are especially important in our electronic age, when so many of us are hoping to do business over the internet with people we've never met.

So, can you toss a coin across the internet fairly? Amazingly you can, and it's the mathematics of prime numbers that makes it possible. Primes, the indivisible numbers, fall into two piles. If I divide a prime number by 4 it obviously doesn't divide exactly; I either get a remainder of 1 or remainder of 3. For example, 17 on division by 4 leaves a remainder of 1, while 23 on division by 4 has a remainder of 3. (There is one exception to this, namely the prime 2. Mathematicians call 2 an odd prime — the kind of joke we find amusing.)

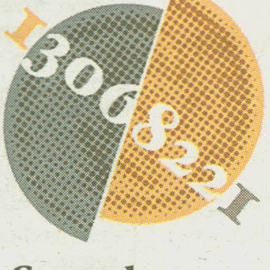
One of the great successes of the 19th century was to prove that primes are evenly split in terms of remainders. Look at the primes below any particular number and approximately half of them will have a remainder of 1 on division by 4 and the other half a remainder of 3. So whether a prime leaves a remainder of 1 or 3 on division by 4 is no less biased than whether a fair coin lands heads or tails. So let's associate heads with the primes with a remainder of 1 on division by 4 and tails with primes that have a remainder of 3 on division by 4.

Now here comes the clever bit. If I take two prime numbers, such as 17 and 41, from the heads pile (the ones that have a remainder of 1 on division by 4) and multiply them together, the answer also has a remainder of 1 on division by 4. For example: 41 x 17 = 697 = 174 x 4 + 1. But if I take two primes, such as 23 and 43, from the tails pile (the ones that have a remainder of 3 on division by 4), a curious thing happens when I multiply them together. They also give a number that has a remainder of 1 on division by 4. In this case: 23 x 43 = 989 = 247 x 4 +1. So the product of the primes gives no hint as to whether they are from the heads or tails pile. It's this that I can exploit to play internet "heads or tails".

If I toss a coin and it lands heads, I choose two primes in the heads pile and multiply them together. If it lands tails I choose two primes in the tails pile and multiply them together. So I've tossed my coin, done my calculation and now I send you the answer, which in this case is 6,497. Because the answer will always have remainder 1 on division by 4 it is impossible for you to tell without knowing the primes whether the two primes I chose were in the heads or tails pile. So now you are in a position to call heads or tails. Made your choice?

To see if you won I just need to send you the two primes I chose. In this case it was 89 and 73, two primes in the heads pile. No other primes will multiply together to give you 6,497. So I have given you enough information with the number 6,497 to make sure that I don't cheat, but not enough information that you can cheat. Actually, there is a way you can cheat. If you can crack 6,497 into 89 x 73 you know to call heads. But provided I choose the big enough primes it is almost impossible with current computing power to crack large numbers into primes. A similar principle is used in the codes that protect credit cards sent across the internet.

Of course, if you have an infinitely powerful computer you'll be able to hack my game of "heads or tails". So how can you play against someone with unlimited computing power? It's not primes this time that come to the rescue but the mysterious world of quantum physics. But an explanation of how polarised light can guarantee you a fair toss of a coin will have to wait until *The Times* commissions a Foxy Physics column alongside its Sexy Maths spot.



Conundrum

I've tossed a coin. I've taken two primes from the heads or tails pile and multiplied them together to give me the number 13,068,221. Did the coin land heads or tails? Try answering this with a calculator, but without using a computer.

Answer

It landed heads. 13,068,221 = 3,613 x 3,617. Both 3,613 and 3,617 are primes that have remainder 1 on division by 4. There is a way to factorise this particular number quickly using a method discovered by Fermat. The trick is to try to capture the number as the difference of two square numbers. If you square 3615 you get 13,068,225, which is 4 away from 13,068,221. 4 is also a square. Now use a bit of algebra that tells you how to factorise the difference of two squares: $A^2 - B^2 = (A + B) \times (A - B)$. So $13{,}068{,}221 = 3615^2 - 2^2 = (3615 + 2) \times (3615 - 2) = 3613 \times 3617$.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAR

Tails of a submariner

Tails of a submariner

I have just received an e-mail from an American sailor about last week's column, in which I explained how you could use prime numbers to play heads or tails on the internet. Given that the Marine was deployed leagues under the sea on a submarine and I was sitting above ground, I thought that he might be challenging me to a game. But not at all. It was a curious property of the primes I was using that intrigued him.

The way the game works is to divide prime numbers into "heads" primes and "tails" primes. The "heads" primes are those that have a remainder of 1 when divided by 4. The "tails" primes are the ones that have remainder 3 on division by 4. Being in a submarine gives you lots of time to think and he had turned his attention to whether my prime number coin might be biased.

He'd decided to count how many "heads" primes and "tails" primes there were. For example, up to 10 there are two tails primes (3 and 7) and one heads prime (5). So more tails than heads. Up to 20 there are two more tails primes (11 and 19) and two more heads primes (13 and 17). So tails are still in the lead. As you keep counting, something rather strange seems to happen. Tails always seems to be in the lead. My US Marine is not the first to notice this apparent bias. It was first spotted by Pafnuty Chebyshev, a 19th-century Russian mathematician, and is called the Chebyshev bias.

So do tails always stay in the lead? Well, no. Eventually the heads primes do pull it back — the first time they take the lead is when you get to the prime 26,861. But they immediately lose it again since the heads prime 26,861 is followed by a tails prime 26,863. The next time heads takes the lead isn't until you hit the prime 616,841, but again the lead is immediately snatched back by tails. Although heads seems to do badly, John Littlewood, a British mathematician, proved in 1914 that heads will take the lead an infinite number of times, despite never holding the lead for very long.

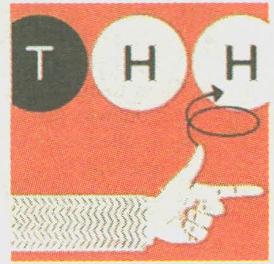
A similarly unexpected phenomenon occurs when you toss a real coin. Most people's intuition would tell them that, with a fair coin, heads and tails should share the lead equally often. But this turns out to be wrong. It is likely that heads or tails will be in the lead for most of the time. But the bias in the prime number coin is even more significant than the apparent bias in a real coin and it is only in the past few decades that mathematicians have truly understood the bias.

Two mathematicians, Michael Rubinstein and Peter Sarnak, proved that my US Marine and Chebyshev were right. They found that, measured in the right way, the tails primes lead the heads primes more than 99 per cent of the time. This is different from a fair coin. With this, although heads or tails will often hold a big lead over a long stretch of tosses, in time these leads will get evened out.

But the Chebyshev bias does not contradict the fact that you can use the primes to play a fair game of coin tossing. If you continue to count the primes, then you will find that the number of heads gets closer and closer to 50 per cent. In fact, the difference between them remains so small as to become negligible, so it won't affect the fairness of the prime number coin.

I'm not sure how long my US Marine's tour of duty is but, if it's long enough, there are lots of other prime number challenges out there that we haven't sorted out. In fact, Rubinstein and Sarnak's analysis works only if one of the big conjectures about prime numbers is true. It's called the Riemann hypothesis, and there is a million-dollar prize for the first person to crack that prime number mystery. So I'll be checking my e-mail over the coming months to see what further inspiration 20,000 leagues under the sea might bring.

Marcus du Sautoy will be opening the Oxfordshire Science Festival on Saturday in Oxford. The festival runs until March 15



Conundrum

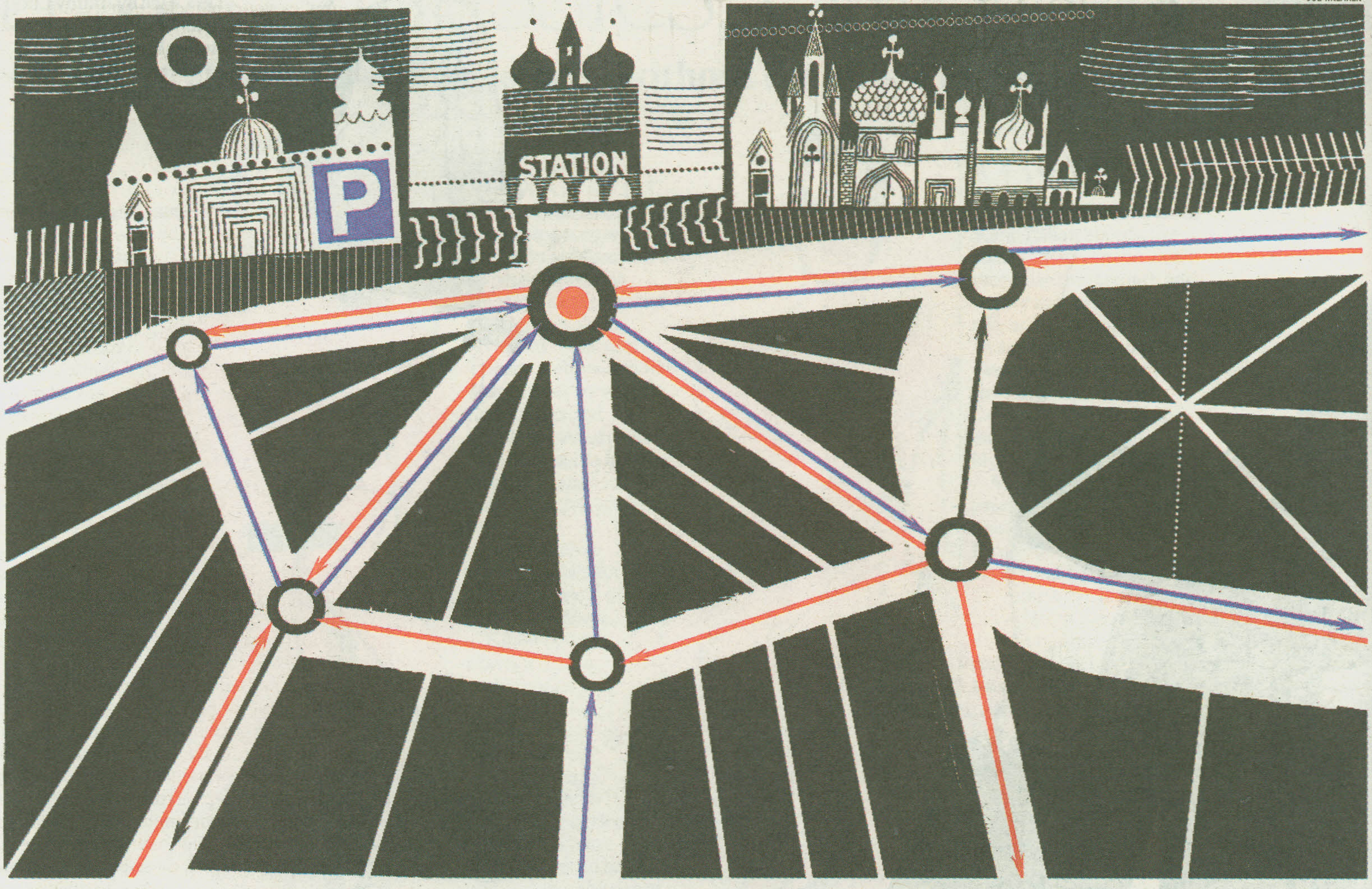
Let's play a game. I'll toss a coin. If the first three tosses come up heads, heads, tails, I'll pay you £20. However, if it's tails, heads, heads, then you pay me £10. Should you play?

Answer:

No. It is three times more likely that tails, heads, heads appears. There are four possibilities for the opening two tosses: heads, heads; heads, tails; tails, heads; tails, tails. In the case of heads, heads, I can't beat you. You just have to wait for a tail to appear. However, in the other three cases you can't beat me. The first occurrence of heads, heads must be preceded by a tail, giving me the win.

Marcus du Sautoy's Sexy maths

JOE McLAREN



You'll never be lost if you follow the colour code

You'll never be lost if you follow the colour code

I recently visited a friend in a neighbouring city, but wasn't sure where he lived so asked him for directions. He asked where I would be entering the city. I wrote back to say that I wasn't sure, as I didn't know the city at all. His reply quite shocked me. Instead of a lot of different directions that depended on where I was coming from, he just sent me a map of the city and one set of directions. "Don't worry," he wrote. "If you follow these instructions, wherever you are in the city you'll eventually arrive at my house."

I couldn't quite believe that one set of instructions was going to be enough. I could be anywhere in the city. How would taking the second right followed by the first left work wherever I was? But when I opened up the package with the map and instructions, I realised that these weren't the usual directions that you hear the lady on the GPS blurting out. My friend, after all, is a mathematician.

Mathematicians like to throw away unnecessary information and get to the bare bones of a problem. The city consists of lots of roads meeting at various junctions. The distances between each of the junctions are unimportant. Once I have started driving down a road, I will end up at the next junction and need a new instruction. So the map just consisted of lots of dots, the junctions, and lines emanating from these dots, representing the roads meeting at this junction.

In fact, anyone travelling around London or any other major city with an underground system will be quite used to this sort of map. A physical map showing the geographic locations and routes on the London Underground is not a very helpful picture for negotiating your way around the city. Instead Harry Beck's iconic map of the London Underground, which he developed in 1933, isolates the way in which the network is connected, while ignoring physical dimensions. The fact that the same length of line is used to represent the connection between Covent Garden and Leicester Square as that between Finsbury Park and Seven Sisters does not mean that the actual distances between them are the same. For a commuter, knowing there is such a connection is much more important than knowing the distance between stations.

This is an example of a new way of looking at the world that was introduced in 20th-century mathematics. The word geometry comes from the Greek for measuring the Earth. But often the exact distances between objects are not important. It is how they are connected that is key to the identity of the shape. This new way of looking at the world is called topology.

> "Amazingly, wherever I start in the city, if I obey these instructions the journey will end outside my friend's house"

Each road in the map that I had was represented by two lines with arrows on: one line represented the side of the road on which traffic was oncoming, the other line was the lane going in the opposite direction. If the road was one-way, there was only one line with an arrow indicating which direction to travel.

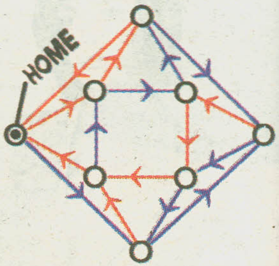
Then came the really clever bit. My friend had indicated each of these lines in different colours. The instructions consisted of the following directions: first take a blue road; at the next junction take a red road; at the next junction take a blue road. Remember, I can go up a road only if the direction allows me. Amazingly, wherever I start in the city, if I obey these instructions the journey will end outside my friend's house.

Whether such instructions were possible for certain types of maps was a subject of conjecture until last year, when a 63-year-old former security guard in Israel came up with an elegant proof. Avraham Trakhtman gives the lie to the belief that maths is only for the young.

Of course, not only cities contain networks of roads. The internet too can be considered a huge network of interconnected hubs and links. Just imagine an e-mail that has been lost in the system and can't find its recipient. Thanks to Trakhtman's theorem, wherever the e-mail is in the system, one set of instructions could steer it through the network so that it would arrive at its intended destination.

So you stick with your GPS or your A-Z if you want to. Me, I'm getting out my coloured pens.

Marcus du Sautoy is appearing with Sir Peter Cook at the Barbican, London, tomorrow at 7pm. They will talk about maths and architecture.



Conundrum

Can you come up with a set of instructions that will always get you home if you start from any circle in the diagram above?
Answer on page 21

Pat yourself on the back if you spotted the misprint in last week's conundrum. It should have read as follows:

Conundrum: Let's play a game. I'll keep tossing a coin. As soon as the sequence Heads Heads Tails appears, I'll pay you £20, unless Tails Heads Heads appears first, in which case you pay me £10. Should you play?
Answer on page 21

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN

A number-munching celebration

A number-munching celebration

When Homer Simpson's superhero alter-ego the Pie Man first threw a fruit-filled pastry in anger a few years ago, Springfield's local news anchor was moved to observe: "Pie! Tasty dessert, tricky math thing, and now sword of righteousness."

He wasn't the first to make the connection between sticky puddings and the most intriguing number in maths — since 1988 mathematicians across the land have been celebrating pi day each year by tucking into a feast of them. The number has obsessed generations of mathematicians for millennia, and not because it's an excuse to eat pudding.

Pi is related to one of the most important geometric objects in nature: the circle. The number defines the ratio of the circumference of a circle to its diameter. So if a circle is one metre across, the number of metres it takes to go round the outside of the circle is 3.14159... and then the numbers spiral off to infinity in a dizzying dance of digits. And it's the digits of pi that define the date for the celebration of pi day. The first digit gives you the month: 3 for March. The next two give you the date: the 14th. The next three give you the time that celebrations kick off, hence 1.59. Most mathematicians interpret the time as pm rather than am (the afternoon is a better time for pie).

Calculating an exact value has obsessed mathematicians since ancient times. The Rhind Papyrus, written by the Egyptian scribe Ahmes in about 1650BC, approximates pi as 256/81 or roughly 3.16. Not bad for a first estimate.

As mathematics developed, so more cultures had a go at trying to capture this important number. The Ancient Greek mathematician Archimedes used a 96-sided shape to estimate that pi lay between 223/71 and 22/7. This is where we get the approximation that most engineers use for pi of 22/7. In fact, engineers celebrate pi day on July 22, but then they never did care quite so much about precision as mathematicians.

It was in Kerala in the south of India in the 15th century that a mathematician called Madhava came up with an exact formula for pi. By successively adding and subtracting different fractions Madhava discovered that he could capture pi precisely. If you start with the number 4 and then take away four thirds then add four fifths then take away four sevenths then add four ninths and you kept on doing this, each time alternating between 4 divided by the next odd number, when you've done this infinitely many times you'll hit pi exactly. The formula, although beautiful, is not very practical. You have to add up a lot of fractions before it starts honing in on pi.

> "Pi has obsessed generations of mathematicians for millennia, and not because it's an excuse to eat pudding"

A curious way to calculate pi was discovered in 1777 by the Compte de Buffon, a French naturalist. If you take a needle and a page of lined paper, on which the lines are separated by the same distance as the length of the needle, then if you continually throw the needle on to the page, the proportion of times the needle will cross a line when it lands is 2 divided by pi. This gives you a physical way of calculating pi. Keep chucking down the needle and the more you toss the better your estimate for pi should be.

Mathematical methods and computing power mean that we know pi to a staggering trillion digits. Of course no one needs to know so many values for the practical purpose of calculating circumferences. You need to know only 39 digits to calculate the circumference of a circle the size of the observable universe to the precision comparable to the size of a hydrogen atom.

The use of decimal numbers is anyway a very human construct, dependent on the fact that we have ten fingers. If we had evolved with a different anatomy, say with eight fingers like Homer Simpson, pi would still be the same expression of the unchanging ratio between the circumference and diameter of a circle but using powers of eight rather than powers of ten as our natural base would mean pi began 3.110375...

So while we'll have to wait till Saturday for a mathematical excuse to tuck into tarts, the Pie Man should be celebrating today. But even if he's metaphorically number-munching on a different day, as mathematicians we'll all be agreeing with him when his eyes glaze over and he sighs, "Mmm ... pi ...."

Marcus du Sautoy presents *Sunday Feature* on Radio 3 at 9.30pm on Sunday, on the art and science of the Baroque

Conundrum

A duck sits in the middle of a perfectly circular pond, the radius of which is 40 metres. On the edge of the pond is a hungry fox that can't swim. The fox can run four times faster than the duck can swim. Will the fox always be able to catch the duck the moment it reaches the shore?

Answer: Not if the duck swims just short of 10m from the centre of the pond, and then swims round a circle concentric with the edge of the pond. As its circumference is just less than a quarter of the circumference of the pond, the duck can swim round its circle faster than the fox runs round the edge. The duck then keeps swimming until it is diametrically opposite to the fox, before heading to shore. It must cover just over 30m. The fox in that time can run just over 120m. But to reach the duck it must run a distance of pi x 40m — approximately 125m. So the duck (just) makes it first. Of course, it could have flown.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN

In search of the poetry of symmetry

In search of the poetry of symmetry

With school half-term fast approaching my kids are bracing themselves. Unlike most children, they get a little nervous about an impending holiday. With a mathematician for a dad they often find themselves embarking on some nerdy, maths-inspired adventure.

It all started one year when we were on holiday in Andalusia. My family were happy to join the tourists who flock to the Alhambra in Granada, Spain. For me, the Moorish palace is one of the meccas of mathematics. If I had to choose one building to which to be banished, as a man obsessed with symmetry, I would choose the Alhambra. What gets my blood racing are the symmetrical tiles that cover every available surface of the palace.

Muslim texts forbid the depiction of things with souls. So the Moorish artists found more geometric ways to express themselves. Armed with a sophisticated mathematical intuition, they began to cover the ceilings, walls, floors and gardens of their palaces with tiles of different geometric shapes and colours.

Each wall was like a new canvas, challenging the artist to create a different, original way of covering the façade. The tiles repeat themselves left and right, up and down. Sometimes the tiles have left-right reflectional symmetry such as that found in the human face. But there are also more subtle symmetries at work in the walls of the Alhambra.

Symmetry for a mathematician means being able to pick up all the tiles, move them around and then set them back down again so that they fit into the original outline. Symmetry is like a magic trick: close your eyes, I move the tiles and when you open your eyes the tiles look as they did before I moved them.

But can you make a science out of this art? Is there a way to say that two walls have the same underlying group of symmetries although they look physically very different? Are the symmetrical possibilities endless, or are there limits to what symmetries can exist?

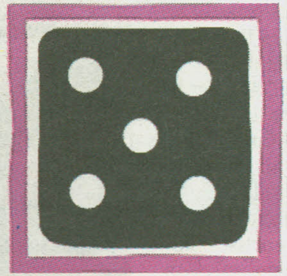
It took until the beginning of the 19th century for the creation of the mathematics that would reveal the symmetrical secrets of the Alhambra. A young revolutionary called Evariste Galois developed a language called group theory that allowed mathematicians to articulate the story of symmetry. His life, alas, was cut tragically short before he could fully realise the potential of his discovery. He was shot dead, aged 20, in a duel over love and politics. But by the end of the 19th century mathematicians used the ideas of Galois to prove that, however hard you tried, there are only 17 underlying groups of symmetries that one can depict on the walls of the Alhambra.

> “Galois's group theory allowed mathematicians to articulate the story of symmetry. He was shot at the age of 20”

The drawings of M.C. Escher , the Dutch artist, were inspired by visits that he made to the palace. Not bound by the Muslim prohibition of drawing things with souls, his designs are full of angels and demons, lizards and fish. But, however hard Escher tried, the mathematics of Galois proves that there cannot be an eighteenth group of symmetries that he could squeeze out of these pictures.

The task for my family's half-term visit to the Alhambra was to discover whether the Moorish artists had found examples of all 17 different sorts of symmetries. We set out on our mathematical treasure hunt, trying to find as many symmetries as we could. Often we'd think that we'd got a new design only to discover that it was an old one in new clothes.

We spent two days combing the walls as groups of tourists flew past us unaware of the hidden mathematical beauty in front of them. By the end we'd discovered examples of all 17 . . . well, nearly. There was one group of symmetries that proved quite elusive. Eventually we found a wall that almost gave us the missing one. The only trouble was that some of the tiles were the wrong colour. Once we'd painted the blue tiles black we had our seventeenth symmetry. But before I get inundated with letters complaining that I've defaced the Alhambra let me reassure you that the repainting was done in our minds. Mathematicians don't like to get their hands dirty.



Conundrum

How many different ways can you put a dice down inside a square outline on a table?

Answer

24. There are 6 faces that can be showing on the top of the cube. Each of these faces can be rotated in 4 different orientations. So that makes a total of 24. These correspond to the rotational symmetries of the cube.

Marcus du Sautoy's Sexy maths

JOE McLAREN

Twists and turns that make a rollercoaster ride

Twists and turns that make a rollercoaster ride

I love rollercoasters. It's not just the thrill of the ride. If you're a nerdy mathematician like me, it's the buzz of all the geometry and calculus that has gone into constructing a ride that pushes things to the limit. But there is one rollercoaster in Europe that gets my blood racing more than any other: the Grand National in Blackpool. When you race round this track you are experiencing one of the most exciting shapes for mathematicians: the Möbius strip.

The Grand National is a race between two trains. When you get in your carriage at the top of the ride there seems to be two parallel tracks. Riders in one train can touch those in the other as they pass through features named after some of the jumps of the famous horse race. But as the trains make the dash for the winning post something strange happens. The trains arrive at the stations opposite to the ones from which they embarked. Very curious. The tracks never meet or seem to cross each other. How on earth did the designers create this feat?

The effect is achieved at the infamous Becher's Brook jump, where one track races over the top of the other and from that point the tracks have swapped sides, so that by the end of the ride the trains arrive at the opposite station.

And it is this simple twist at Becher's Brook that is the key to the Möbius strip, the beautiful mathematical shape that underpins the design of this track. To construct your own Möbius strip, tear off a long strip of paper from the bottom of the newspaper — about 2cm in width. Now join the ends up in a loop, but before you do so make a twist of 180 degrees in one end. If you imagine a piece of paper running between the two tracks of the Grand National ride then at Becher's Brook the paper is twisted 180 degrees as the tracks run under and over each other before they are joined to the track at the beginning of the ride.

The Möbius strip has some very curious properties. The shape has only one edge. Put your finger on the edge of the strip and follow it round. You will be able to reach any other point on the edge. This means that the rollercoaster at Blackpool is just one continuous track rather than two parallel tracks.

Extraordinarily, the shape also has only one side. Take a pen and start tracing a line along the length of the shape without taking the pen off the paper. Eventually you come back to the place where you started. Not so surprising given that the paper is a loop. But the unexpected discovery is that when you look you find that there is no side of the paper that hasn't been drawn on. That's because the twist has ensured that this shape has only one side. The truly bizarre consequence of making this twist appears when you take a pair of scissors and cut the shape down the line that you've just drawn. Instead of the two loops of paper that you'd expect by cutting something in half, the result is a single piece of looped paper with two twists in it.

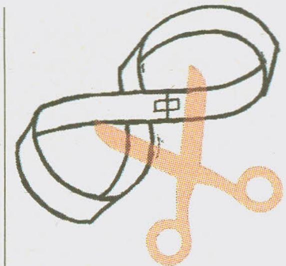
> “The Möbius strip has some very curious properties. The shape has only one edge and, extraordinarily, only one side”

The properties of this shape have not been exploited only by the rollercoaster industry. Conveyor belts are often built in the shape of the Möbius strip because having just one side means that the entire surface of the belt gets the same amount of wear and so the belt lasts longer. The recycling symbol is based on a Möbius strip. And the least you'd expect of the Oxford University Mathematical Society is that its scarf be knitted into the shape.

It even has its own movie. The Argentine film *Möbius* concerns the disappearance of a train in the labyrinthine Buenos Aires subway system. The mathematically minded young detective assigned to the case concludes that the countless additions made to the network over the years have unwittingly created a Möbius strip on which the train is now trapped.

The disappearance of the train is a metaphor for the political activists who went missing in Argentina during the 1970s, but the maths of why a Möbius strip should make trains disappear is never fully explained. Next time you are in Blackpool, however, remember to count your friends on and off the Grand National rollercoaster. Maybe the Argentinians have discovered something that we mathematicians don't know about the shape.

Marcus du Sautoy will give the Geary Lecture tomorrow at 7pm at City University, London, on his book *Finding Moonshine*



Conundrum

If I take a Möbius strip and cut it lengthways down the middle I get just one looped piece of paper. But what happens if I cut this new loop lengthways down the middle?

Answer
You get two loops of paper but they are interlinked. Each loop has four twists in it.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

Mathematics that is out of this world

The winner of the Abel Prize this year was announced recently by the Norwegian Academy of Science and Letters in Oslo. Although a relative newcomer to the accolades that mathematicians can receive, the Abel Prize is quickly becoming the Nobel prize that Alfred Nobel never endowed. The recipient of the million-dollar prize this year is the Russian-born mathematician Mikhail Gromov, who is being rewarded for his revolutionary contributions to geometry.

For 2,000 years the world of geometry was dominated by one name: Euclid. In the greatest textbook of all time, *Elements*, he set down basic axioms about the way points and lines behave. For example, it seems self-evident that between any two points you can draw a line. From these principles, Euclid began to draw conclusions about the shapes that you could draw with these points and lines. For example, he discovered that the angles in a triangle always add up to 180 degrees, something that all schoolchildren are taught.

But as time went on suspicion began to arise over one of Euclid's "self-evident truths", the so-called parallel postulate. Euclid believed that if you draw a line and a point to one side of that line, you can draw a second line through the point that is parallel to the first line, ie, it will never meet the first line. He also believed that there was only one parallel line through that point and that any other line you drew would eventually intersect the first line.

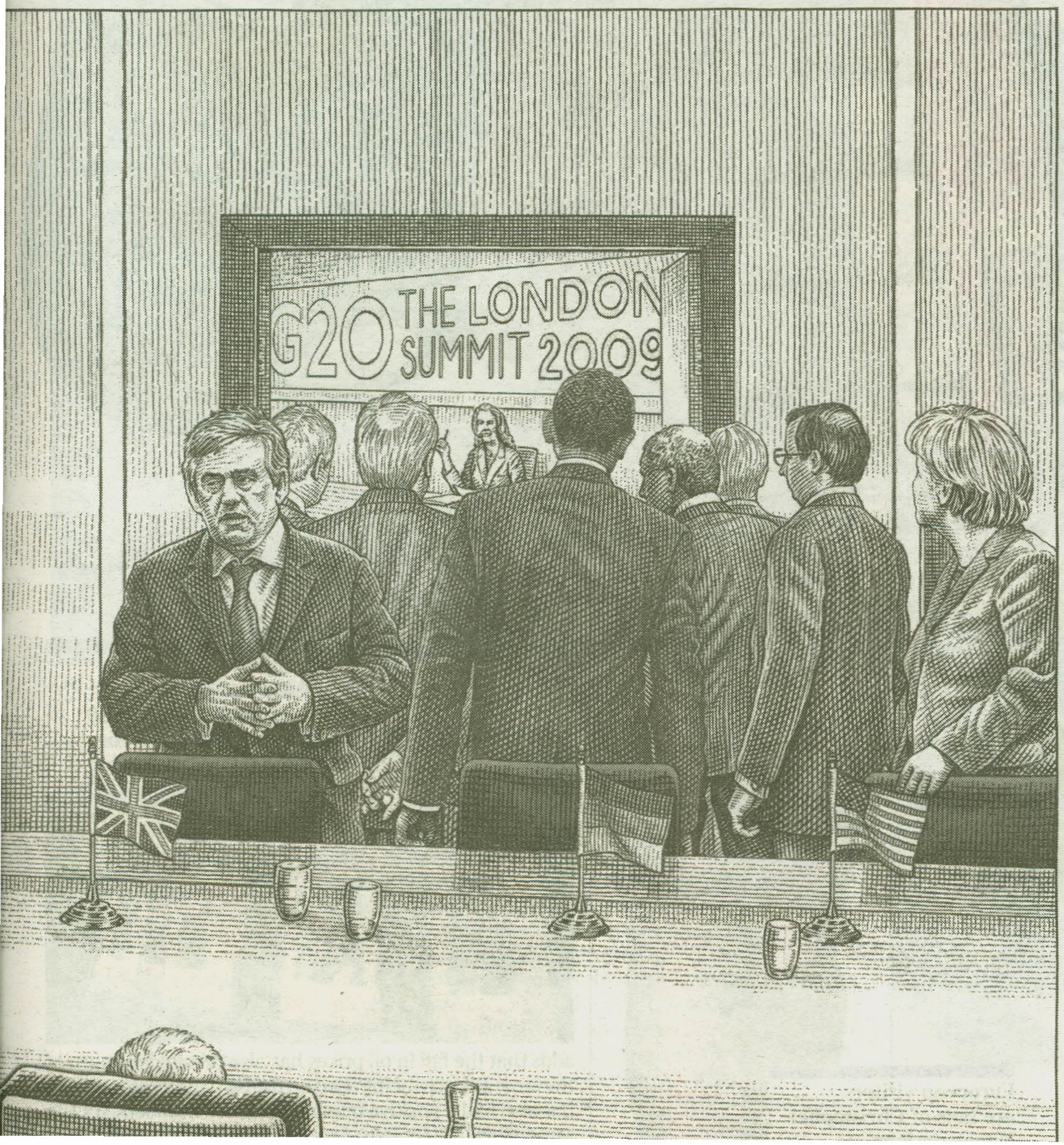
It took a long time for mathematicians to realise that there are geometries in which the parallel postulate is false. Eventually, in the 19th century, Riemann and Gauss realised that Euclid's geometry wasn't the only one.

The word geometry is Greek in origin and means measuring the Earth. If you don't travel far, the Earth appears flat, which gives rise to the geometry that Euclid captured. His geometry is the one that works on a flat piece of paper in an exercise book. But as soon as you start to navigate the surface of the Earth, another geometry appears.

Anyone who has flown from London to San Francisco may have been shocked

Conundrum

A hunter left his camp and walked six miles south, then six miles east and shot a bear. He then walked north for six miles and was back at his camp. What colour was the bear?



" In the geometry of the Earth's surface, angles in a triangle add up to more than 180 degrees "

out of this world

Conundrum

A hunter left his camp and walked six miles south, then six miles east and shot a bear. He then walked north for six miles and was back at his camp. What colour was the bear?

the way points and lines behave. For example, it seems self-evident that between any two points you can draw a line. From these principles, Euclid began to draw conclusions about the shapes that you could draw with these points and lines. For example, he discovered that the angles in a triangle always add up to 180 degrees, something that all schoolchildren are taught.

But as time went on suspicion began to arise over one of Euclid's "self-evident truths", the so-called parallel postulate. Euclid believed that if you draw a line and a point to one side of that line, you can draw a second line through the point that is parallel to the first line, ie, it will never meet the first line. He also believed that there was only one parallel line through that point and that any other line you drew would eventually intersect the first line.

It took a long time for mathematicians to realise that there are geometries in which the parallel postulate is false. Eventually, in the 19th century, Riemann and Gauss realised that Euclid's geometry wasn't the only one.

The word geometry is Greek in origin and means measuring the Earth. If you don't travel far, the Earth appears flat, which gives rise to the geometry that Euclid captured. His geometry is the one that works on a flat piece of paper in an exercise book. But as soon as you start to navigate the surface of the Earth, another geometry appears.

Anyone who has flown from London to San Francisco may have been shocked halfway through the flight to look out and see ice below. The map in the in-flight magazine doesn't show any ice on the line joining London and San Francisco and you worry that you've got on the wrong flight. But, because the Earth is curved, the shortest path between London and San Francisco is not the line you'd draw on a map but a great circle, like a line of longitude, with London being one of the poles. In the geometry of the surface of the Earth, lines are actually these great circles because they are the shortest paths from one point to another. This results in some surprising properties of triangles drawn in this geometry.

> "In the geometry of the Earth's surface, angles in a triangle add up to more than 180 degrees"

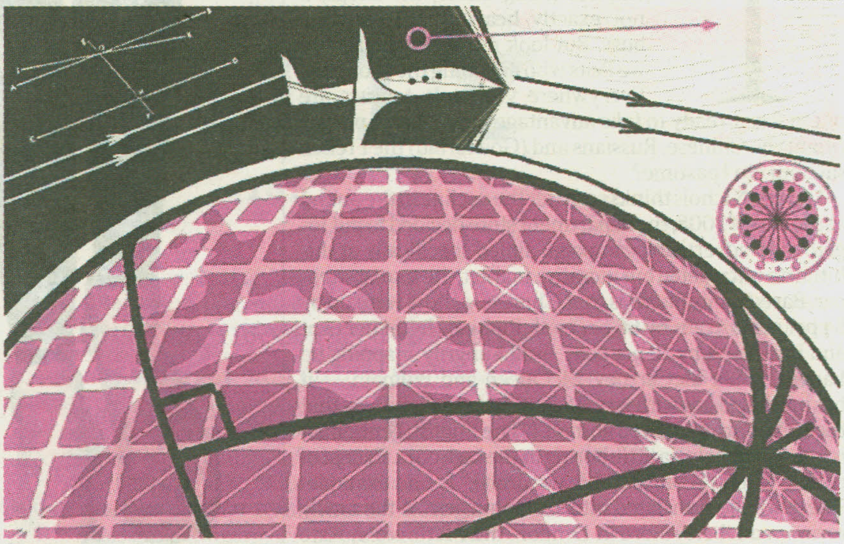
Mark a point at the North Pole and two points on the Equator. Now draw three straight lines between the three points to make a triangle. A straight line between the two points on the Equator is just a line running along the Equator. The two lines joining these points to the pole are the two lines of longitude running from the pole to the points. These lines of longitude meet the Equator line at an angle of 90 degrees. So already the two angles at the Equator add up to 180 degrees. Add on to this the angle between the lines of longitude and the angles in a triangle add up to more than 180 degrees. Does that mean that Euclid got it wrong? Will the whole of mathematics collapse because of this apparent contradiction?

Not at all. This is a geometry that doesn't satisfy the parallel postulate. In fact, in this geometry there are no parallel lines. Take a great circle and a point off that circle: any great circle through this point must meet the first great circle at two places, polar opposite to one another.

Even stranger geometries were discovered at the beginning of the 19th century by a Transylvanian mathematician called Janos Bolyai and a Russian, Nikolai Lobachevsky. In these geometries, angles in triangles add up to less than 180 degrees, and there are many parallel lines through each point.

Mikhail Gromov has been awarded the Abel Prize for his work making sense of the range of geometries that appeared in the 19th century and he is a worthy successor to the likes of Euclid, Riemann and Gauss, Bolyai and Lobachevsky.

JOE McLAREN

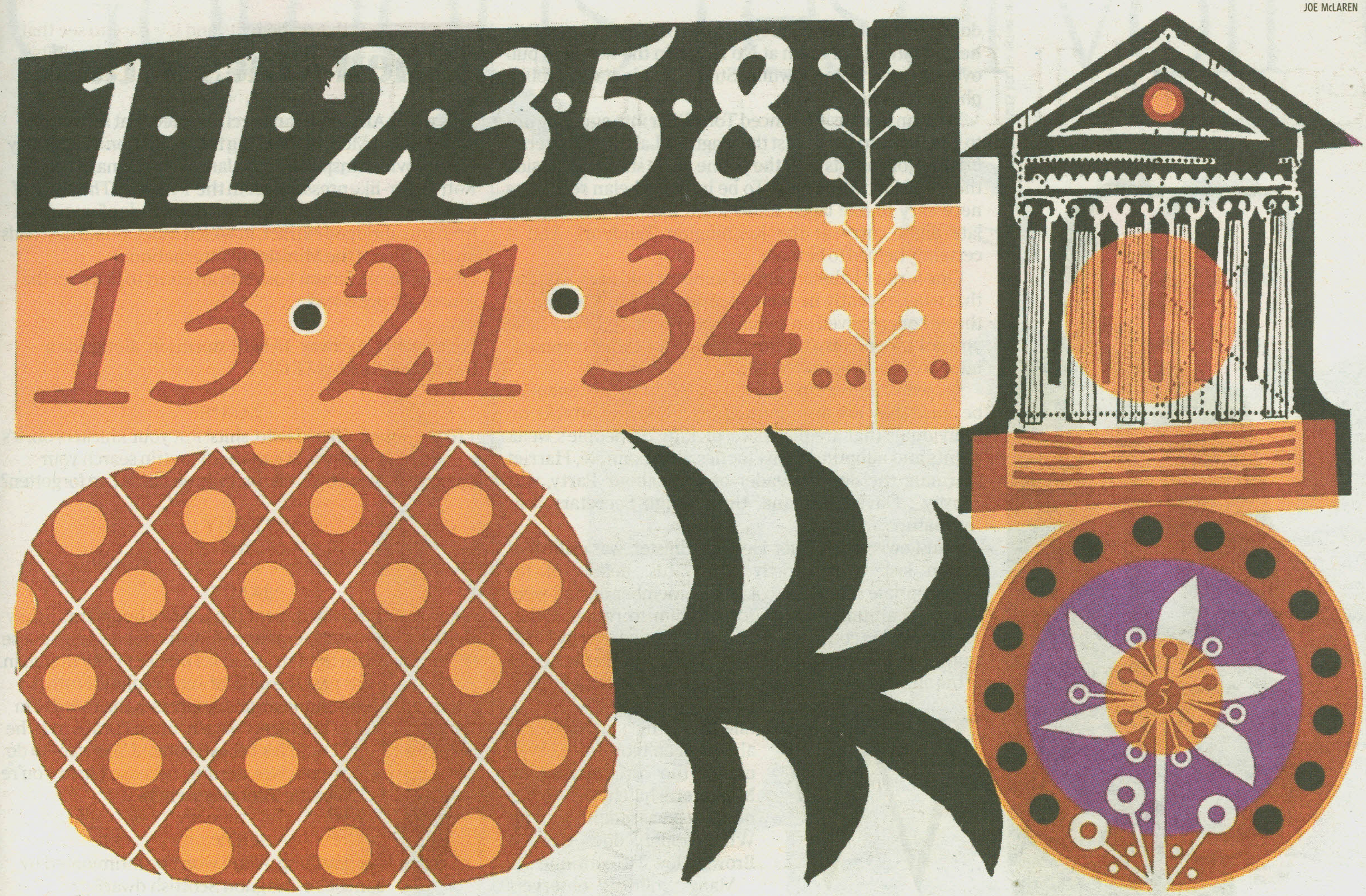


Answer

The bear was white. His camp must be at the North Pole, which is the only place where you satisfy the conditions of the puzzle.

Marcus du Sautoy's Sexy maths

JOE McLAREN



Unlocking the secrets of the sequence

Unlocking the secrets of the sequence

What's the next number in this sequence: 1, 1, 2, 3, 5, 8, 13, 21 . . .? Anyone who has read Dan Brown's *The Da Vinci Code* will know that the answer is 34. The sequence is one of the first codes that readers are challenged with in the thriller. Even if you haven't read Dan Brown, spotting the underlying pattern is not too difficult. You get the next number in the sequence by adding together the two previous numbers. So 5 + 8 gives you 13, for example.

These are some of Nature's favourite numbers. They can be found all over the natural world. Take a pineapple and count the number of cells climbing up the side of the fruit, then count down one of the other spirals and you'll find two numbers in the sequence. Count the number of petals on a flower and it is nearly always either one of these numbers or twice one of these numbers (some flowers are built as if they are two flowers, one on top of each other).

The sequence is named after the great 13th-century Italian mathematician Fibonacci, who spotted the importance of the sequence when he was investigating how the number of rabbits evolves from one generation to the next. But he wasn't the first to reveal the importance of these numbers — a 6th-century Indian poet called Virahanka was perhaps the first to single them out as significant. Virahanka discovered that these numbers count rhythm patterns.

Virahanka was interested in rhythms that you can make out of long and short notes. A short note lasts one beat, while a long note lasts two beats. For example, how many rhythms can you make that are four beats long by making different combinations of short and long beats? You could do short, short, short, short, or long, long, or short, short, long, or short, long, short, or, finally, long, short, short. That's five different rhythms.

If you now analyse the number of rhythms with five beats, you'll get eight different rhythm structures, the next number in the Fibonacci sequence.

The connection with the Fibonacci sequence becomes clear when you realise that, if you want the number of rhythms with N beats, then there are two ways to get them: take the rhythms with N-2 beats and add a long note; or take the rhythms with N-1 beats and add a short note. The total number of rhythms therefore consists of simply adding the two previous numbers in the sequence together.

These numbers have not only obsessed Indian poets, Italian mathematicians and thriller writers. Similar sequences of numbers are at the heart of Le Corbusier's theory of Modulor architecture.

Le Corbusier recognised that these numbers were related to proportions in the human body and believed that the vibrancy of a building depended on capturing proportions in human forms. But there was another reason why he was drawn to these numbers. Hiding behind all these sort of sequences is a very important number in theory of aesthetics: the golden ratio.

> “These numbers have obsessed Indian poets and French architects”

Consider the fractions that you can make by dividing one number in the sequence by the previous number. For example, with the Fibonacci sequence we get a list of fractions: 1/1, 2/1, 3/2, 5/3, 8/5 . . . Then, as you work your way through these fractions, they get closer and closer to this special number called the golden ratio, which starts 1.61803 . . . and then, like pi, the decimals go on for ever without any pattern. The golden ratio is considered by many to be the perfect proportions in art and architecture. The proportions of the Parthenon are typically meant to capture this mathematical ratio. The fractions built from Le Corbusier's numbers also home in on the golden ratio.

But despite being studied for more than 1,000 years, these numbers retain many mysteries. For example, is there an infinite amount of prime numbers in the Fibonacci sequence? Mathematicians know that every Fibonacci number that is not in a prime position in the list cannot be prime. For example, the sixth Fibonacci number is 8, not prime. So if a Fibonacci number is prime, it must be in a prime position. For example, 13 is prime and it's the seventh Fibonacci number. But unfortunately, if we look at a Fibonacci number in a prime position it doesn't always give a prime.

It is still a mystery whether you can get infinitely many Fibonacci primes. But then mathematics would be boring if we knew all the answers.

1 1 2 3 5
8 13 21
34 55 89
144 233

Conundrum

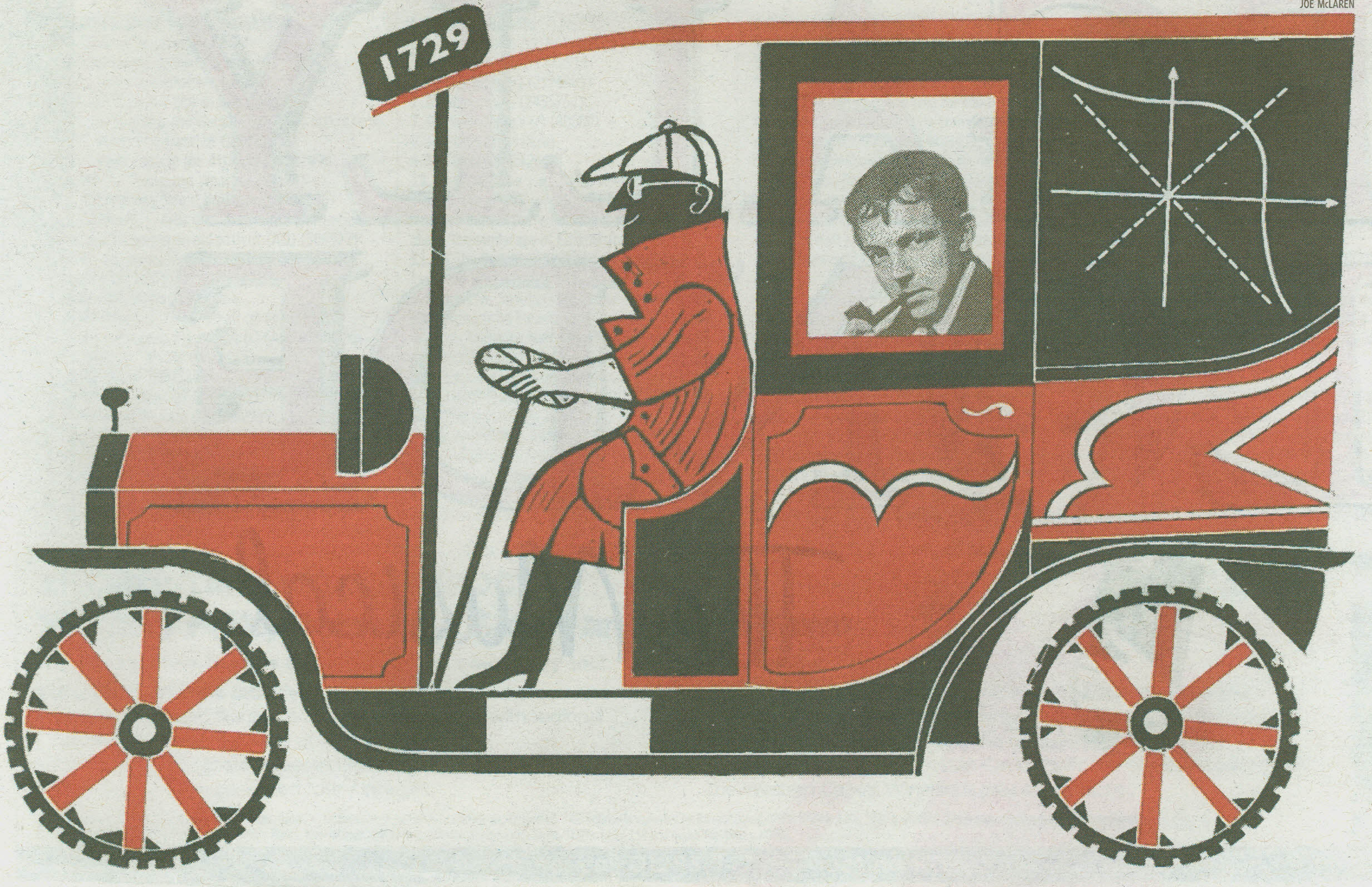
Find a Fibonacci number, other than 1, that is in a prime position in the sequence but that isn't prime.

Answer
The 19th Fibonacci number is 4,181 = 37 x 113.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN



The sum of all that's unique about 1,729

The sum of all that's unique about 1,729

The number 1,729 has been appearing in some curious places recently. In the animated sitcom *Futurama* Bender was the 1,729th robot to be manufactured by its creator. The *Nimbus BP-1729*, the space craft captained by Zapp Brannigan also pays homage to the number. The Complicite's hit show *A Disappearing Number*, features a man obsessed with getting his telephone number changed to include the digits 1, 7, 2 and 9. And in David Auburn's play *Proof*, the protagonist Catherine calculates at the beginning of the play that if every day she'd lost to depression was a year it would work out at a total of 1,729 weeks.

The thread that runs through all these strange occurrences of the number 1,729 is that the scripts were written by authors obsessed with mathematics. Because 1,729 isn't any old number but has some very interesting mathematical properties.

Not that every mathematician thought so. Indeed, the reason that 1,729 has such resonance for those obsessed with mathematics is because the great Cambridge mathematician G. H. Hardy once famously declared that he thought it a rather dull number. He said this while visiting his great collaborator Srinivasa Ramanujan at a London nursing home.

Hardy had discovered this untrained Indian genius some years earlier in 1913 after Ramanujan, a Hindu clerk at the Port Authority in Madras, had sent the Cambridge mathematician letters full of wild and unimaginable formulas. Hardy immediately arranged for Ramanujan to be brought to Cambridge where they'd worked together proving amazing theorems. But Cambridge couldn't accommodate Ramanujan's Brahmin customs. He had been used in India to someone handing him food as he calculated away. Suffering malnutrition, he fell gravely ill and depressed and eventually found himself confined to the nursing home in Putney.

Hardy sat next to his sick friend. But, being mathematicians, both were hopeless at small talk. So Hardy ventured 1,729, the number of the taxi he'd arrived in, as an example of a rather dull number.

Ramanujan's eyes lit up. "No, Hardy," he replied. "It is a very interesting number. It is the smallest number expressible as the sum of two positive cubes in two different ways."

He was right. Many numbers can be written as two cube numbers added together. For example $35 = 2^3 + 3^3$. But 1,729 is the smallest number that has two different ways that it can be split into cube numbers. One way is to write it as $12^3 + 1^3 = 1{,}728 + 1$. But you can also express 1,729 as $10^3 + 9^3 = 1{,}000 + 729$. And it was Ramanujan's extraordinary ability to recognise the special character of this number that sealed its place in mathematical folklore.

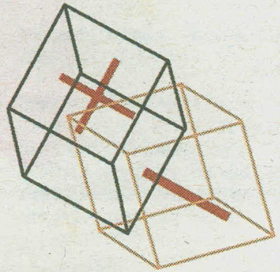
> "The ability of Ramanujan to recognise the special character of this number sealed its place in folklore"

The story is frequently told to illustrate Ramanujan's special mathematical mind. He would often say that his mathematical discoveries came to him in dreams delivered by his family goddess Namagiri. A colleague of Hardy's in Cambridge said that Ramanujan seemed to know every number as if it were an intimate personal friend. But for me this is not the sign of a strange autistic mind, but an indication that Ramanujan was thinking about some of the deep problems that have fascinated mathematicians for millennia. Because the story is related to one of the central topics of mathematics: solving equations.

Take any number N and consider the equation $x^3 + y^3 = N$. These are examples of some of the most mysterious equations in mathematics, called elliptic curves.

One of the holy grails of mathematics is a problem called The Birch-Swinnerton-Dyer Conjecture, which tries to understand whether equations such as these have solutions or not. So important is the conjecture that there is a $1 million prize for the first person to crack it. But it isn't just important for mathematics. Some of the cutting-edge codes being used in industry exploit properties of these equations. Indeed, air-traffic control uses codes based on elliptic curves to keep information about flight paths secure from hackers. The number 1,729 is just the tip of one of the most mysterious topics in mathematics.

So next time you take a cab, spend the journey trying to unlock the interesting properties behind your taxi's number. As Ramanujan revealed to Hardy, there is no such thing as a dull number.



Conundrum

The cubes Ramanujan used to express 1729 were both positive. What is the smallest number that can be written as the sum of two cubes in two different ways if the cubes can be positive or negative?

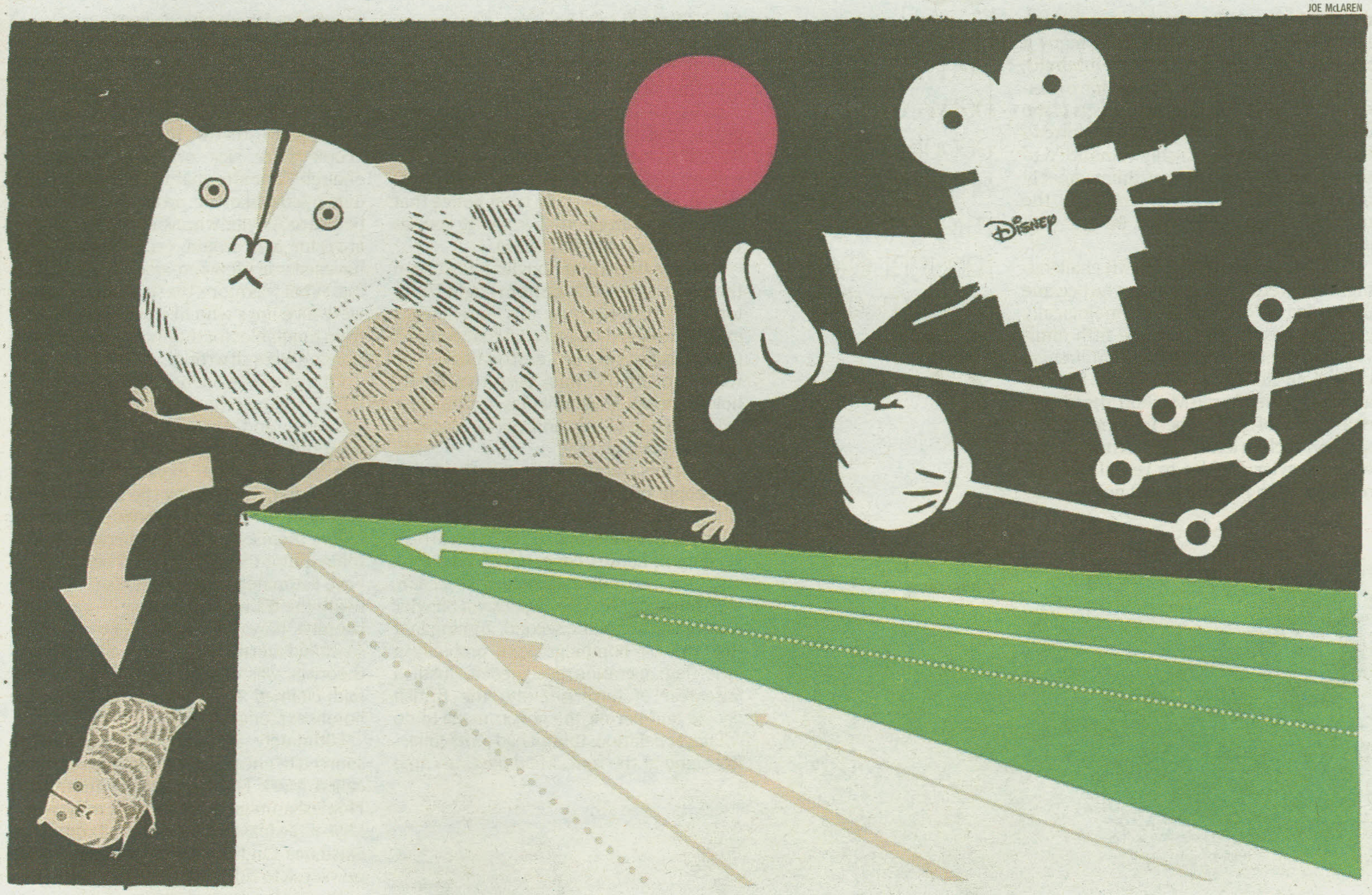
Answer

$91 = 3^3 + 4^3 = 6^3 - 5^3$

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN



Pushing the lemming theory over the edge

Pushing the lemming theory over the edge

A natural history question to kick off this week's Sexy Maths. Which animal throws itself off a cliff every four years in a mass suicide pact, nearly wiping out a generation of the animal? "A lemming" I hear you cry. Certainly there is evidence that periodically the numbers of this Arctic rodent plummet dramatically. But throw themselves over a cliff?

The popular myth about lemmings derives from a 1958 Disney documentary called *White Wilderness*, which included footage that purported to show this mass suicide taking place. So convincing was the evidence that the term "behaving like a lemming" found its way into modern parlance, becoming shorthand for those who follow the masses unquestioningly, with potentially disastrous consequences. The animal's conduct even spawned a video game in which the player would attempt to save the lemmings from their own mindless march to the cliff face.

But it was revealed in the 1980s that the film crew of *White Wilderness* had faked the whole scene in landlocked Alberta, Canada. According to a Canadian TV documentary, the lemmings Disney took there to film apparently refused to leap over the cliff on cue, so members of the film crew encouraged them over the edge.

So if it isn't some mass suicide pact that causes a sudden drop in the number of lemmings from one generation to the next, then what is the explanation? It turns out that mathematics has the answer. A simple equation determines how many lemmings there will be from one generation to the next: $K - K \times L \div 100$, where L is the number of lemmings that survived the previous season, K is the population of the colony after its annual births, and $K \times L \div 100$ is the number that die during the season. The equation assumes that, because of environmental factors such as food and predators, there is a maximum population attainable (in this equation we have assumed, for convenience, that the maximum lemming population is 100).

This equation, although simple, has some surprising properties. In the simplest scenario, we'll presume that the lemming population doubles each spring (ie, $K = 2L$), then $2L \times L \div 100$ will die. In such a situation, whatever the original population is, it will eventually home in on half the maximum number, where it will remain. So, once you hit 50 lemmings, the number doubles to 100 during the season, but by the end of the year $100 \times 50 \div 100$ will have died, leaving a population of 50 lemmings again.

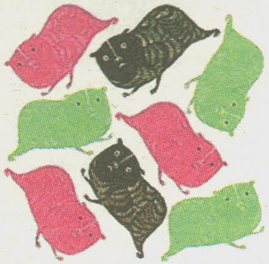
> "The lemmings refused to leap over the cliff on cue, so some of the film crew encouraged them over the edge"

But what if the lemmings are more productive? If the population slightly more than triples from one season to the next, the equation doesn't tend to a stable population but instead flips between two values. In one season numbers are quite high, but then drop in the following season.

As the productivity of the lemmings increases further, even stranger dynamics for the numbers from one generation to the next occur. If the population increases by a factor of 3.5, then the equation for the number of lemmings oscillates between four values. And this is where we see that in one out of four years there can be a significant drop in the number of lemmings. Not because of a mass suicide pact but because of the maths.

The really interesting behaviour in the population dynamics occurs when the lemmings increase by a factor of just over 3.57. Then the numbers from one generation to the next jump around seemingly without any rhyme or reason. One season the numbers can be high then they plummet, remain low for a few seasons and then suddenly the next season there are lemmings everywhere. This is an example of one of the most important themes in modern mathematics: chaos. Beyond this 3.57 threshold, the dynamics of the population numbers are almost impossible to predict.

Ironically, it transpires that when lemming populations do get really large, they migrate in large numbers in search of new habitats, and do occasionally find themselves stuck on the top of a huge cliff with their mates forcing them to tombstone over the edge. If the Disney film crew had waited for the right year, and gone to the right country, they may even have captured the real thing on film.



Conundrum

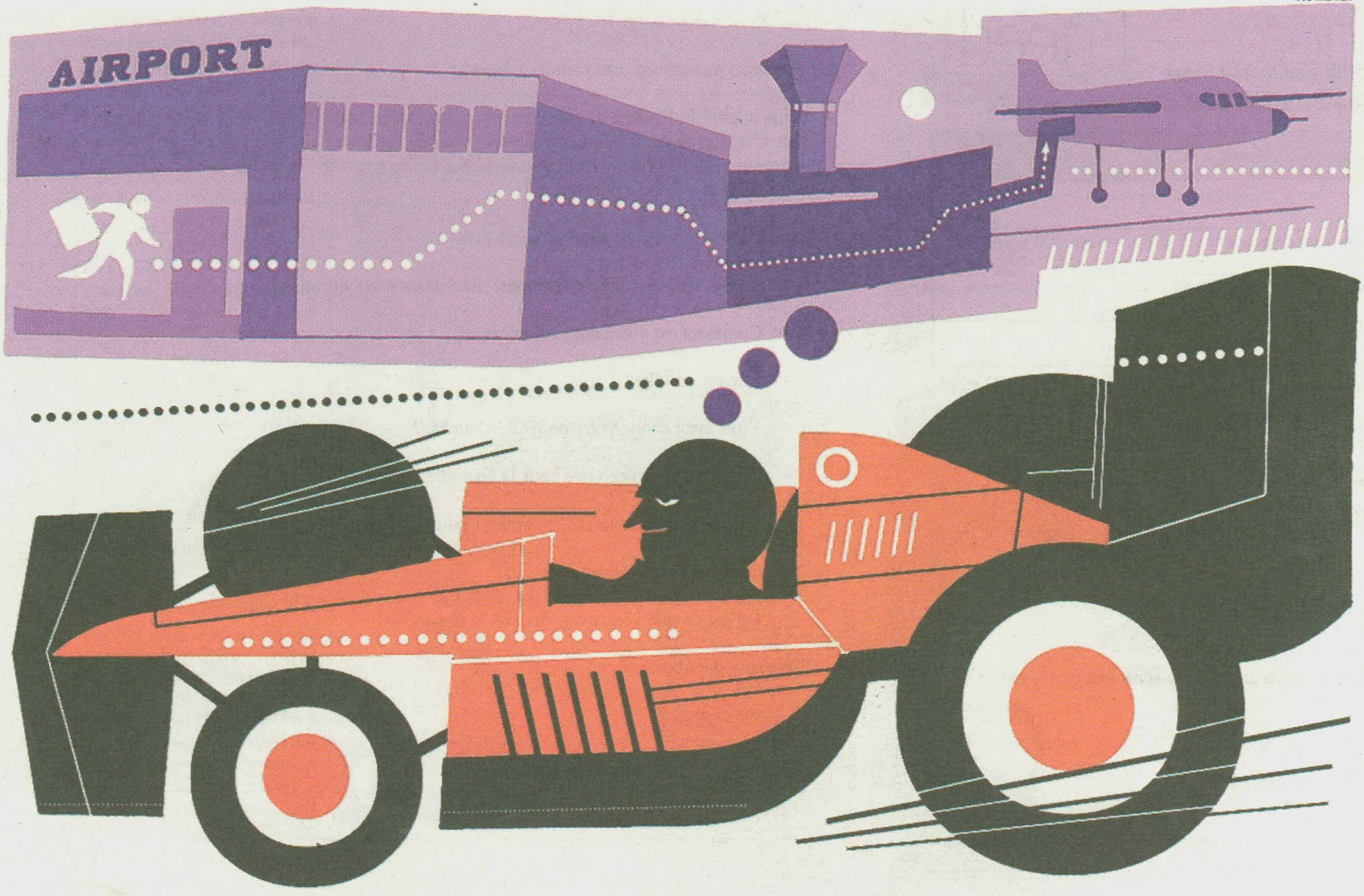
Suppose that each season the number of lemmings triples. If there were N lemmings last season and 3N at the end of this breeding season, then $3N \times N \div 100$ don't survive until the following year. Starting with N=30 lemmings, how does the number of lemmings change from one season to the next? (Fractional lemmings get rounded up or down to the nearest whole number.)

Answer

The population goes from 30 to 63 to 70 and then ping-pongs back and forth each year between 63 and 70 lemmings.

Marcus du Sautoy's Sexy maths

JOE McLAREN



Formula won: the key to boosting faster travel

Formula won: the key to boosting faster travel

This year Formula One teams that agree to race within a strict budget rather than relying on unlimited funds to develop their cars are being rewarded with a number of perks under a scheme called the Kinetic Energy Recovery System. And one of the bonuses includes a booster button that can be pushed for a maximum of 6.7 seconds during a race.

This has led to some interesting mathematical questions. If you are behind the wheel of a Formula One car and you have this booster button, when is the best time to press it and release the power? Should you do it to help overtaking, or should you use it to get through a slower section of the course?

The designers of Formula One cars already rely on a huge amount of mathematical input. But the question of optimising the use of such a button is the perfect opportunity for the mathematician to take the driving seat.

Indeed, a similar question has been exercising minds on the blog of one of the world's leading mathematicians. Terence Tao won a Fields medal, the mathematician's Nobel prize, in the latest round of awards along with Grigori Perelman, the Russian recluse who solved the Poincaré Conjecture. Although Perelman turned his down, Tao has been revelling in the limelight and writes a popular blog (among maths geeks anyway), discussing a range of complex mathematics from partial differential equations to low-dimensional topology.

But a recent discussion concerned a more mundane problem that turned out to be close to the heart of the globe-trotting mathematicians as they jet between international conferences.

You arrive at an airport for a flight connection but the timing is tight. You discover that your next plane is leaving from the far end of the airport. There is a moving walkway for part of the journey. You've got enough energy to do a short burst of running, but otherwise you'll walk at a constant speed. However, one of your shoelaces is undone so you're going to need to stop at some point to do it up.

You want to get to the gate as quickly as possible, so the question is: when should you use your burst of energy? Should you run on or off the moving walkway? And that shoelace? Should you tie that on the moving walkway? Or should you stop on the portion of the journey without a moving walkway? Or does it make no difference where you run or stop?

A mathematician's instinct is to reach for algebra to unravel the problem, and that is a sure way of getting the right answer. But sometimes a little lateral thinking can save you the ink. One blogger had twins setting off simultaneously. As they approached the moving walkway one twin stopped to do up his lace, the other stepped on and did his lace on the moving walkway. It is clear that by the time both have finished tying their lace, the twin who tied on the walkway is well ahead of his brother. So shoelaces should be tied on the walkway.

> “Formula One teams should monitor the blogs of leading mathematicians — it might give their drivers the edge”

But what about the burst of running? Our twins are walking together towards the walkway when one twin decides to press the boost button to get him to the walkway before his brother. He reaches it D metres ahead of his brother. But as soon as he steps on to the walkway the distance between them begins to increase even more. When the second twin steps on to the walkway he presses his booster. Since both are on the walkway, this will allow him to catch up only D metres on his brother. But his brother is more than D metres away by now. So the boost button should be hit off the runway.

There is an economic maxim that underlies both decisions: a worker should spend as much time as possible on the most efficient machine. In this case, staying on the walkway for as long as possible is the best strategy. Stop to tie the shoelace on the walkway but don't press the boost button as it will get you off the walkway quicker. Save that for the bit of the airport without the moving walkway.

Of course, if you wait until you reach the walkway before you tie your lace, you may discover one disadvantage — tripping flat on your face! So, while Formula One teams would be well advised to monitor the blogs of the world's leading mathematicians, they should also bear in mind good old-fashioned common sense.

Conundrum

A biker travels to work at 30mph. Coming home, the roads are clearer so he travels at 45mph. What is his average speed?

Answer

36mph (not 37.5mph). Suppose the distance from home to work is 45 miles. It takes 1.5 hours in the morning and 1 hour on the way home. So the average speed is (45+45) ÷ 2.5 = 36mph.

Marcus du Sautoy's Sexy maths

JOE McLAREN

A game of 12 pentagons

A game of 12 pentagons

It was the moment of which every schoolboy dreams. The call-up to play football for England against the Germans. The publication of my recent book, *Finding Moonshine*, qualified me last year for inclusion in the illustrious England writers football team. But a 6-1 drubbing at the hands of the Germans has left us licking our wounds ... me in particular, as I had my writing hand broken by a hulking German playwright.

He certainly knew where to hit a writer where it hurts most. So we are all waiting for the chance to make amends at the rematch, which will take place in England on June 13. This time we have a secret weapon. Aidy Boothroyd, the manager who took Watford to the Premier League and the semi-final of the FA Cup, has volunteered to help the English cause and last week gave our team two full days of training. The sessions were a revelation. Boothroyd exposed the game for what it is: geometry in motion. We learnt a whole range of shapes to apply at different points in the game: the banana, the diamond, the flower and the hedgehog. The more philosophical among the team were disappointed not to have a fox to go with the hedgehog, although I don't think Boothroyd's bedside reading includes the essays of Isaiah Berlin.

I'm certain it must include books on higher mathematics, though, because the shapes that were flying across the pitch left my mathematical brain racing. Kids are often taught to "make triangles" when playing football, so that the player on the ball always has two passing options, but under Boothroyd's tutelage when one player had the ball the whole team had to adjust its formation to allow the ball to move freely through the team. It was, I realised, the sporting equivalent of a mathematical network: the 11 players represented the hubs, while the passing options that didn't risk being intercepted by opposing players were the links between them.

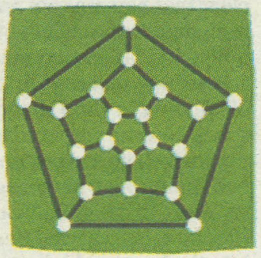
William Hamilton, the 19th-century mathematician, was intrigued by such networks, so much so that he patented a game based on them. The Irishman's puzzle took a dice-like shape made from 12 pentagonal faces, but with one of the faces removed, so that it could be flattened on to a two-dimensional board. Starting at one corner, or vertex, of the shape, players had to trace a circuit round the edges of it that visited all the other 19 vertices only once and returned to the starting point.

Hamilton sold the game to a dealer in 1859 for £25, but it didn't capture the public's imagination and flopped. The puzzle, though, intrigued mathematicians, who began to investigate which networks had these Hamiltonian circuits. It turned out that you can trace a path round the vertices of any Platonic solid such as the dodecahedron and even the Archimedean solids, shapes that have a selection of different symmetrical faces.

More intriguingly, it was discovered that 11 is the smallest number of points for which there exist "polyhedral" networks that have no Hamiltonian circuit (the word polyhedral here refers to a network that can actually be wrapped around to make the faces of a ball). Transferring such a network to the field of play, there is no way to pass the ball around all the players without one team member making two passes.

So, from a mathematical point of view, there is something intrinsically interesting about the 11-a-side game that doesn't occur for five-a-side or seven-a-side matches. Boothroyd probably knows this already, but it was an exciting revelation for someone who has spent years with little connection between my right foot and my right brain. The great Dennis Bergkamp was right when he said that every kick of the ball requires a thought. Then again, as Cristiano Ronaldo has shown with his careful inspections of the ball before placing it for a free kick, the leather sphere itself is also worthy of contemplation.

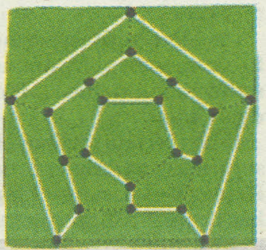
During our break for orange segments, I began obsessively trying to trace a Hamiltonian circuit round the football's network of seams. The classic football is constructed from 12 pentagons and 20 hexagons. What Boothroyd had been trying to get our team to knock gracefully around the park was, perhaps, the most famous Archimedean solid of all.



Conundrum

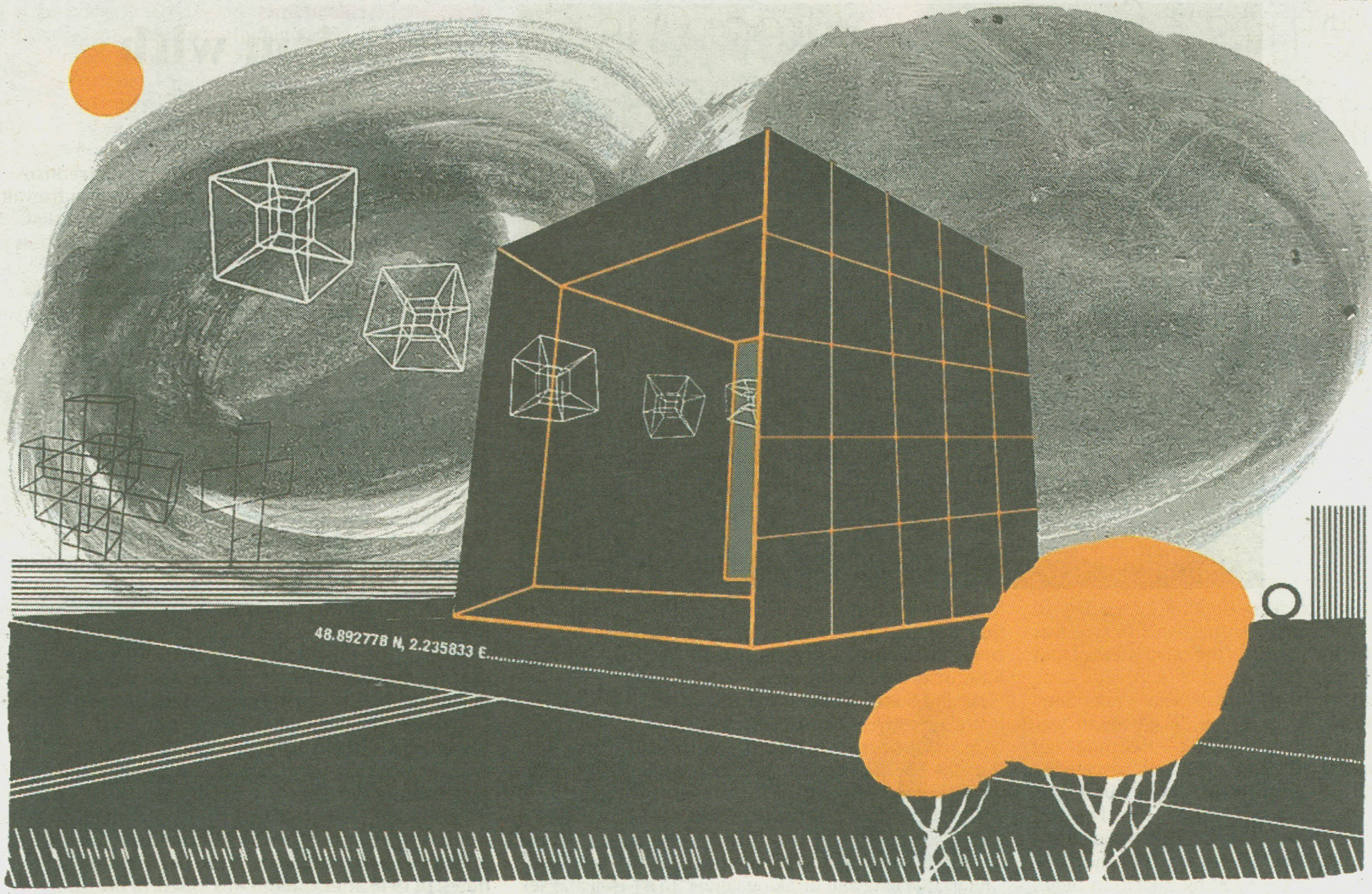
Can you solve Hamilton's Icosian Game? Trace a circuit round the diagram visiting all 20 vertices exactly once. You must return to the place you started.

Answer



Marcus du Sautoy's Sexy maths

JOE McLAREN



Go fourth … into another dimension

Go fourth ... into another dimension

Every day thousands of travellers take the Eurostar to a strange and foreign land. No, not Paris; the Fourth Dimension. Although many visitors to Paris don't realise it, at the heart of the city is a portal to hyperspace. As you emerge from the Paris subway into the financial district at La Défense you are greeted by a huge four-dimensional cube.

Well, it's not quite a four-dimensional cube. The architect of the Grande Arche, Johann Otto von Spreckelsen, was restricted to building in the three-dimensional world in which we live. What he has done instead is to create a shadow of a 4-D cube. The Danish architect used a trick developed by Renaissance artists to depict a three-dimensional cube on a 2-D canvas. Artists would draw a square inside a larger square and, by joining up the corners, they would create the illusion of seeing three dimensions. Von Spreckelsen built a smaller cube inside a larger cube and joined up the corners of the two cubes to create this hypercube with its 16 corners.

He wasn't the first artist to experiment with the mysteries of the hypercube. Salvador Dali used a different method to unravel the fourth dimension in our 3-D world. If you want to build a 3-D cube, you can start in two dimensions by cutting out a net consisting of six squares joined in a cross shape and fold up the shape to make a cube.

In Dali's painting, *Crucifixion (Corpus Hypercubus)*, he depicts Christ being crucified on what a 4-D cube would look like if it was unfolded into our 3-D world. It consists of eight cubes, four stacked in a tower and four arranged around the faces of the third cube in the stack. It looks like two intersecting crosses. For Dali the sense of a dimension beyond our physical world resonated with the spiritual world that he was trying to capture.

But if you want to see in four dimensions, it isn't the world of the architect or artist that will give you that vision but the world of mathematics. Descartes' invention of Cartesian geometry creates a dictionary that turns geometry into numbers. And it's using these numbers that ultimately allows the mathematician a passport to hyperspace.

We are all familiar with how this dictionary works. Every position on the surface of the planet can be turned into a set of two numbers. One number indicates the location's longitude, the other its latitude. So, for example, the co-ordinates (48.892778 N, 2.235833 E) give the precise location of the Arche at La Défense. I can use this dictionary of numbers to describe a 2-D square by numbers. It is a shape whose corners are located at the positions: (0,0), (1,0), (0,1) and (1,1). If I want to describe a 3-D cube then I need to add an extra co-ordinate to keep track of the height off the ground. So the eight corners of the 3-D cube are located at (0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1) and (1,1,1).

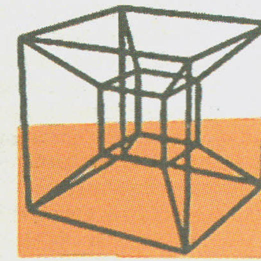
> "If you want to see in four dimensions, it is the world of mathematics that will give you that vision"

So what's a 4-D cube? Since we have only three physical dimensions, the geometric side of Descartes' dictionary runs out. I can't physically construct a 4-D cube. But the numbers on the other side of the dictionary don't run out. To move into the fourth dimension I simply add another co-ordinate to keep track of how far I'm moving in this new direction. So the 16 corners of a 4-D cube? They start at (0,0,0,0), which is connected to (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) and then we move all the way through to the extremal point at (1,1,1,1).

Using the pair of 4-D spectacles provided by Descartes' language of co-ordinates, mathematicians can explore the geometry of these shapes in hyperspace. But if you want a glimpse of this world, a Eurostar ticket will start you on your journey.

But a little warning. Whenever I've visited the Arche there seems to be a howling wind that sucks through the centre of the arch, even on a calm day. By building a hypercube at the heart of Paris it feels sometimes as though the architect has opened a gate into the fourth dimension. You begin to wonder whether it really is the suburbs of Paris that you see through the centre of the arch. So approach the hypercube with caution. You never know where you might end up when you pass through the Arche.

Marcus du Sautoy takes Alan Davies into the fourth dimension in *Horizon*, BBC Two, Tuesday, March 31, 9pm



Conundrum

How many edges does a four-dimensional cube have?

Answer

32. Each corner has four edges emerging from it, one in each direction. There are 16 corners in the hypercube. So that looks like 4 x 16 = 64. But actually you've counted each edge twice: once as an edge emerging from the point at one end and also as an edge emerging from the point at the other end of the edge. So the total number of edges in the four-dimensional cube is 16 x 4 ÷ 2 = 32.

Marcus du Sautoy's Sexy maths

JOE McLAREN

The shape of things already here...

The shape of things already here...

It seems a bit crazy to reinvent the wheel, but that is precisely what the Chinese inventor Guan Baihua has done. Although his new set of wheels have a little twist to them. The 50-year-old military officer from Qingdao has devised a rather curious new bicycle. Instead of circular wheels the bike has a pentagonal wheel at the front and a triangular wheel at the back.

He believes that people will be drawn to the bike because it requires more work to cycle and therefore will provide more exercise for the cyclist than a conventional bike.

Those who have tried it have been surprised at how smooth the ride is.

That is because the edges of the pentagonal and triangular wheels are not perfectly straight. The sides of the shapes bulge outwards in such a way that the wheels share an important feature with the circle: the diameter across the shapes is the same which ever way that you measure it.

This is clear for the circle, which is defined as the shape with a fixed radius from the centre. But the wheels of Guan's bike also have the property that if you take two parallel lines that touch either side of the shape then the distance between the lines is the same regardless of how you turn the shape between the two lines. The shape will never suddenly extend beyond the confines of these lines.

This means that as I cycle, although the wheel itself does not have a fixed centre of rotation, the saddle of the bike doesn't bob up and down as you might expect but stays a fixed distance from the ground. To get the exact contours for the shapes, the curved edges are drawn so that they are part of a circle centred on the vertex opposite the edge. They are called Reuleaux polygons after the 19th-century German engineer Franz Reuleaux, who did pioneering work on machines that turn one type of motion into another.

Guan's bicycle isn't the first to exploit these shapes — they have found themselves being exploited by urban planners. They have been used as manholes in the street. The most common shape for a manhole is of course the circle. But why do they tend to be circular as opposed to square shaped?

The reason is that the square has the annoying property that the diameter from one corner to the other is bigger than the diameter halfway along an edge. This means that if you put the square manhole on its side, it is very easy to drop the square cover down the manhole.

With a circular hole this is impossible.

> “Guan's bicycle isn't the first to exploit these shapes — they have been exploited by urban planners as manholes”

There is no way a worker can accidentally drop the cover through a circular hole in the road. Some cities, bored with the circular manhole, have exploited the same shapes that are behind the wheels of the Chinese bike.

For example one can find on the streets of San Francisco triangular manholes with the property that the diameter is constant. Like the circular manholes, a worker need not worry about these triangular covers dropping through the manholes.

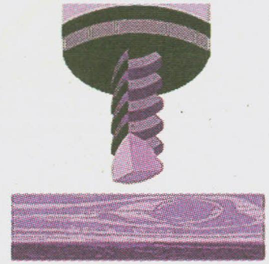
But as well as walking over these shapes in the road, most of us in the UK walk around every day with these shapes in our pockets. Both the British 50p piece and the 20p piece are coins with seven sides. But if you examine the coins carefully you'll see that, as with wheels of the Chinese bicycle, the sides are not perfectly straight.

They are designed in such a way that they too have a constant width across the coin. This means that whatever way you drop a 50 pence piece into a slot machine, the mechanism can measure the diameter and recognise the validity of the coin.

Although the 20p and 50p pieces have been a great success it doesn't look likely that Guan's Flintstone-esque bike will be going into mass production in the near future.

But Guan is not put off by the lukewarm response to his first invention. It is rumoured that he is now working on ideas for a new social networking site. Twitter and Facebook had better watch out.

To see a picture of the bike: tinyurl.com/chinabike
For more on the shapes: tinyurl.com/concurves



Conundrum

If you make a drill piece in the shape of a Reuleaux triangle, what shape hole will it drill?

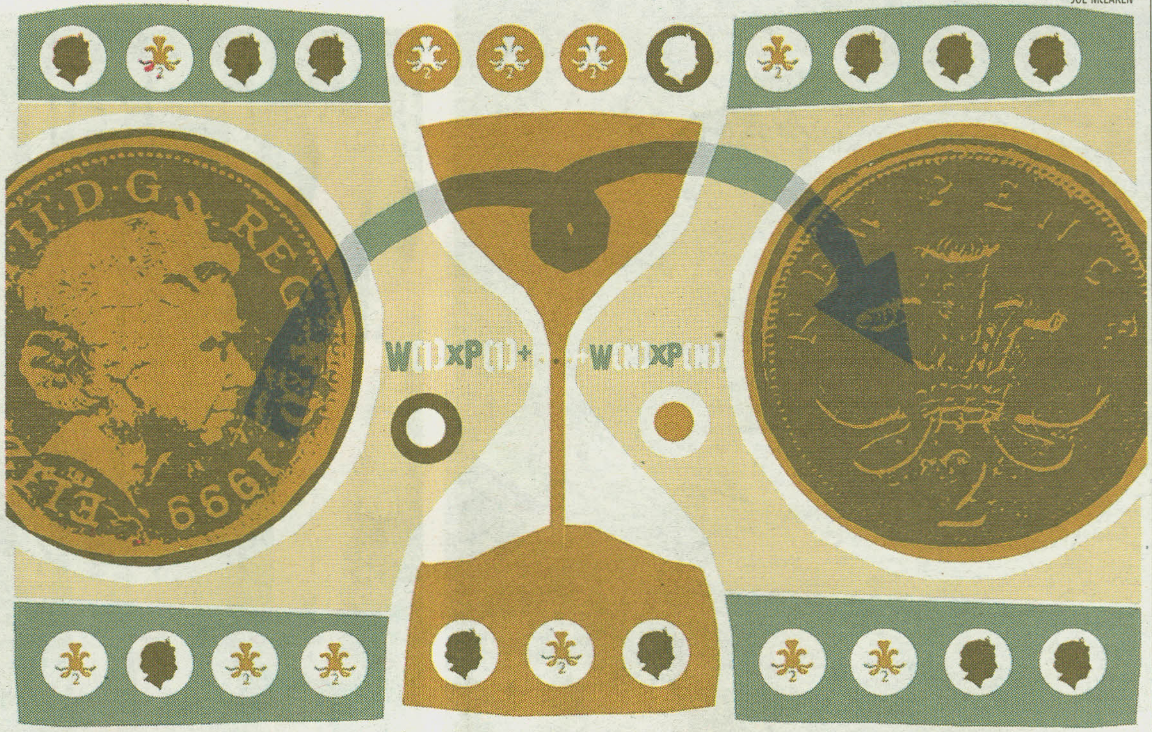
Answer

A Square. Well, almost. The corners of the square are slightly rounded.

Marcus du Sautoy's Sexy maths

When does it pay to play a game of chance?

JOE McLAREN



When does it pay to play a game of chance?

Let's start by playing a game. I roll a dice and pay you in pounds the number that appears on it. How much would you be prepared to pay to play? If you pay £1 you cannot lose, and if you pay £6 you cannot win but at what point do the odds tip from my advantage to yours?

It was the correspondence between two of the greats of mathematics, Fermat and Pascal, that led to the discovery that you could apply mathematics to analyse games of chance. Previous generations had not dreamt of such a connection. Mathematics is a subject of certainties and truths. How can it apply to the analysis of chance and randomness? The mathematics of chance crystallised with the publication by Swiss mathematician Jakob Bernoulli of *Ars Conjectandi*, or *The Art of Conjecture*. It is here that you find the formula for the fair price that you should pay for any game.

Suppose there are N possible outcomes (in our dice game, N=6). You win W (1)

> The formula says the average payout will be infinity pounds! It's worth playing whatever the cost

pounds if outcome 1 occurs (ie, £1 for a roll of 1). This happens with probability P (1) (in this case, 1/6). Similarly, outcome 2 occurs with probability P (2) in which case you win W (2) pounds (in our game, £2 for a roll of 2, again with a probability of 1/6). On average, a game earns you W (1) x P (1) +...+W (N) x P (N) pounds each time you play, which in our dice game equals £1 x 1/6 + £2 x 1/6... + £6 x 1/6 = £3.50. So if I offered you less than this to play, then you're going to be the winner in the long run.

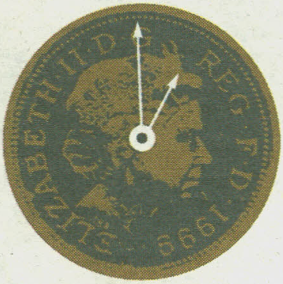
The formula seemed sound until Jakob Bernoulli's cousin Nicolaus, in an almost oedipal act, came up with the following game: I toss a coin. If it lands heads I pay you £2 and the game ends. If it lands tails then I toss again. If the second toss is heads I pay you £4. If it is tails I toss again. Each time I toss, the payout doubles. So if I toss six tails followed by a head I'll pay you 2 x 2 x 2 x 2 x 2 x 2 x 2 = 2^7 = £128. How much would you be prepared to pay to play Nicolaus's game? Four pounds? Twenty pounds? One hundred pounds?

Well, there's a 50 per cent chance that you'll win only £2. After all, the probability that it lands heads on the first toss is 1/2. So P (1) = 1/2 and W (1) = 2. But you're hoping for a long run of tails followed by a head to get as big a prize as possible. The probability that you get a tail followed by a head will be 1/2 x 1/2 = 1/4. But this time you win £4. So the second outcome has P (2) = 1/4 but W (2) = 4. As you keep going the probabilities get smaller but the payout bigger. For example, six tails followed by a head has a probability of $(1/2)^7$ = 1/128 but wins you 2^7 = £128.

If you stopped the game after seven tosses then you would lose only if there were seven tails in a row. Using Jakob's formula the average payout would be W (1) x P (1) +...+ W (7) x P (7) = (1/2 x 2) + (1/4 x 4) +...+ (1/128 x 128) = 1 + 1 +...+ 1 = £7. It is worth playing the game, therefore, if anyone offers you less than £7 to play.

But here is the sting. Nicolaus is prepared to play the game indefinitely until a head appears. You're a winner every time. So how much will you pay to play the game?

There are infinitely many options now. The formula says that the average payout will be 1 + 1 + 1 +... namely infinity pounds! If anyone offers to play this game with you, it's worth playing whatever the cost to play. In the long run the maths says



Conundrum
If you could play a game a second, how long would it take to play 2^{60} games? This is the number of games you might expect to play to break even in the Petersburg game if the entry price was £60.

that you will come out on top. But why is it that most of us wouldn't play the game for anything more than about £10?

It's called the St Petersburg Paradox after Nicolaus's cousin Daniel who, while working at the Imperial Academy of Sciences in St Petersburg, came up with the first explanation of why no rational person would pay any price to play the game. The answer is what any billionaire will tell you. The first million you earn is worth so much more than the second million. You shouldn't put in the formula the exact amount you win but what that prize is worth to you. In this way the price to play this game will vary according to how you value the outcomes. Daniel's resolution goes far beyond just the curiosity of a mathematical game: it is essentially the foundation of modern economics.

Go to www.mathematik.com/Petersburg/Petersburg.html for an online simulation of the game

Answer
More than 36 billion years. The universe is at most 14 billion years old — another explanation for why most people wouldn't pay an arbitrary price to play the game.

Marcus du Sautoy's Sexy maths

JOE McLAREN

Arithmetic eases swine flu worries

Arithmetic eases swine flu worries

As I lay in bed in a state of high fever I couldn't help googling "swine flu" for any other telltale symptoms that might add to my state of hypochondria. With the increase of cases in Australia, the World Health Organisation declared last week that swine flu, or the H1N1 virus as it's also called, was officially a pandemic.

The biologists are analysing the composition of the virus and trying to create a vaccine to combat the disease, but it is mathematics that now allows the scientists to understand how virulent it is.

Each virus is assigned a number, which is a measure of how quickly it spreads. The number for Aids is between 2 and 5. Measles is much more contagious with a number between 16 and 18. Estimates for the number corresponding to swine flu is somewhere between 1.2 and 1.6. This is much smaller than normal influenza, which is between 1.5 and 3.

Even with a small infection rate, it is remarkable how quickly the disease can spread. If a contagious person infects 1.2 people each day then it takes only 125 days to infect the whole population of the Earth. Such is the power of exponential growth. Of course we don't expect everyone in the world to contract swine flu, and that is why you need a more subtle mathematical model that takes into account other factors in the spread of such a disease.

For a start, once people have the disease they will either die or recover and become immune. So they will remain contagious only for a fixed period rather than continually infecting people. As the disease spreads, more of the population will become immune. To run the model it is important to know the proportion of the population who are likely to be susceptible to the disease. This means that the number corresponding to a spreading virus will change over time. Once it drops below 1, the mathematics implies that the disease will die out. If it stays above 1 then the model shows that eventually the number of infected people stabilises to a fixed amount. In this case we say the disease has become endemic. For example, chicken pox in the UK is an endemic virus.

It is by using such mathematical models that you can assess the effect of immunising the population against the spread of a virus. A vaccine decreases the proportion of the population susceptible to the disease. Immunise enough people and you can see the infection rate drop below 1 leading to the eradication of the disease. Smallpox had an infection rate of 4 but by immunising at least three quarters of the population the disease has died out.

The reason that measles has started to spread again in the UK is because the mathematical balance has been upset between the proportion of the population immunised and the high infection rate for measles at between 16 to 18. On the other hand, the model reveals that only 5 per cent of the population need be treated with antiviral drugs to counter the pandemic effect of a flu virus with contagion rate of 1.9.

The model is also helpful in assessing the effect of introducing travel restrictions to try to contain the spread of diseases. Research last year on the possible spread of avian flu revealed that drastic travel limitations would have the effect of delaying the pandemic's evolution only by a few weeks with almost no effect on the mortality rate. Compared with the huge economic disruption that such travel restrictions would inflict, the mathematics has led to the conclusion that travel restrictions are of little use in combating viruses such as swine flu.

So it was reading about the mathematics of the spread of the virus that proved the best antidote to my bout of hypochondria last weekend. And sure enough, by the beginning of the week, the fever had dropped and I was feeling fine.

A spoonful of maths is all it took to see me through the worst of it.

Conundrum

An abbot visits a monastery and says that at least one of the monks there has a deadly but non-contagious disease. The only symptom is a red dot that will appear the following day on the forehead of those infected. As they will die an agonising death within months, he urges those infected to kill themselves as soon as they know. But they have sworn an oath not to communicate with each other and there are no mirrors and they meet only once a day, at lunchtime. One month later, the abbot returns to find that all the infected monks killed themselves on the same day. How did they know they were infected and why did they die on the same day?

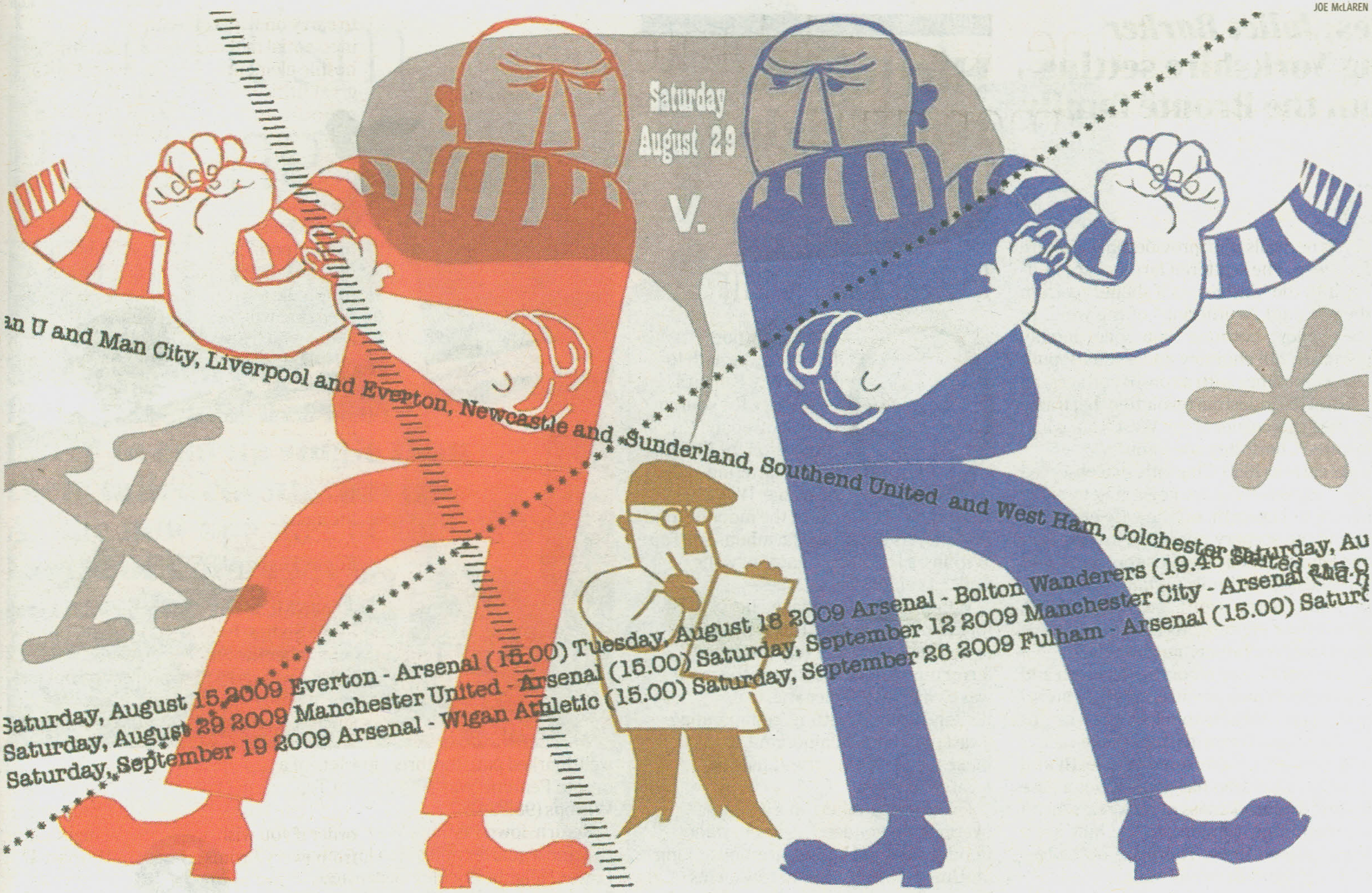
Answer

Suppose there was only one monk who had the disease. On the day after the abbot leaves the monk sees no one else with a red dot on his forehead. Since at least one person is infected he knows it is him and he kills himself. Suppose two monks are infected. The infected monks see only one other monk with a red dot. But since they are still alive on the second day, they realise that there must be another monk infected, or else the infected monk they can see would already have killed himself. Since the infected monk can see only one other monk infected he deduces that he must also be infected. So on the second day they both kill themselves. The logic extends from 2 to 3 to N monks. It takes N days to realise that N monks are infected.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN



How to avoid a grudge match

How to avoid a grudge match

This time last week I was awaiting the announcement of the fixture list for the next football season. Who would Arsenal be playing on the opening weekend of the season? Would our run-in next year be a series of easy games? When would we play the "big three"? And would we be at home on Boxing Day? For the past four seasons Gooners have been denied the chance to walk off their turkey with a trip to Highbury or the Emirates.

So a cheer went up in our house at the news that we have Aston Villa at home on December 26. But for those spectators still cursing their team's fixtures, don't be too hard on the scheduler. The task of constructing fixtures timetables is a complex mathematical problem.

There are many constraints. No team is allowed to play more than two home or two away matches in succession. Every five matches must have at least two away and two home games. Away fans should not have to travel too far from home on Boxing Day. There should be variety from one season to the next (something that Liverpool will be doubting as, for the sixth year in succession, they start their campaign away from Anfield). And the most complex issue of all: if Arsenal are playing at home, then our arch-rivals Spurs have to be away to avoid fans clashing in North London.

In fact, every club is allowed to nominate teams who should not be playing at home when they are. There are, of course, the classic local rivalries: Manchester United and Manchester City, Liverpool and Everton, Newcastle and Sunderland (which is not a problem this season, after Newcastle's relegation to the Championship). But there are more curious pairings that need to be avoided. Southend United believe that their home crowd falls dramatically if West Ham play at home on the same weekend. Colchester United share stewards with Ipswich Town, which makes hosting home games on the same weekend impossible. Such factors led this season to 11 clubs being involved in a circle of teams that had to avoid each other.

Trying to pick the teams to play at home each weekend is like trying to arrange party invitations so that you don't invite anyone who won't get on. Suppose you have 200 friends. You want to invite 100 to your party. But there are certain combinations that won't work: you can't invite Andrew if you invite Bryony because they split up last week. You can't invite Carlo if you invite Dave because they always get into a fight. Given all these forbidden combinations, is there an efficient way to find 100 people whom you can invite to the party without the evening ending in tears?

> "There is a million-dollar prize on offer for anyone who can unravel the complexity of this sort of challenge"

One way is systematically to work your way through all the combinations of 100 people whom you could pick from your 200 friends, then check that there are no bad pairings. So how many different parties are there with 100 guests? Well, you could pick one of 200 people for the first guest. The second guest is chosen from the remaining 199. So that's already 200x199 possibilities — although you have to divide this by two, because inviting Andrew and then inviting Carlo gives you the same party as inviting Carlo, then Andrew. So the total possible number of parties is 200x199x198 . . . 101 divided by the number of times you've invited the same party, which turns out to be 100x99x98x . . .x2x1. So the number of possible parties you have to check has a staggering 59 digits. If the parties were held night after night, you would have got through only about 5 trillion parties — a number with 13 digits — since the Big Bang. Even a computer checking a party every nanosecond would take longer than the lifetime of the Universe to check through every possible party.

The trouble is that mathematicians haven't found a much more efficient way to produce a party that won't end in a fight, beyond checking one possible party after another. In fact, mathematicians believe that there may be no simple algorithm to find the party that works beyond a trial-and-error method, and a million-dollar prize is on offer for anyone who can unravel the complexity of this sort of challenge, which is called the NP versus P problem.

So if you are still cursing the scheduler for next season's football fixtures, give him a break. He is doing battle with one of the great unsolved problems of mathematics.

Conundrum

At the party with 100 guests, everyone shakes hands with everyone else. How many handshakes take place?

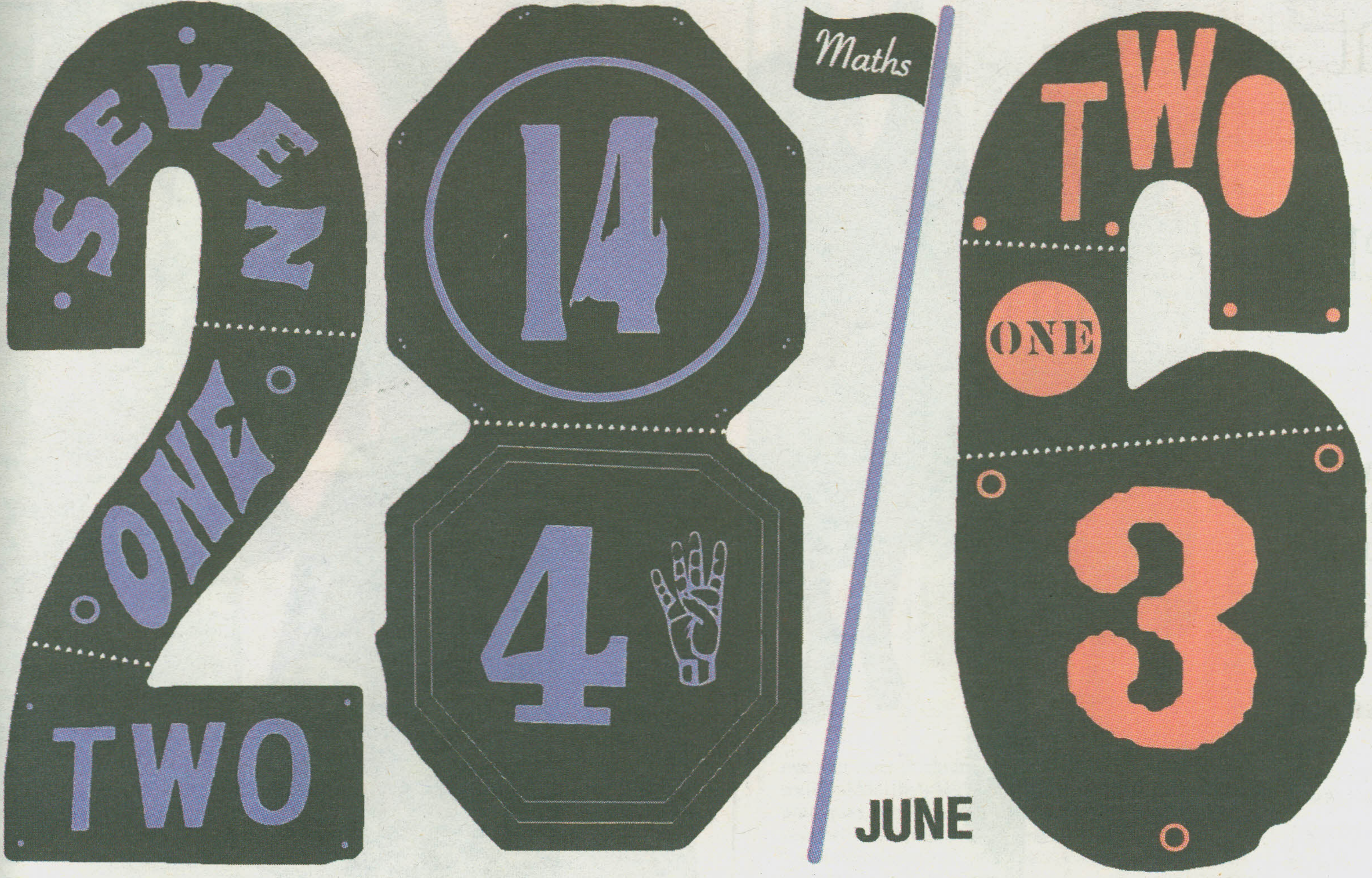
Answer

Each of the 100 guests shakes 99 other people's hands. That's 100x99. But we have counted each handshake twice. Andrew shaking hands with Carlo is the same as Carlo shaking hands with Andrew. So the total is 100x99÷2=4,950 handshakes.

timesonline.co.uk/thewaywelive

Marcus du Sautoy's Sexy maths

JOE McLAREN



Oh, it's such a perfect day

Oh, it's such a perfect day

I don't know whether you noticed but June seemed to have more "World ---- Days" than any other month, with the subjects varied from Oceans (June 8), Blood Donors (June 14) and Child Labour (June 12) to Falling Over (June 23), which was organised by Help the Aged. The month has become so crowded that, on the 20th, jugglers, surfers, badgers and refugees were all trying to persuade us to celebrate their World Day.

As a mathematician I was beginning to feel a bit left out. We've got Pi Day, which we celebrate on March 14 (3.14 being the first few digits of pi), but how about adding a World Maths Day to the June calendar? On Sunday, when I looked at the date, it suddenly dawned on me why 28/6 would be the perfect day to celebrate World Maths Day — literally the perfect day because 28 and 6 are what mathematicians call perfect numbers.

A number is called perfect if, when you take all the smaller whole numbers that can be used to divide the number and you add them together, you get the original number. For example 6 is divisible by 1, 2 and 3. Add these together and you get 6. Similarly, 28 is divisible by 1, 2, 4, 7 and 14, which add up to 28. These are the first two perfect numbers. The next is 496.

Perfect numbers have been studied since ancient times and were regarded as having mystical significance. The 4th-century philosopher Saint Augustine believed that God created all things in six days precisely because the number 6 is a perfect number. In Jewish mysticism, it was because 28 was a perfect number that the mystics believed that God chose it as the number of days that it takes the Moon to travel round the Earth, so defining the length of the Jewish month.

But it was the great Greek mathematician Euclid who discovered the exciting connection between perfect numbers and another important sort of number: the primes. By adding up powers of 2 he found that whenever the answer was a prime number then there was a way to use that prime number to get a perfect number. For example, 1 + 2 + 4 = 7, which is a prime number. If you multiply that prime by the last power of 2 in the sequence you get a perfect number. In this case 7 x 4 = 28. The next time you get a prime by adding up powers of 2 is 1 + 2 + 4 + 8 + 16 = 31. Multiply 31 by 16 and you get 496, the third perfect number.

Euclid was able to prove that this always worked. Whenever the powers of 2 add up to a prime, you get a perfect number by multiplying the prime by the last power of 2 you added. But was every perfect number discovered this way? It wasn't until nearly 2,000 years later that mathematicians were able to give a partial answer to this. Both Descartes and Fermat wrote to the French monk Mersenne with their discovery that every even perfect number must come from the primes that are sums of powers of 2.

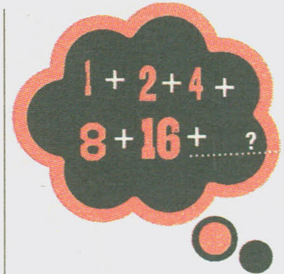
> "So far we have discovered 47 perfect numbers, the largest of which has nearly 26 million digits"

It is these strange connections between seemingly unrelated bits of the mathematical world that make mathematics such an exciting subject. Primes and perfect numbers don't look at first sight as though they have anything to do with each other. But Euclid, Descartes and Fermat's work reveal that they are two sides of the same equation.

Mersenne acted as something of a medieval internet hub, broadcasting discoveries across Europe, and it is his name that is now given to the primes that are got by adding powers of 2. Whenever a new Mersenne prime is discovered it leads to the discovery of a new perfect number. So far we have discovered 47 perfect numbers, the largest of which has nearly 26 million digits.

But there are still many mysteries that surround these perfect numbers. Are there infinitely many of them? Or is the 47th perfect number the last one? Thanks to Descartes and Fermat, we know how to get even perfect numbers. But it is still unknown how to get an odd perfect number. Indeed most mathematicians believe there is none.

So when June comes round again next year, and it's time to give thanks for the badgers and the oceans, why not celebrate a bit of mathematics on the perfect day: June 28.



Conundrum

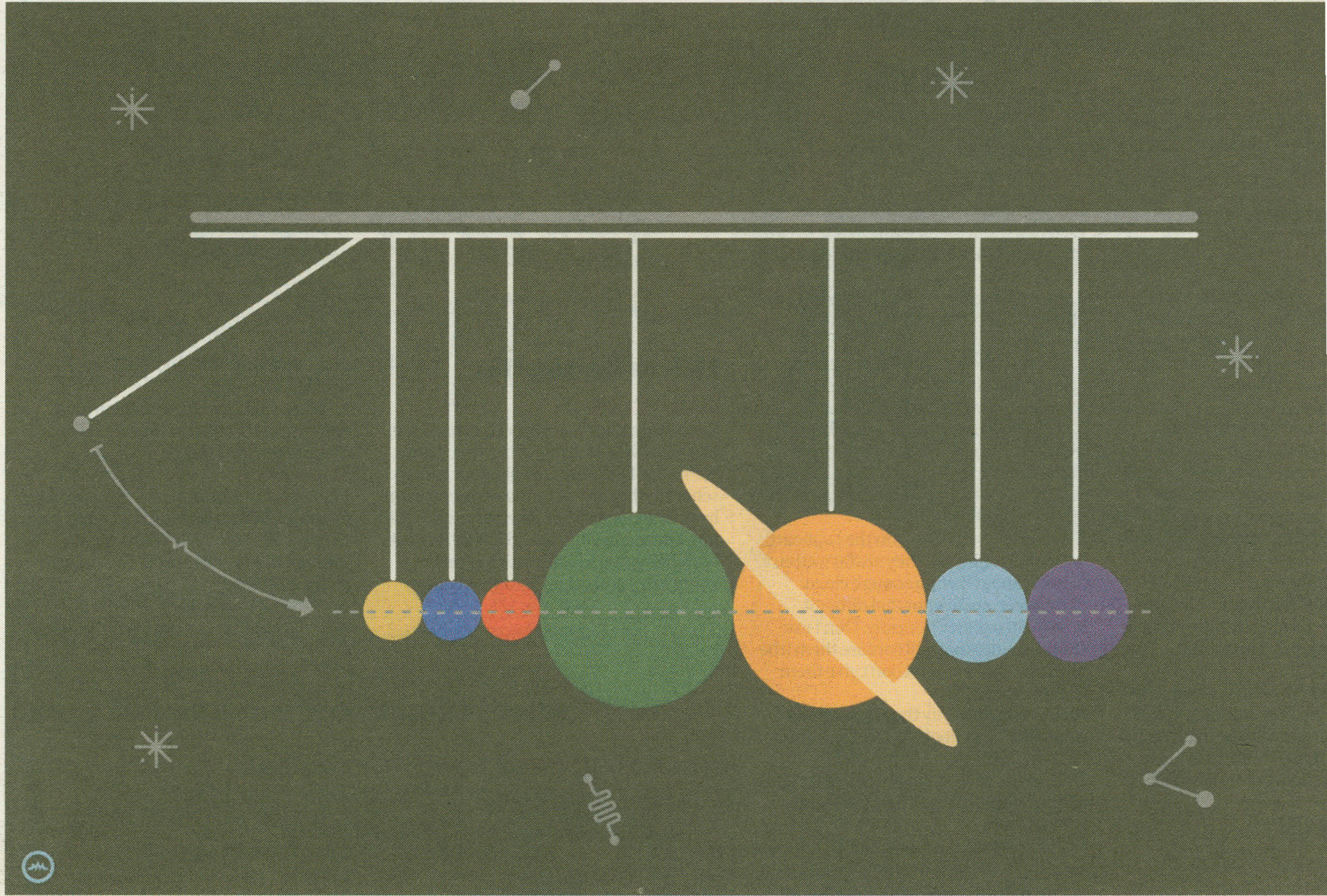
Use Euclid's discovery to find the fourth perfect number.

Answer

1 + 2 + 4 + 8 + 16 + 32 + 64 = 127, which is a prime number. So 127 x 64 = 8,128, which is the fourth perfect number.

Marcus du Sautoy's sexy science

LEANDRO CASTELEO / DUTCH UNC



If Mercury wobbles, hold on tight

If Mercury wobbles, hold on tight

For nearly five billion years the planets in our solar system have been orbiting the Sun like a beautifully tuned clock. Eclipses come and go with precise regularity. The transit of the planets across the Sun can be predicted with stunning accuracy. Indeed time itself has for millennia been measured by the regularity of the night sky. But how can we be sure that at some point the planets in the solar system don't fly apart in some catastrophic solar blow out?

In roughly five billion years the planets are due to be engulfed by the Sun as it runs out of fuel and evolves into a red giant but two French astronomers, Jacques Laskar and Mickael Gastineau, have been investigating whether the fate of the solar system might be sealed long before the Sun hits its expiry date. Because despite the stunning regularity of the planets to date, there is evidence that at some point the orbit of one planet, Mercury, could destabilise the solar system with devastating effect.

If our solar system consisted of just one planet orbiting the Sun then, Newton proved, they would process around each other tracing out ellipses that repeated themselves to the end of time. But when Newton tried to analyse three bodies, such as the Sun, Earth and Moon, he was stumped. He wrote "to consider simultaneously all these causes of motion . . . exceeds, if am not mistaken, the force of the human mind".

It was the French mathematician Henri Poincaré, who discovered at the beginning of the 20th century that, when it came to a solar system with more than one planet, good behaviour in the past was no guarantee of its future conduct. The reason three or more bodies are so difficult is that even very small changes in the position and motion of the planets can result in huge deviations from current orbits. It was the discovery of one of the most important mathematical concepts of the past 100 years: chaos theory.

The mathematics of chaos underpins many natural phenomena: from the planets to population dynamics, from economics to the weather. Indeed the dramatic implications of chaos have been captured by the emotive expression "the butterfly effect". A small change in the Earth's atmosphere created by the flapping of the wings of a butterfly could result in a hurricane hitting the other side of the planet.

So is there the chance that a butterfly could destabilise our solar system? As Newton wrote, the problem is so complex that it is beyond the resources of the human mind. So Laskar and Gastineau have enlisted the help of the computer to run several thousand models of the future evolution of our solar system. And their experiments have identified a potential butterfly: Mercury.

> "Even a close encounter with a planet could be disastrous for Earth"

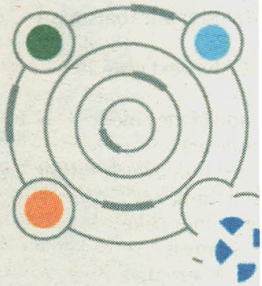
The simulations start by feeding in the records we have of the positions and motion of the planets to date. But it is difficult to know these with 100 per cent accuracy. So each time they run the simulation they make small changes to the data. Because of the effects of chaos theory, just a small change could result in a large deviation in the outcomes.

You might expect that if the solar system were going to be ripped apart it would be one of the big planets such as Jupiter or Saturn that would be the culprit. But actually the orbits of the gas giants are extremely stable. It's the rocky terrestrial planets that are the trouble makers. In 1 per cent of the simulations tiny Mercury posed the biggest risk. The models show that Mercury's elliptical orbit could start to extend owing to a certain resonance with Jupiter, with the possibility that Mercury could collide with its closest neighbour, Venus. In one simulation, a close miss was enough to throw Venus out of kilter with the result that it ended up colliding with the Earth.

Even close encounters with the other planets would be enough to cause such tidal disruption on Earth that the effect would be disastrous for life on the planet.

But before we abandon ship, the simulations show that it will take several billion years before Mercury might start to misbehave. For the time being, human interaction and not Mercury poses the most serious threat to the planet's survival.

Professor Marcus du Sautoy is Simonyi Professor for the Public Understanding of Science at the University of Oxford

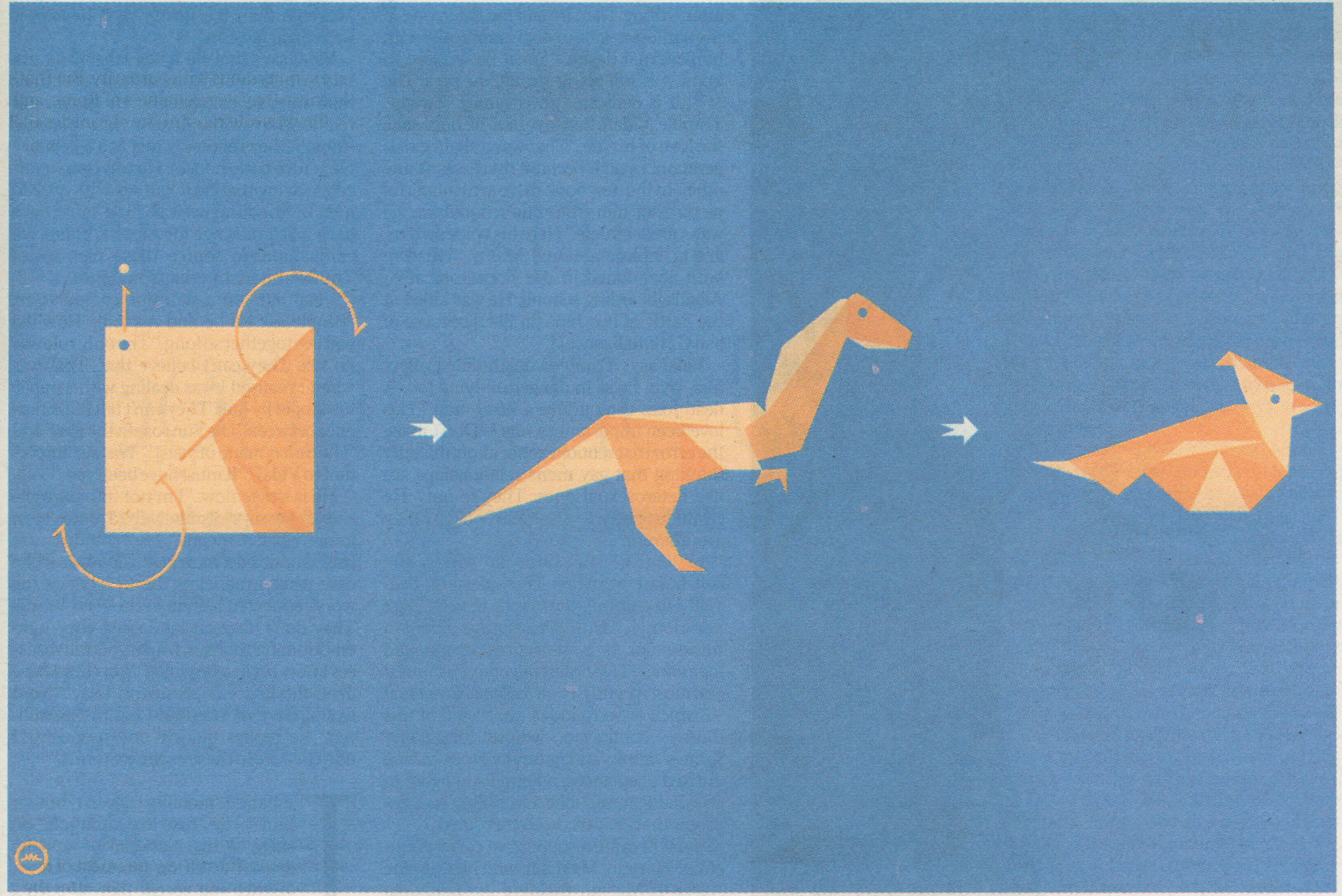


Cool fact

Catastrophe has already hit another solar system. Astronomers have observed three planets orbiting the Sun at the heart of Upsilon Andromedae, a star located 44 light years from our solar system. But the strange orbits indicate that in the past these planets combined to eject an unlucky fourth planet from its solar system.

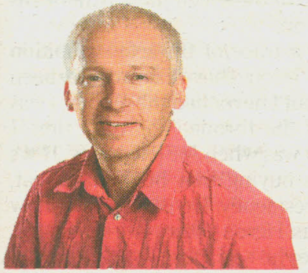
Marcus du Sautoy's sexy science

LEANDRO CASTELEO / DUTCH UNCLE



Of chickens and eggheads

Of chickens and eggheads



It has been a good few weeks for birds. New fossils discovered in China by Xing Xu, the Indiana Jones of palaeontology, has filled in vital gaps that link birds directly to dinosaurs in the evolutionary tree. The fossils help to solve an annoying problem that evolutionists have had in trying to make a convincing case for the link.

Embedded in the wing of a bird are the remains of an ancestral hand. The fingers seem to derive from the second, third and fourth digits of a hand which originally had five digits. The trouble has been that the three fingers on the closest dinosaur relative, the theropod, seemed to derive from the first, second and third digits. What has been missing was any evidence that fingers one, two and three in the dinosaur had transformed into fingers two, three and four hidden inside the bird wing. Xu's discovery plugs the gap.

Dating from 155 million years ago, the fossil is of a small theropod whose first digit has become stunted while the wrist bones show a developed fourth finger. Xu's fossil provides a snapshot of the transformation from dinosaur hand to bird wing. But it isn't the only bird breakthrough that ornithologists are celebrating.

A second advance on another bird mystery was announced this month in the journal *Nature*. It solves a problem that has baffled scientists for years: bird sex. The sex of birds, like that of human beings and other mammals, is determined by specific chromosomes. In humans, two X chromosomes make you female while an X chromosome combined with the degenerate Y chromosome make you male. But in birds it's the opposite. It's the male bird that has two Z chromosomes while female birds have a mixed ZW chromosome. The mechanism for how the Y chromosome in humans leads to male sex organs has been understood for some years. The Y chromosome contains a specific gene called SRY which is responsible for triggering testis development. It's like a switch that throws the embryo into male mode.

But in birds the mechanism was a mystery. There didn't seem to be any specific gene on the single W chromosome that was specifically linked to determining the female sex of a bird. The best candidate for determining bird sex was a gene on the Z chromosome called DMRT1 but since both male and female birds had the Z chromosome it wasn't clear how it could work.

Recent groundbreaking research on chicken embryos carried out in Australia by Craig Smith and a team of scientists has provided convincing support for the DMRT1 gene being responsible for sex in birds. The key was to realise that one copy of the Z chromosome could not produce enough gene product to make a testis. Two copies of the Z chromosome were enough to push the chicken embryo away from the default female pathway. Instead of a switch, it is more like a set of scales that need enough weight before they will tip to the male side.

> “In experiments, gonads looked more like ovaries than testes”

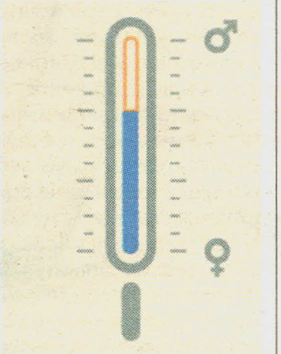
The Australian-based team demonstrated the mechanism by taking embryos with ZZ chromosomes and inserting genetic material that would inhibit the effect of the DMRT1 gene. Sure enough, what they observed in the subsequent development of the embryo was that the gonads looked more like ovaries than testes.

The experiment is related to the recent news stories about the sexual status of the South African athlete Caster Semenya. People can be born with XY chromosomes but problems with the SRY gene on the Y chromosome can result in the testes not developing fully and the person being left with feminine characteristics.

The two breakthroughs on the science and evolution of birds are not unrelated. The mechanism in birds for determining sex appears to have ancient roots. It is also found in some reptiles and in egg-laying mammals such as the platypus that diverged from all other mammals in the evolutionary tree some 166 million years ago. In fact the human mechanism for determining sex involving the action of one gene on the Y chromosome seems to have evolved quite recently.

Although Cole Porter was right that humans and birds, and even educated fleas all do it, it transpires that we do it in very different ways.

Professor Marcus du Sautoy is Simonyi Professor for the Public Understanding of Science at the University of Oxford

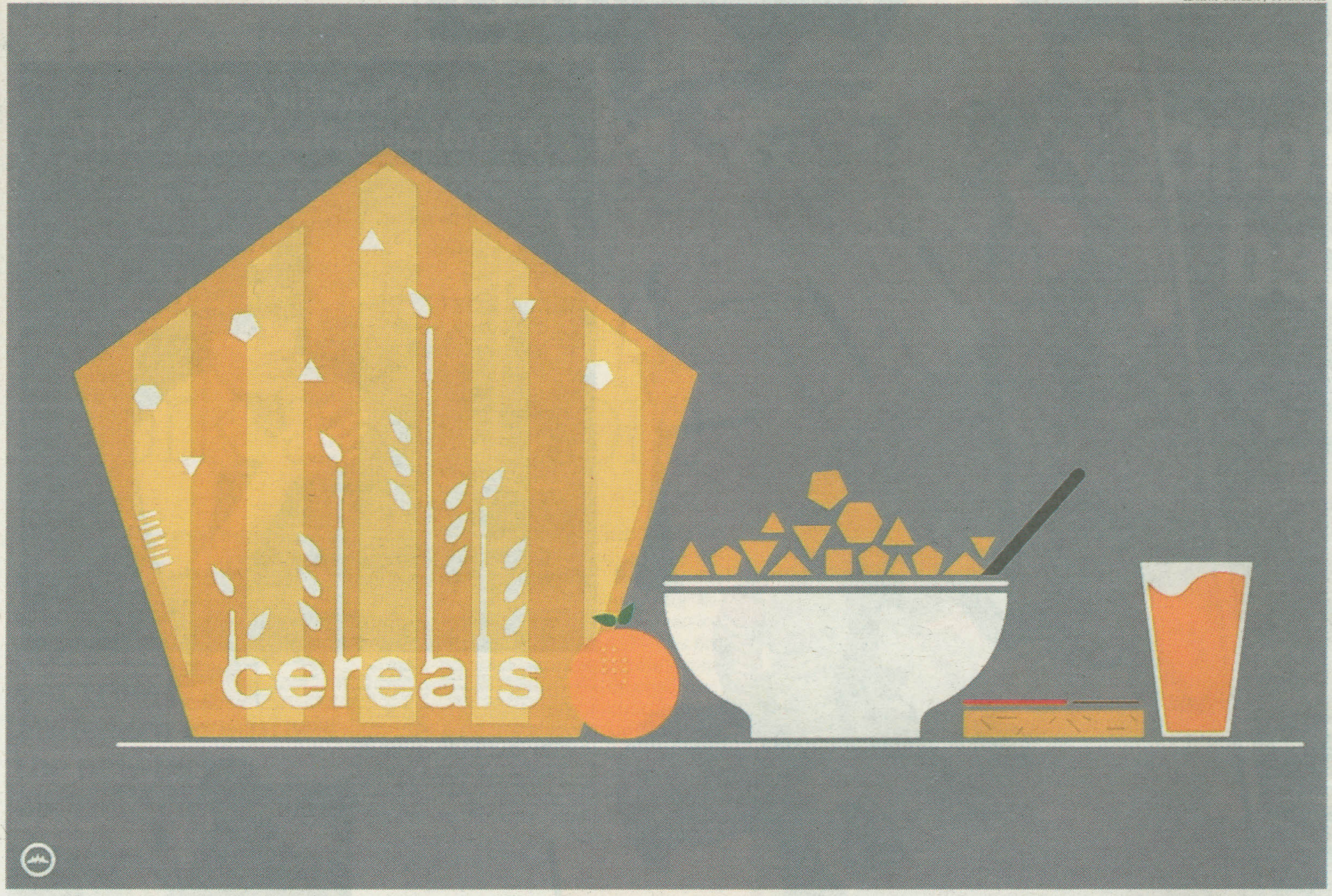


Cool fact

In turtles it is the temperature of the eggs during incubation that determines the sex of embryos. Cool eggs and you get male turtles. Female turtles arise from hot eggs. In crocodiles, females arise from hot or cold eggs, while male crocodiles thrive in eggs between these two extremes.

Marcus du Sautoy's sexy science

LEANDRO CASTELAO / DUTCH UNCLE



The shape of things to pack

The shape of things to pack

While perusing *The Times* at the breakfast table this morning your eyes might have drifted over to the box of breakfast cereal. As well as being packed with nutrition information, among the small print you will also find the following phrase: "sold by weight, not volume; settling of contents may occur during transit". What you might not realise is that knowing just how much the cereal can pack down is a problem that has challenged geometers for centuries.

Recent research by Salvatore Torquato and Yang Jiao, two scientists at Princeton University, has made significant progress at determining the most efficient way that certain shaped cereals can be arranged.

If your cereal is perfectly spherical, for example chocolate balls of cocoa puffs, then the famous 17th-century astronomer Johannes Kepler conjectured that the best way to pack the balls into the box is to use the hexagonal stacking used by the grocer to pile up oranges, which fills a little more than 74 per cent of the cereal box.

Incredibly, it took until 2005 for Thomas Hales to publish a proof that no other configuration could pack the cereal box better than Kepler's guess four centuries earlier. With Kepler's conjecture proved, Torquato and Jiao have turned their attention to different-shaped cereal. Instead of spherical balls, they have been considering how to pack symmetrical shapes called Platonic solids.

Platonic solids have symmetrical faces all the same shape arranged in a perfectly symmetrical fashion. They are the ideal shape for dice because no face is favoured over any other. The ancient Greeks proved that there are five possible dice: the cube, the tetrahedron (a pyramid made up four equilateral triangles), the octahedron (made from eight triangles), the icosahedron (consisting of 20 triangular faces) and the dodecahedron (a shape with 12 pentagonal faces).

Plato believed that they were so fundamental that he associated them with the building blocks of matter: earth, wind, fire and water. Although the Ancient Greeks were mistaken in their understanding of what matter was made from, nevertheless modern scientists have found these shapes at the heart of how molecules are put together. From diamond to the Aids virus, the Platonic solids are key to many of the shapes seen through the microscope. And this is why the question of how efficiently these shapes pack is not just important to cereal manufacturers but is of fundamental scientific interest.

" Platonic solids are key to many shapes, from diamond to the Aids virus "

One of the Platonic solids is easy to analyse. Cubes can be packed so that they leave no space unfilled, a fact that probably influenced Oxo's choice of the shape for its product. But what about the other four shapes? How much space can Tetley fill with its tetrahedral pyramid-shaped tea bags?

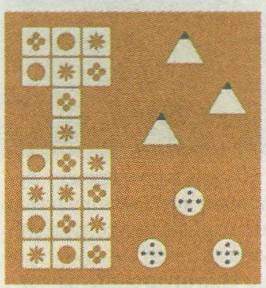
In the case of the octahedron, dodecahedron and icosahedron, Torquato and Jiao's analysis could not beat previous records. Stacked cleverly enough, it has been discovered that these shapes will fill respectively about 94.7 per cent, 90.4 per cent and 83.6 per cent of space.

Their surprise came with the simple tetrahedron. This shape has a slightly different feature to all the other Platonic shapes. If you spin any of the other shapes by 180 degrees, they look the same. But with the tetrahedron, the point of the pyramid points in the opposite direction. This lack of what is called central symmetry gives the tetrahedron more interesting ways to pack.

Torquato and Jiao's computer analysis revealed a new packing that beats all previous records. It improves on a configuration discovered by Elizabeth Chen last year, in which five tetrahedron are fused along an edge to create what Chen called a wagon wheel. By cleverly piecing together these wagon wheels Torquato and Jiao found that they could fill just over 78.2 per cent of space using tetrahedrons.

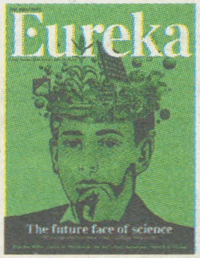
It is now conjectured that these arrangements are the best possible but it could still take another 400 years before a proof is discovered that nothing can beat them. Who knows how long we'll have to wait before someone successfully analyses what's happening in a box full of irregular cornflakes?

Marcus du Sautoy is the author of *Finding Moonshine* (Harper Perennial)



Cool fact

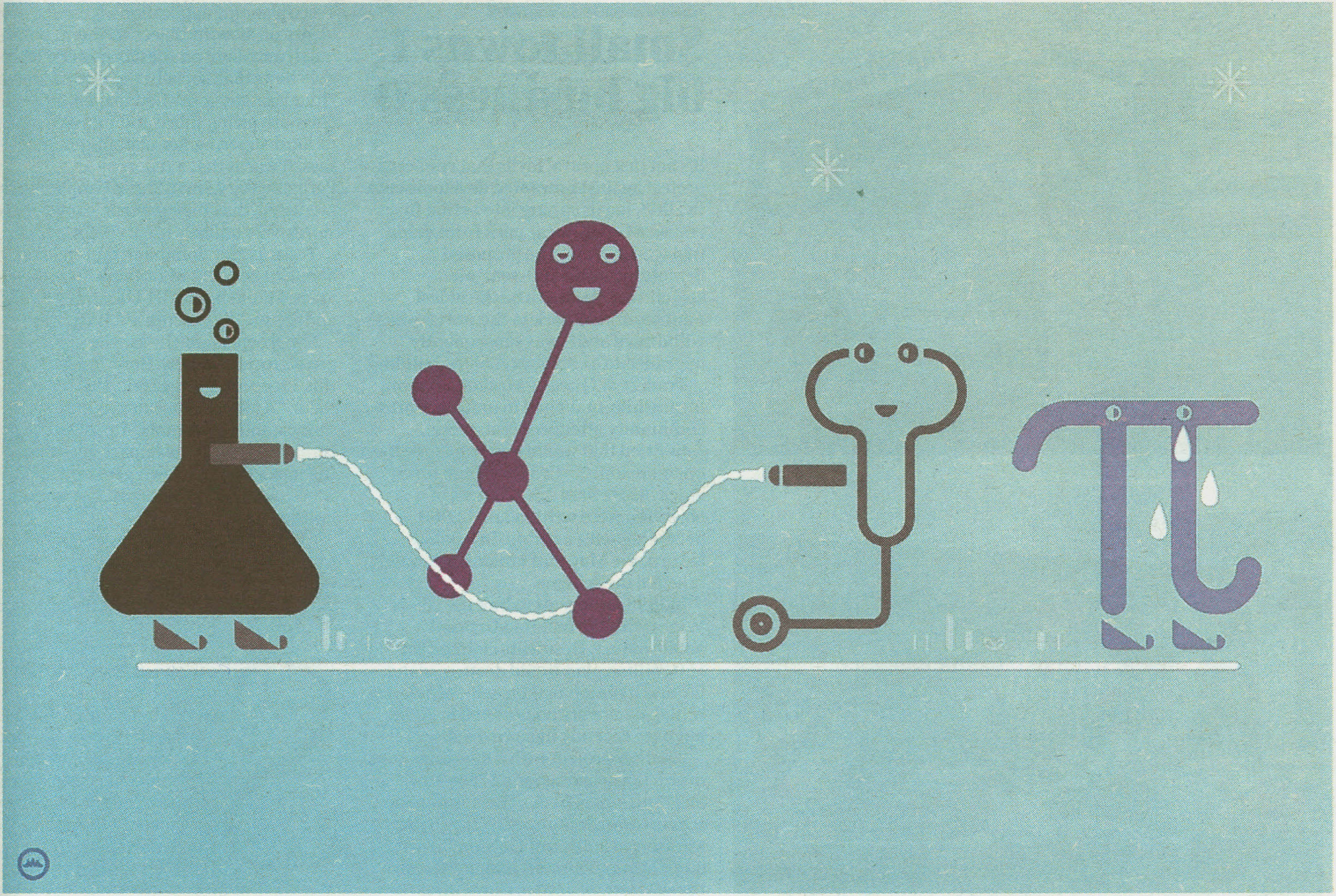
The first dice used in history was not a cube but a tetrahedron. Housed in the British Museum, the Game of Ur, dating back to 2600BC, used tetrahedrons with two of the four corners coloured black. To score, players would throw a pile of tetrahedron and count how many landed with a painted corner pointing up.



Eureka, a new magazine on life, science and the planet

Marcus du Sautoy's sexy science

LEANDRO CASTELAO / DUTCH UNCLE



Unleashing the power of the code

Unleashing the power of the code

As a mathematician I always feel rather envious of my scientific colleagues at this time of year. As the Nobel prizes were announced last week, Chemistry, Physics and Medicine each had its day in the limelight. But the omission of mathematics means that its own worthies have to sit on the sidelines while the rest of science is celebrated.

But as I read the Nobel citations, I couldn't help but pick out the interesting mathematical theme that ran through the three scientific awards: the power of codes to communicate information from one place to another.

Two of the prizes are related to one of the most important codes in science: DNA. The molecualr sequence — indexed by the letters A, C, G and T arranged along the length of the double helix of DNA — encodes all the information needed to create life. Understanding the mechanism of how this code works, though, has thrown up many puzzles.

The key component that transforms the passive code of As, Cs, Gs and Ts into active life is called ribosome. The structure of ribosome and how it translates the coded string of letters into different proteins in the body is at the heart of this year's Nobel Prize for Chemistry. The Nobel Prize for Medicine rewards a discovery that explains why this code doesn't get degraded every time DNA replicates itself. As the two strands of DNA in the double helix separate, the strands are not symmetrically replicated. One strand is copied continuously in the direction from which the DNA is being pulled apart. The other strand is copied in the other direction in fragments. But there was a problem: at the end where the DNA finally separates, it wasn't clear how the strand that is built in fragments could complete the last piece. There should always be a bit left uncopied.

The solution is that the ends do not have complex new information, but have a standard bit of code that can always be added on to complete the DNA. An enzyme called telomerase is responsible for adding the bit of code to the ends of the new pieces of DNA, making sure that the clumsy replication that happens down one strand does not degrade the DNA.

The Nobel Prize for Physics also celebrates the power of codes. For years scientists dreamt of using light waves to send information, rather than radio waves. The higher frequency of light means that it can transit more information than radio. But how to send light from one location to another without it being absorbed on the way? The solution is something called internal reflection. When light travels from water or glass into air it bends towards the surface from which it emerges — which is why your legs look shorter in a swimming pool. Bend it enough and at a critical angle the light doesn't appear out the other side but gets reflected back.

> “If Nobel were alive now he'd leave money for a prize for mathematics”

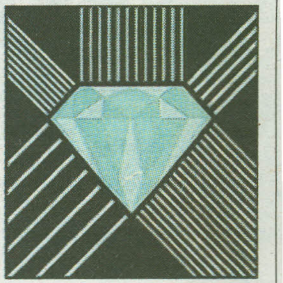
The fibre-optic cables below the surface of the Atlantic exploit the fact that light can bounce back and forth, never emerging from the cable until it reaches the end. To ensure that it keeps going the fibre has to have no impurities. Half of the Nobel Physics Prize goes to understanding how to make the fibre pure enough.

The other half of the Physics prize rewards a process that changes this light into a code, leading to the creation of the technology at the heart of digital cameras.

Some have criticised this year's choice of the Nobel Prize for Physics because it rewards the invention of technology rather than fundamental new insights into physics. But Alfred Nobel wanted the prizes to reward science that would benefit humanity. It is perhaps the reason that he omitted mathematics because, at the time, the subject did not seem that directed into helping society.

Yet many of the breakthroughs rewarded last week exploit fundamental mathematical techniques. Unravelling the geometry of the X-ray crystallographic images of ribosome; decoding the role of different strings in DNA; transmitting codes on light and turning light into codes — all are dependent on ideas of mathematics. If Nobel were alive today I'm sure he would have left money for a mathematical prize. Not that I'm biased, of course.

Marcus du Sautoy is appearing at *The Times* Cheltenham Literature Festival on Friday at 10am.

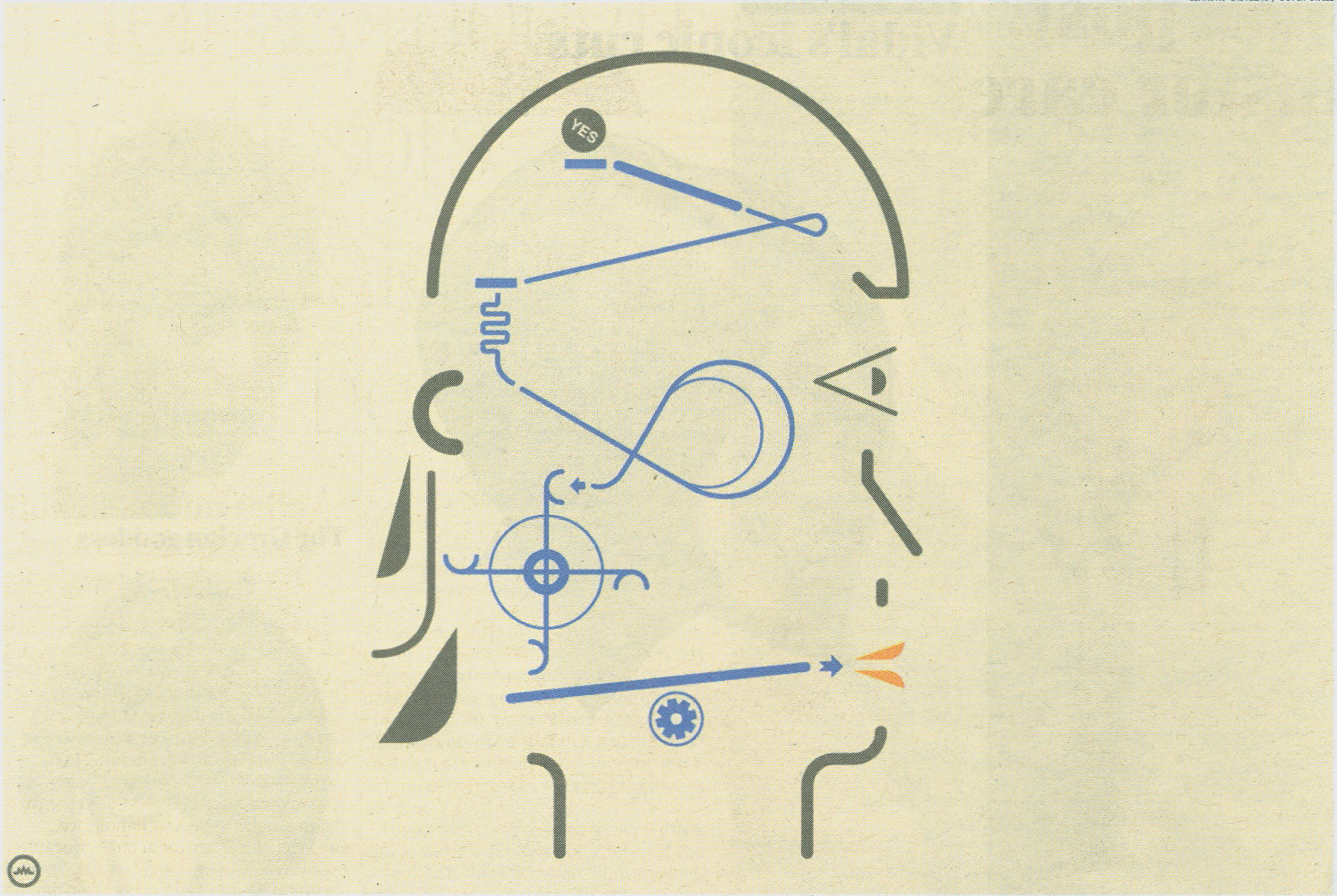


Cool fact

The reason a diamond sparkles so much is because the angle at which light is bent as it emerges from the stone is very large. If the diamond is cut correctly a lot of light gets trapped inside the stone and emerges only out of certain faces — giving it that sparkle

Marcus du Sautoy's sexy science

LEANDRO CASTELAO / DUTCH UNCLE

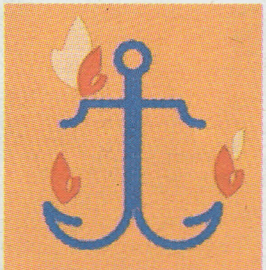


Is free will just an illusion?

machine scanned my brain activity as I made these random conscious decisions. This extraordinary piece of equipment has done for neurophysiology what Galileo's telescope did for astronomy 400 years ago. Just as the far reaches of our galaxy have come into view with ever more sophisticated telescopes, the fMRI scanner has allowed scientists such as Haynes to peer inside our heads and see what the brain is doing.

100 per cent certainty yet but the predictions that he is making are clearly above the hit-rate that you'd get if you were trying to guess. And Haynes believes that with more accurate imaging it might be possible to get close to 100 per cent accuracy.

It should be stressed that this is a very particular decision-making process. Obviously if you are in an accident your brain makes decisions in a split second



Is free will just an illusion?

I'm sure that you had the impression you were asserting your free will in your choice to read *The Times* today. That it was a conscious decision to stop on this page and start reading the Sexy Science column. But before you read on, let me warn you that the story I am about to tell could fundamentally change your perspective on whether you are freely making decisions or not. Because recently I took part in an experiment which has rocked my sense that I am consciously making the decisions I take. Free will it appears is just an illusion.

The experiment took place in Berlin in the laboratory of the British-born scientist John-Dylan Haynes. I was given a little hand-held console with two buttons, one to be activated with my right hand and the other with my left. I was asked to go into a Zen-like relaxed state and whenever I felt the urge I could choose to press either button.

While performing the experiment, I wore goggles containing a tiny screen on which a random stream of letters was projected. Each time I pressed a button, I would then be asked to record which letter was on the screen at the point that I had consciously made the decision, to press the right or the left button.

The experiment was conducted while I was lying in a functional magnetic resonance imaging scanner (fMRI) The machine scanned my brain activity as I made these random conscious decisions. This extraordinary piece of equipment has done for neurophysiology what Galileo's telescope did for astronomy 400 years ago. Just as the far reaches of our galaxy have come into view with ever more sophisticated telescopes, the fMRI scanner has allowed scientists such as Haynes to peer inside our heads and see what the brain is doing.

And what he discovered is that, by analysing my brain activity, he is able to predict which button I am going to press six seconds before I am consciously aware of which one I choose. Six seconds is a huge length of time. My brain decides which button I am going to press. Left or right. Then one elephant, two elephant, three elephant, four elephant, five elephant, six elephant. Now my brain throws the decision into my conscious brain and gives me the feeling that I am consciously making the decision.

Haynes can see which button I will press because there is a region in the brain that is lighting up six seconds earlier preparing the motor activity. A different region of the brain lights up according to whether the brain is preparing my left finger to press the button or my right finger. Haynes is not able to predict with 100 per cent certainty yet but the predictions that he is making are clearly above the hit-rate that you'd get if you were trying to guess. And Haynes believes that with more accurate imaging it might be possible to get close to 100 per cent accuracy.

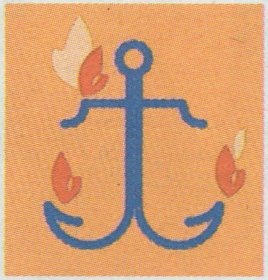
> "The brain has made a decision long before we realise it"

It should be stressed that this is a very particular decision-making process. Obviously if you are in an accident your brain makes decisions in a split second, making your body react without the need for any conscious brain activity. Many processes in the brain occur automatically and without our consciousness being involved, thus preventing our brain being overloaded with routine tasks. But whether I pressed the right or left button was not a matter of life or death. I sat there for about 20 seconds and then I freely made a decision to press the left button.

In practice it takes the research group in Berlin several weeks to analyse the data from the fMRI so I was safely back in the UK before Haynes actually knew what I was going to do next. But as computer technology and imaging techniques advance, by looking at the images from the fMRI, the potential is there for Haynes's consciousness to know which button I am going to press six seconds before I am consciously aware of the decision I think my free will is taking.

Although the brain seems unconsciously to prepare the decision a long time before we think we are asserting our free will, it is still not clear where the final decision is being made. Maybe we can still override the decision that our brain has prepared for us. If we don't have "free will", maybe we still have "free won't".

Marcus du Sautoy presents BBC Horizon's *The Secret You*, available to view on BBC iPlayer

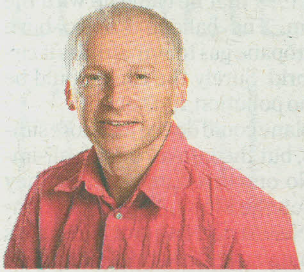


Cool fact

I was asked before I went into the fMRI whether I had any tattoos on my upper body (no, I don't have the Navier-Stokes equations tattooed on my arm). The reason is that the paint in older tattoos contains metal traces which can cause the tattoo to heat up in the scanner.

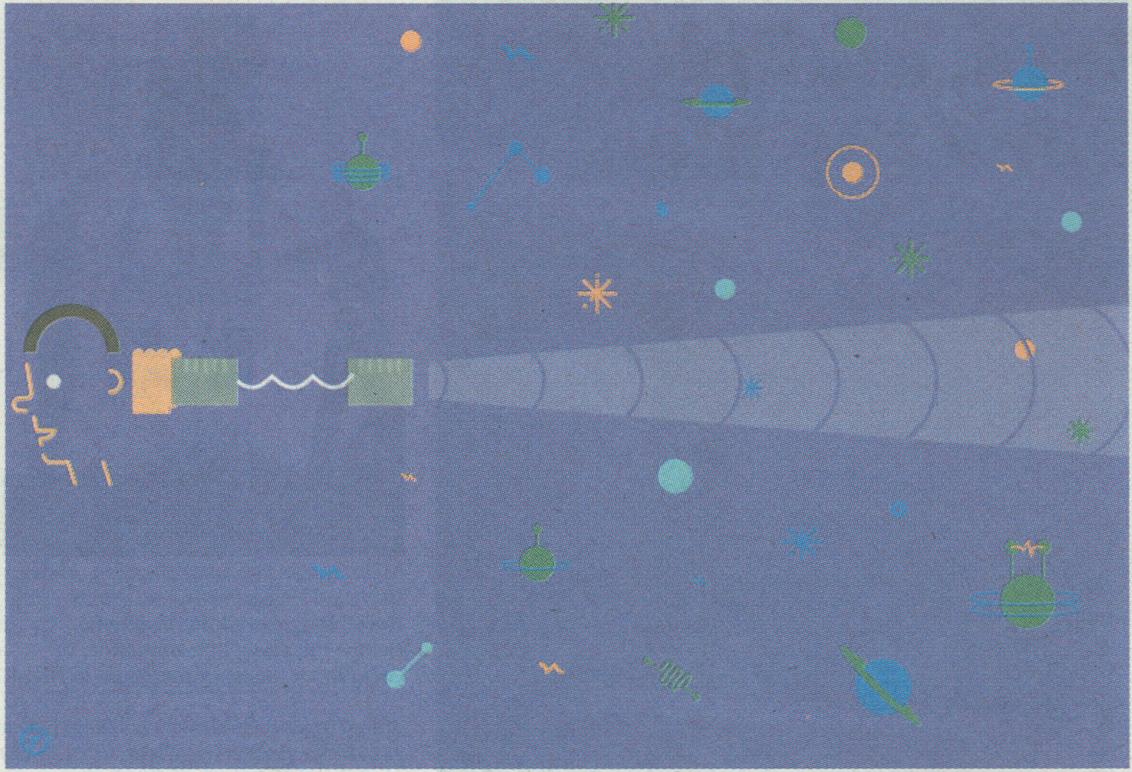
Marcus du Sautoy's sexy science

Waving hello to the little green men



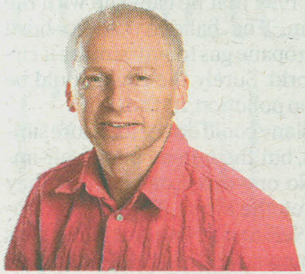
My favourite moment of science fiction comes

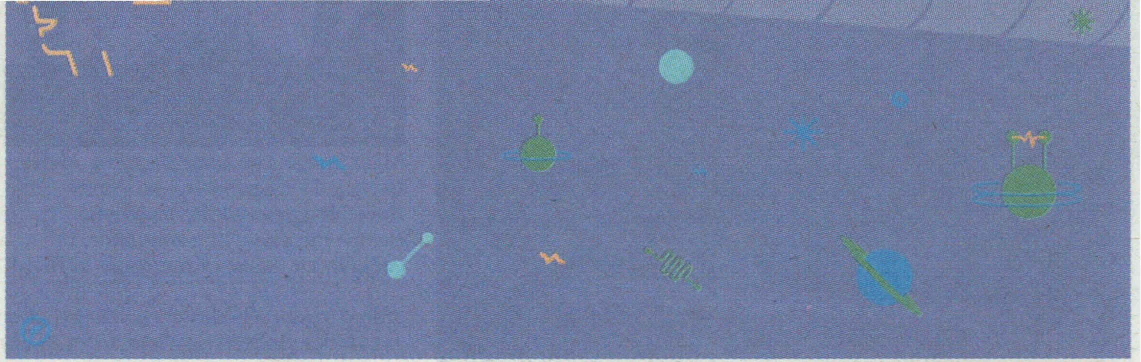
LEANDRO CASTELAO FOR DUTCH UNCLE



count the number of pulses: “ 59, 61, 67

funding in 1993. But this summer, thanks





My favourite moment of science fiction comes from Carl Sagan's novel, *Contact*. Ellie Arroway is a young scientist working at SETI (the Search for Extraterrestrial Intelligence). She spends her time listening to the background hiss of the Universe picked up by the large satellite dishes pointing into the deepest recesses of space. Like listening to an untuned radio, she hopes that at some point she may pick up a more interesting station.

As she sits listening to the white noise she is suddenly struck by an unusual beat, like a pulsing drum. The Universe is full of throbbing sounds. Some neutron stars spin crazily and emit a regular beam of radiation like a lighthouse, leading to these stars being christened pulsars. They were first observed in 1967 by Jocelyn Bell Burnell and Antony Hewish, who initially dubbed the first pulsar they detected in July 1967 LGM-1, thefirst part of which stands for little green men.

They never seriously believed that the beat was the dialling tone on a telephone from alien life trying to contact Earth. The regular pulse was a result of the rotation of the star every few seconds.

The explanation put paid to any fanciful suggestion that it was a message. But in Sagan's novel, Arroway's beat is not so easily explained away. She begins to count the number of pulses: "...59, 61, 67, 71... Aren't these all prime numbers?" Arroway asks excitedly. Sagan chooses the sequence of prime numbers because they are so unlikely to be generated in a natural manner. Sure enough, Arroway's excitement is justified: the primes are a way of the aliens saying hello. But not all the story is fiction; SETI really exists and celebrates its 50th anniversary this year.

For decades radio telescopes at facilities such as Arecibo in Puerto Rico have been scanning the night sky in the hunt for intelligent life. Apart from a strange signal picked up in 1977 that no one has been able to explain, we haven't picked up any evidence of anyone saying hello... yet. But as Jill Tarter at SETI counters, the amount of signal that has been analysed is small when compared with what's out there: "It's like dipping a small glass into the ocean and, on discovering it empty, concluding there are no fish in the sea."

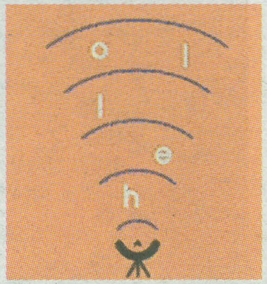
Radio astronomy is not the only science being used to detect life on other planets. Chemical spectroscopy is a process that analyses the light being emitted from distant objects. The wavelengths of light hold clues to the molecular structure of things on its surface. Scientists can use this to look for the fingerprints of life: oxygen, methane and water. But even if life is detected on another Earth-like planet, it will be SETI that has the best chance of determining how intelligent that life is.

The project has suffered for some years, after the US Government rescinded funding in 1993. But this summer, thanks to private donations, most notably from the Microsoft co-founder Paul Allen, a new array of 42 six-metre dishes in the high desert in California has begun combing the centre of our Milky Way for alien messages. Rather than a small glass, these dishes will be able to scoop up a much larger part of the sea of noise that permeates space. Over the next decade, the array should be able to scan more than a million stars.

In the 1960s, Frank Drake developed a mathematical formula that famously calculated how much intelligent life may be out there. The formula estimated that in the Milky Way alone there could be as many as 10,000 civilisations that may be broadcasting messages. The new array of dishes at Hat Creek, California, may just stand a chance of picking up someone saying hello.

And what about the chances of someone picking up radio broadcasts from Earth? It is almost 90 years since we started filling the airwaves with news bulletins, music and light entertainment. These broadcasts set out from Earth at the speed of light, expanding like a bubble of radio chatter. There are several thousand stars that are within that range, including 50 close enough to pick up radio broadcasts from 15 years ago.

Maybe even now there are little green men tuning in, trying to decode the shipping forecast or wrestling with the rules of *Just a Minute*.



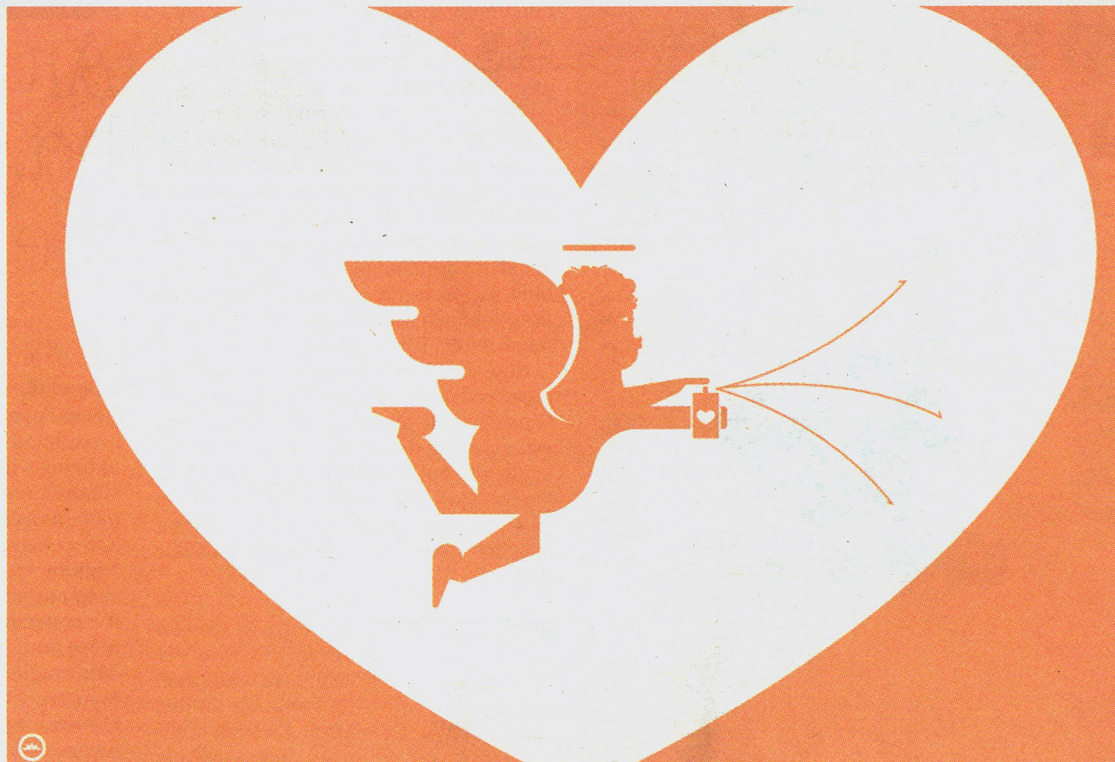
Cool fact

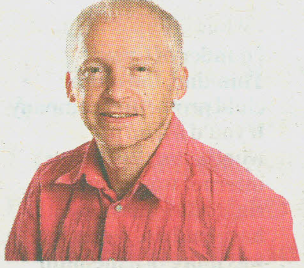
In 1974 the Arecibo radio telescope in Puerto Rico broadcast a message towards M13, the globular star cluster. M13 was chosen because of the huge number of stars that make up the cluster, increasing the chance that the message might reach intelligent life. To decode the message, however, aliens need to know their prime numbers.

Marcus du Sautoy's sexy science

Love is . . . a matter of sniffs and niffs

LEANDRO CASTELAO FOR DUTCH UNCLE





The manufacturers of Lynx deodorant know what they are doing. It's every nerdy boy's fantasy: a quick spray of scent and suddenly the opposite sex can't resist you.

The power of smell to attract mates has been known for millennia. The Ancient Greeks found that bitches secreted scent that alerted male dogs from miles around, but the science of chemical communication is relatively recent. Pheromones, the term for chemicals responsible for producing a reaction in other animals, was coined 50 years ago. It derives from the Greek for "transferring excitement".

The word was first used to mark the identification of a molecule responsible for attracting male silkworm moths. The German biochemist Adolf Butenandt analysed more than half a million moths over 30 years before he isolated the one molecule made by the female to attract a mate. It is so powerful, the secretion of just a few molecules can attract males from several hundred metres away.

Male moths need to be careful, though, because it is not only female moths that can manufacture this attractive chemical. The moth-eating Bolas spider has extraordinary powers of chemical synthesis and has learnt how to make exact copies of female moth pheromones.

The insect kingdom has developed an extensive chemical language for the communication not just of sexual information but a range of messages. Smell has advantages over sound and sight because it can operate at distance and around corners, is good at night and can last for a long time. Ants chemically label trails to tell the rest of the colony where food is. If a beehive is disturbed, the first sting by a bee releases chemicals that tell the other bees where to attack — so, if you are stung, always remove the sting: it is like a megaphone shouting to the other bees: "Sting here".

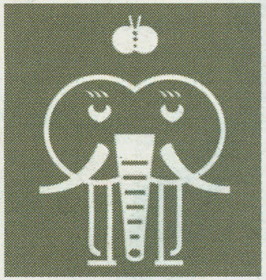
Research published last month in the journal *Nature* reveals how sophisticated a chemical dialogue is being conducted by some insects. Fruit flies produce about 30 pheromones. To understand the function of each, researchers in Canada and the US began by genetically engineering odourless fruit flies, using genetic tools to switch off the cells that secrete these molecules. To their surprise, male fruit flies were still hugely attracted to unscented flies regardless of gender. Basic attraction, it turns out, is not about smell but vision.

Pheromones refine this attraction. Fruit flies typically combine eating with sex — but the compost heap on which the flies feed is usually buzzing with many different species of fruit fly that all look very similar. It is important that a fruit fly does not waste its efforts on a fly of the same sex or the wrong species. The research reveals how a pheromone can be an aphrodisiac to one species but act as a deterrent to males of another species. Once paired up, the male who does not want to share his conquest with other flies will secrete a pheromone on the female during sex that acts to deter other suitors of the same species. However, the female has a counter-pheromone which overrides this chemical chastity belt, so as to increase her chances of getting her eggs fertilised. An extremely complex chemical conversation is at work.

It isn't only insects that use pheromones. Dogs, lobsters, goldfish and salamanders are all at it, and elephants produce odours that are complex enough for family members to identify each other through smell.

Of course, the ultimate challenge is understanding human pheromones. The human armpit produces such a complex cocktail of molecules that it is hard to isolate which one is doing the communicating. To complicate matters, research has found that it is often the combination of several molecules that communicates information; in isolation they do nothing.

Despite all the advertisements that bombard us on the internet and on television, scientists have still not isolated the chemicals responsible for human attraction. But as soon as a breakthrough is announced, you can be confident that the Sexy Science column will be the first to sniff out the story.



Cool fact

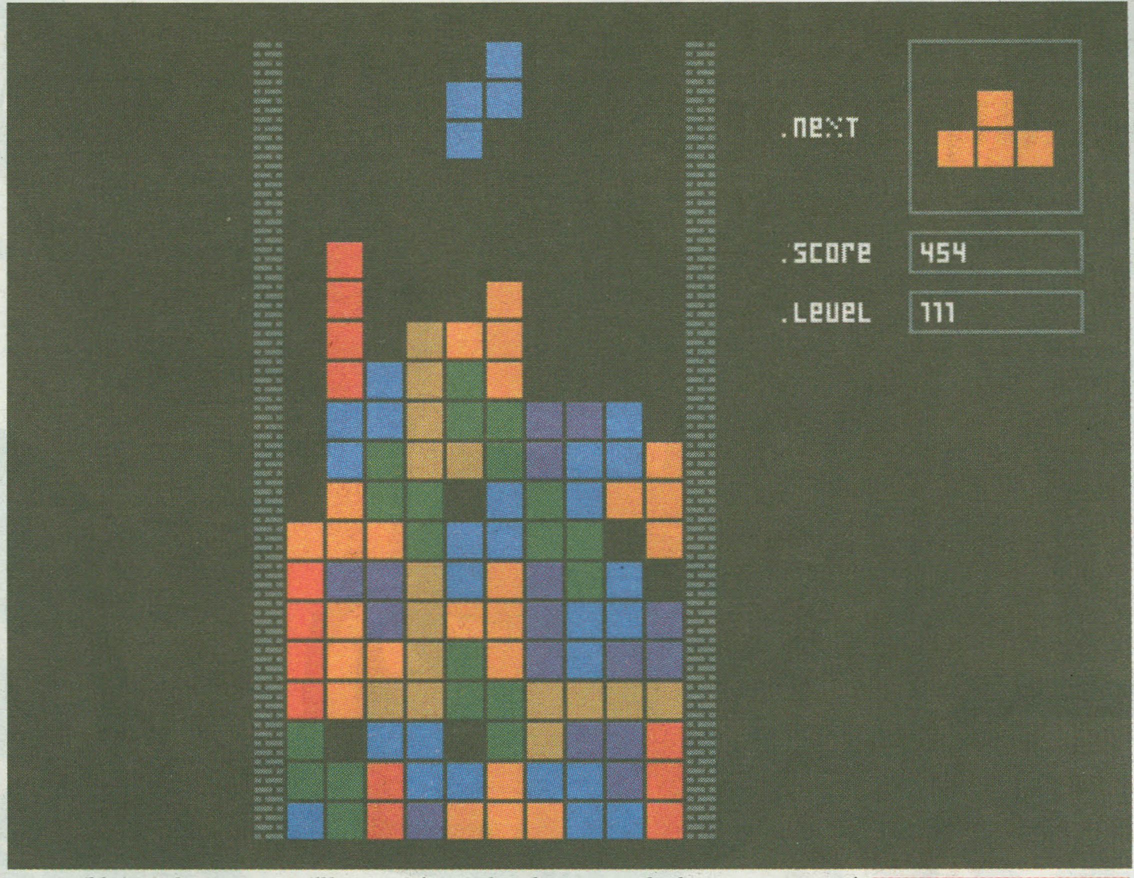
The Asian elephant shares the same female sex pheromone with 140 species of moth. Fortunately, other telltale signs ensure that an elephant and a moth don't end up as mates.

Marcus du Sautoy's sexy science

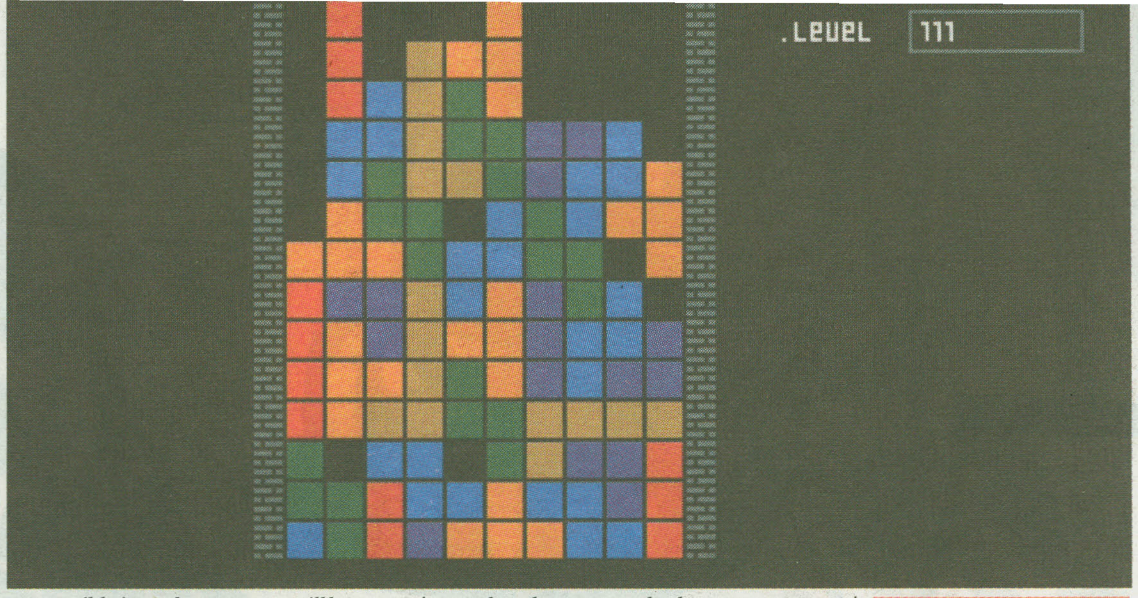
A benefit of being square

Look around you and you will see numerous examples of shapes repeating themselves left-right, up and down, back and forth. Glance down at the pavement and different shaped paving stones are pieced together in repeating patterns across the street. The bricks in the wall are arranged in periodically recurring designs. The tiles in your bathroom are more than likely glued to the wall in a regular fashion.

LEANDRO CASTELAO FOR DUTCH UNCLE



A benefit of being square



Look around you and you will see numerous examples of shapes repeating themselves left-right, up and down, back and forth. Glance down at the pavement and different shaped paving stones are pieced together in repeating patterns across the street. The bricks in the wall are arranged in periodically recurring designs. The tiles in your bathroom are more than likely glued to the wall in a regular fashion.

Nature too exploits similar patterns. The bee's honeycomb uses hexagons to fill the hive. The seeds in a pomegranate pack together in repeating patterns across the fruit. And on the molecular level, you find atoms packing together in regular formations. A diamond, for example, consists of carbon atoms that bind together in recurring patterns across the crystal.

The repetitive character of these structures is fundamental to being able to construct them so efficiently. The bricklayer doesn't want to think about a different rule for each new brick laid. Similarly the hexagons of the honeycomb mean that bees at one part of the hive are working to the same blueprint as their relatives on the opposite side.

If you look inside these different structures then you will see very specific sorts of symmetry. The bricks on the wall can be rotated by half a turn and they will match up. The square tiles in your bathroom will match up after a quarter of a turn. The hexagons of the beehive can be rotated by a sixth and a third of a turn and the cells will realign.

But rotations by a sixth, quarter, third or half a turn are all you will find. A mathematical theorem, discovered at the end of the 19th century, proves that if a structure has more than one axis of rotational symmetry then there is a basic building block that repeats itself in different directions across the structure, and the only rotational symmetries that are possible in such structures will be a sixth, quarter, third or half turn.

So scientists have been excited by the publication last month of research from a team led by Dmitri Talapin, a Chicago physicist, demonstrating that iron oxides mixed with gold naturally assemble themselves into structures that seem to have forbidden rotations of a twelfth of a turn. These so-called quasi-crystals are not new. Similar strange crystals with rotations of a fifth of turn were discovered by Dan Shechtman in 1982, and over the past two decades scientists have manufactured a whole range of exotic crystals with interesting new properties. The surprise in this latest research is that these exotic crystals also seem to occur naturally without scientists forcing their structure.

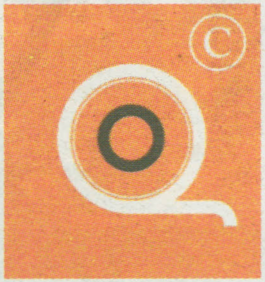
But if mathematicians have proved you can get rotations of only a sixth, quarter, third or half a turn, what is going on? Did the mathematicians get it wrong? The point is that there is only one place where there is an exact symmetry of a fifth or twelfth of a turn. The other rotations are not exact symmetries. Atoms almost line up but not quite. Also, in contrast to the shape of diamond or honeycomb, the atoms in these quasi-crystals do not repeat themselves regularly across the structure.

The mathematical blueprints underlying these crystals were discovered in the Sixties although there is evidence of similar designs in Muslim architecture from 500 years earlier. Roger Penrose constructed two rhombuses, squashed squares, one with internal angles of 72 and 108, the other with angles of 36 and 144, which can tile a wall with no gaps but only in such a way that they never repeat themselves in a periodic fashion. But there is no easy rule for how to stick these tiles together. If you start arranging them there are some configurations that eventually cannot be continued. It is only by globally analysing the shapes as they are laid out that a mathematician knows how to add the next tiles.

So how does Nature know how to build these structures? If the basic building blocks don't repeat themselves periodically across the structure, how can a crystal know what is happening on the other side of the structure in order to continue its growth? This is one of the mysteries that this new research might help to elucidate.

Until we have an answer, if you are thinking of changing those boring square tiles the bathroom and experimenting with some of Nature's more exotic structures, just be sure to have a mathematician on hand to help you to decide which tile to glue on next. But don't expect much more help on the DIY front. Mathematicians don't like getting their hands dirty.

Marcus du Sautoy is a professor of mathematics at the University of Oxford. He will be in conversation with Apostolos Doxiadis, the author of Logicomix, at the Institute of Contemporary Arts in London this Thursday



Cool fact

There is a copyright on the use of Sir Roger Penrose's non-periodic tilings, something Kleenex learnt the hard way when it used the design on toilet paper. It had to settle a case, brought by Penrose, out of court after his wife discovered the designs on rolls she bought from a supermarket.

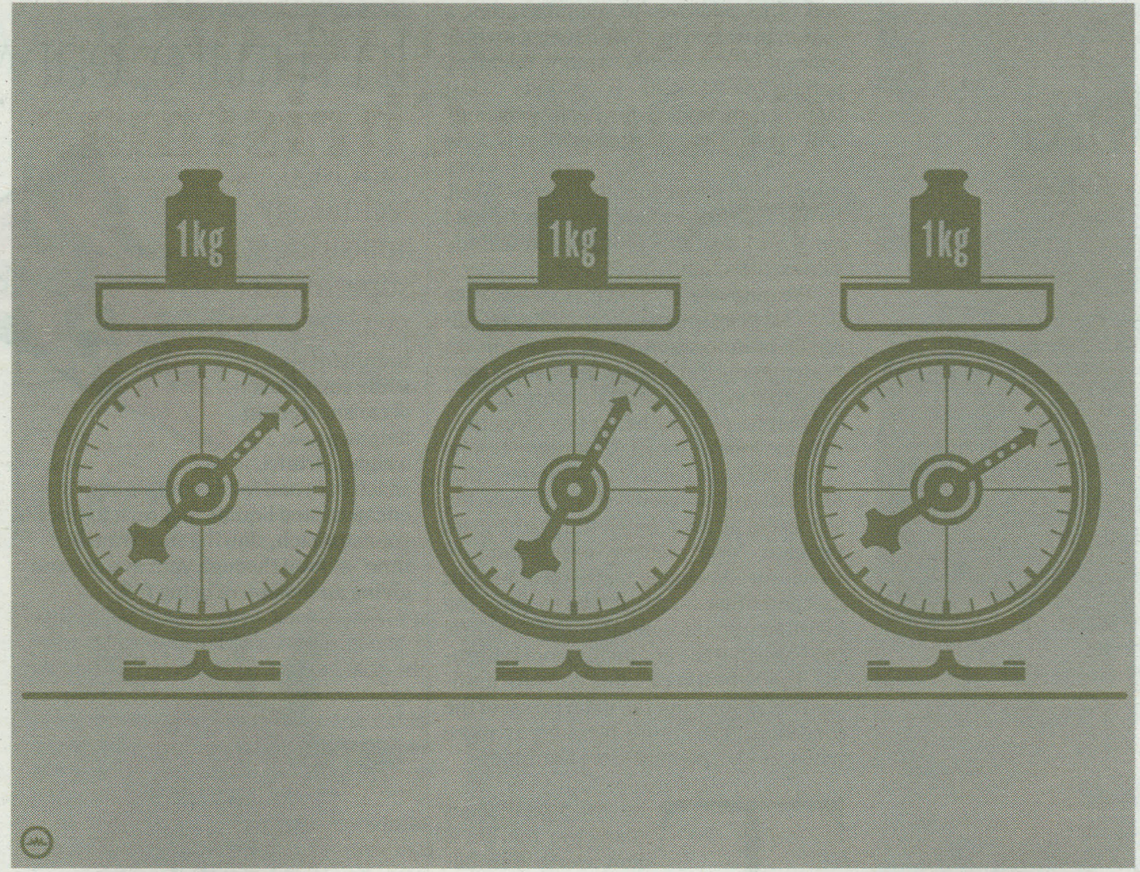
Marcus du Sautoy's sexy science

Measure for vague measure

I am 1.76m tall, weigh 68kg and at noon today will have been alive for 1,395,819,600 seconds. But what precisely is a metre? Who decided how heavy a kilogram is and how long is a second? A meeting in October 2011 of the Consultative Committee for Units at the next Conférence générale des poids et mesures is going to tell us. And the kilogram in particular could be about to change.

The earliest forms of measurement date

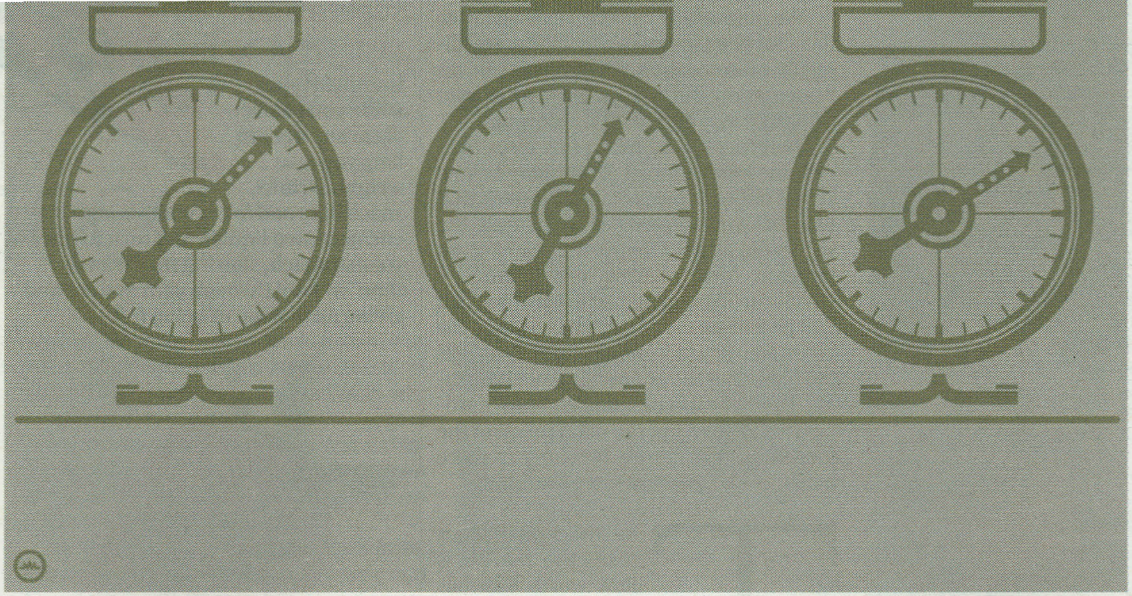
LEANDRO CASTELAO FOR DUTCH UNCLE



times and they had to endure many

second is getting longer over time. So in

Measure for vague measure



I am 1.76m tall, weigh 68kg and at noon today will have been alive for 1,395,819,600 seconds. But what precisely is a metre? Who decided how heavy a kilogram is and how long is a second? A meeting in October 2011 of the Consultative Committee for Units at the next Conférence générale des poids et mesures is going to tell us. And the kilogram in particular could be about to change.

The earliest forms of measurement date back to the Ancient Egyptians when body parts were used as units. An Egyptian cubit was the length from your elbow to your wrist. We see the same use of body parts in pre-metric measurements. The word for inch and thumb is the same in many European languages. A foot is obvious. A yard relates closely to a human pace. But given that we all come in different shapes and sizes, such measurements will vary from one person to another.

King Henry I tried to resolve this problem by insisting that it was his body that was used to standardise the units. He decreed that the yard should be the distance from the tip of his nose to the end of his outstretched thumb. But there are clearly problems if you keep on having to compare lengths to a royal residing in London.

The leaders of the French Revolution believed that a more egalitarian system of measurement should be put in place to which everyone could have access. So it was decided that a metre should be defined as one ten millionth of the distance from the North Pole to the Equator. Although in principle anyone could measure this distance, the impracticality of using this definition soon became apparent.

Two scientists were charged with calculating the distance from the Pole to the Equator. But these were revolutionary times and they had to endure many attacks from locals suspicious of two strangers spying from tops of towers and trees as they measured their way across the country. They succeeded in calculating the distance from Dunkirk to Barcelona, two cities on the same line of longitude and then used astronomical computations to scale up their measurement. Finally, after seven years, they returned triumphantly to Paris with the metre.

A platinum rod was caste whose length corresponded to their calculation and from 1799 the metre resided in the archives in Paris. However, it suffered the same problem as Henry I's yard. Scientists had to journey to France to get a copy of the metre for measurement.

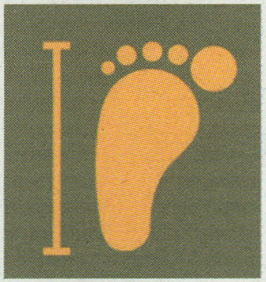
Extraordinarily, it took until 1983 for a more modern definition to be adopted. It was important to find a length that would not vary across time or space. Light travels at the same speed in a vacuum wherever you are in the Universe so this was considered ideal. Today the metre is defined as the distance light travels in a vacuum in 1/299792458 of a second. Of course this just moves the goal posts and begs the question: what is a second?

In 1791 the French commission defined it as 1/86400 of the average time for the Earth to rotate around its axis. However because the friction of the tides is gradually slowing the rotation by a few seconds every year, this means that a second is getting longer over time. So in 1968 a more scientifically robust definition was provided that again could be measured anywhere in the Universe. The definition of a second is related to the frequency of the radiation emitted by an atom of caesium at a temperature of absolute zero. Although the Consultative Committee for Units has sorted out the metre and the second, a kilogram is still proving very difficult to pin down.

The present definition still refers to the mass of a lump of platinum and iridium cast in 1879 and kept in the archives in France. Forty copies were made and distributed to signatories of the accord including one, number 18, that currently sits in a bomb-proof safe at the National Physics Laboratory in Teddington, southwest London. But every time the kilogram is handled it is likely to change very slightly as atoms come away from the lump.

There is no consensus on a more scientific definition of the kilogram. Two opposing camps are fighting it out and the winner will be announced next year at the conference of weights and measures. So stay tuned. You might find that you have unexpectedly lost some weight come October 2011.

Marcus du Sautoy helps Alan Davies to calculate "How long is a piece of string" in this week's *Horizon* available on BBC iPlayer.



Cool fact

A rod is a measure of land dating back to Saxon times when it was defined as the total length of the left feet of the first 16 men to leave church on Sunday morning.

Marcus du Sautoy's sexy science

LEANDRO CASTELAO FOR DUTCH UNCLE

The great numbers enigma

Yesterday was the 150th anniversary of the publication of one of the most important books in the history of science. Darwin's *On the Origin of Species* revolutionised our understanding of where we come from. But November 1859 was also the month that another fundamental paper in the history of science was published, yet it has gone almost unnoticed amid the Darwin celebrations.

One hundred and fifty years ago a shy German mathematician called Bernhard Riemann published a ten-page paper entitled *On the number of primes less than a given magnitude*. The ideas it contained would revolutionise the way we understand the most fundamental numbers on the mathematical books: the primes — indivisible numbers such as 17 and 23.

His teacher, the great mathematician Carl Friedrich Gauss, had a hunch that although the primes get rarer the higher you count, the number of primes thins out in a uniform way. Riemann was trying to confirm Gauss's idea but he couldn't prove it. Riemann had been reluctant to publish at all given that the paper was incomplete. But his election to the Berlin Academy demanded that he publish something to acknowledge the achievement. Since then, trying to fill in the missing details has become the holy grail of mathematics. Called the Riemann Hypothesis, it would explain how the primes are distributed throughout the universe of numbers.

Prime numbers are so important to mathematics because they are the atoms of arithmetic, the hydrogen and oxygen of the world of numbers. Every number is built from multiplying primes together. Despite their fundamental importance, these numbers represent one of the most tantalising enigmas in mathematics. Mathematics is the science of pattern yet when you look at a list of the primes there seems to be no rhyme or reason to be able to predict where the next one will pop up.

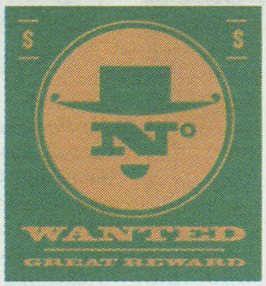
In his 1859 paper, Riemann combined a sophisticated range of mathematics to identify the secret to what makes the primes tick. The proposal he made about the nature of this prime number DNA is now called the zeros of the Riemann zeta-function. If he is right it will imply that the primes are randomly but fairly distributed throughout the universe of numbers. As an analogy one should think of how the molecules of gas are distributed in a room. Although one might not be able to identify where each molecule is, at least you know they are fairly distributed. You won't find a vacuum in one corner with no air. If Riemann is right then the primes are distributed in a similar manner throughout the universe of numbers. Take all the primes with a fixed number of digits and the way they are distributed through all the other numbers looks as randomly scattered as gas particles.

Understanding the primes is of crucial importance to mathematics because so much depends on these basic building blocks. But since the mid-Eighties they are also of commercial importance. All the codes that are used on the internet to keep credit card numbers secret as they are sent through the electronic superhighway are exploiting the mystery of the primes to keep them from prying eyes. That is why if anyone announces any advance on the Riemann Hypothesis it isn't just pure mathematicians who are interested but government security agencies and the world of e-commerce. To underpin the importance of the problem, a businessman in America, Landon Clay, has offered a prize of $1 million for the person who finally completes Riemann's jigsaw.

There is some tantalising evidence that Riemann himself might have known more than he let on in the paper. When he died seven years later his housekeeper decided to clear the house and started burning the mass of incomprehensible scribblings that filled his office. Riemann's colleagues arrived in time to save some of the manuscripts but many went up in flames. The rescued papers contained unpublished results that went far beyond the ideas in his published paper. One can only speculate about how much more Riemann might have unveiled had his housekeeper not been so zealous.

Darwin's book successfully identified the origin of species. But who knows how long we will have to wait until someone completes Riemann's great work started in the paper he published in the same month. Till then the origin of the primes remains one of mathematics greatest enigmas.

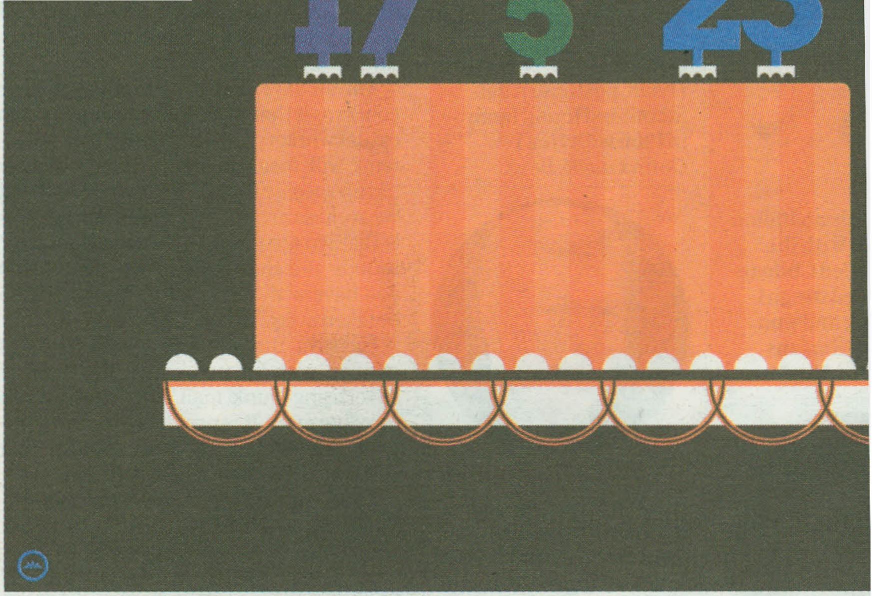
Marcus du Sautoy's *Music of the Primes* (Harper Perennial) is a guide to winning the $1 million prize.



Cool fact

The biggest prime known to date has 12,978,657 digits. There is a $50,000 prize for anyone who discovers a prime with more than 100 million digits.

The great numbers enigma



Yesterday was the 150th anniversary of the publication of one of the most important books in the history of science. Darwin's *On the Origin of Species* revolutionised our understanding of where we come from. But November 1859 was also the month that another fundamental paper in the history of science was published, yet it has gone almost unnoticed amid the Darwin celebrations.

One hundred and fifty years ago a shy German mathematician called Bernhard Riemann published a ten-page paper entitled *On the number of primes less than a given magnitude.* The ideas it contained would revolutionise the way we understand the most fundamental numbers on the mathematical books: the primes — indivisible numbers such as 17 and 23.

His teacher, the great mathematician Carl Friedrich Gauss, had a hunch that although the primes get rarer the higher you count, the number of primes thins out in a uniform way. Riemann was trying to confirm Gauss's idea but he couldn't prove it. Riemann had been reluctant to publish at all given that the paper was incomplete. But his election to the Berlin Academy demanded that he publish something to acknowledge the achievement. Since then, trying to fill in the missing details has become the holy grail of mathematics. Called the Riemann Hypothesis, it would explain how the primes are distributed throughout the universe of numbers.

Prime numbers are so important to mathematics because they are the atoms of arithmetic, the hydrogen and oxygen of the world of numbers. Every number is built from multiplying primes together. Despite their fundamental importance, these numbers represent one of the most tantalising enigmas in mathematics. Mathematics is the science of pattern yet when you look at a list of the primes there seems to be no rhyme or reason to be able to predict where the next one will pop up.

In his 1859 paper, Riemann combined a sophisticated range of mathematics to identify the secret to what makes the primes tick. The proposal he made about the nature of this prime number DNA is now called the zeros of the Riemann zeta-function. If he is right it will imply that the primes are randomly but fairly distributed throughout the universe of numbers. As an analogy one should think of how the molecules of gas are distributed in a room. Although one might not be able to identify where each molecule is, at least you know they are fairly distributed. You won't find a vacuum in one corner with no air. If Riemann is right then the primes are distributed in a similar manner throughout the universe of numbers. Take all the primes with a fixed number of digits and the way they are distributed through all the other numbers looks as randomly scattered as gas particles.

Understanding the primes is of crucial importance to mathematics because so much depends on these basic building blocks. But since the mid-Eighties they are also of commercial importance. All the codes that are used on the internet to keep credit card numbers secret as they are sent through the electronic superhighway are exploiting the mystery of the primes to keep them from prying eyes. That is why if anyone announces any advance on the Riemann Hypothesis it isn't just pure mathematicians who are interested but government security agencies and the world of e-commerce. To underpin the importance of the problem, a businessman in America, Landon Clay, has offered a prize of $1 million for the person who finally completes Riemann's jigsaw.

There is some tantalising evidence that Riemann himself might have known more than he let on in the paper. When he died seven years later his housekeeper decided to clear the house and started burning the mass of incomprehensible scribblings that filled his office. Riemann's colleagues arrived in time to save some of the manuscripts but many went up in flames. The rescued papers contained unpublished results that went far beyond the ideas in his published paper. One can only speculate about how much more Riemann might have unveiled had his housekeeper not been so zealous.

Darwin's book successfully identified the origin of species. But who knows how long we will have to wait until someone completes Riemann's great work started in the paper he published in the same month. Till then the origin of the primes remains one of mathematics greatest enigmas.

Marcus du Sautoy's *Music of the Primes* (Harper Perennial) is a guide to winning the $1 million prize.

Opinion

Without maths we're lost in a dark labyrinth

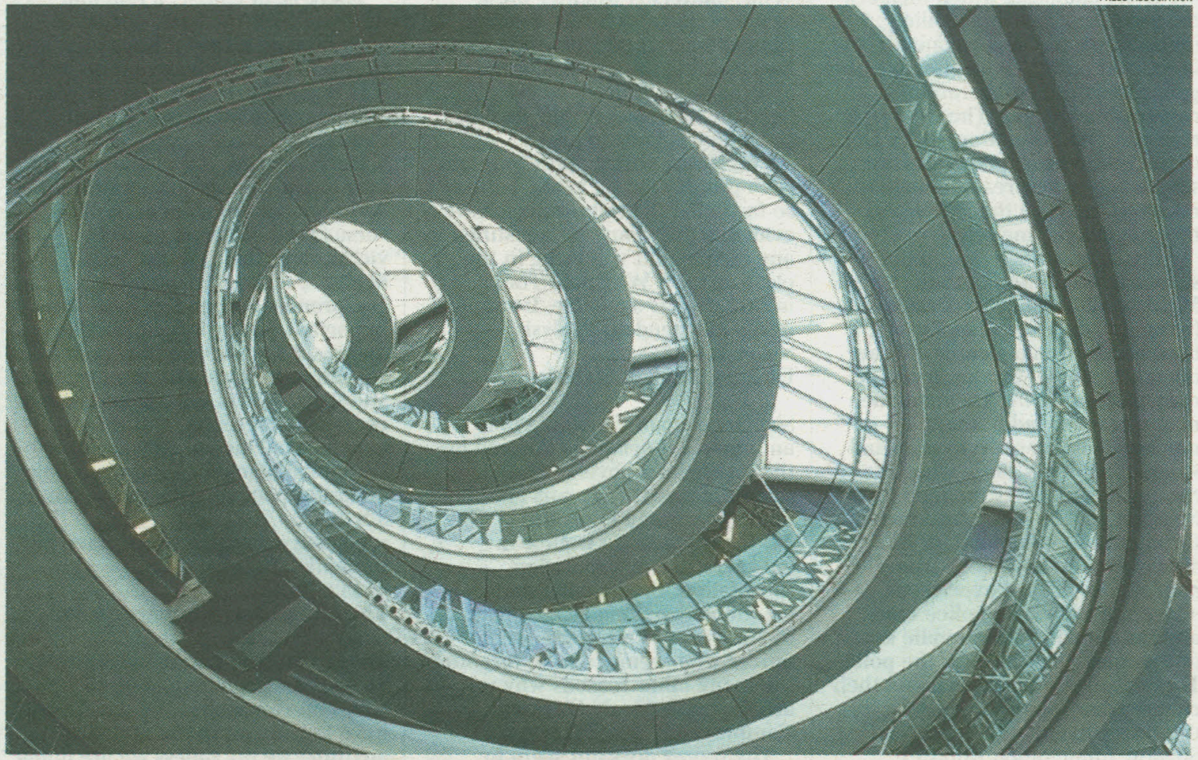
It's the glue that binds scientific and artistic cultures. The language of number and symmetry is spoken everywhere

Marcus
du Sautoy

When I was a kid I hadn't wanted to be a mathematician at all. My dream had been to become a spy. This ambition was fuelled by too many visits to see Roger Moore playing 007 at our local cinema combined with the misconception that my mum, who was once in the diplomatic corps, had been a spy. To realise my dream I decided I would follow in my mum's footsteps and join the Foreign Office.

Speaking foreign languages seemed to be the key to fulfilling my dream, so when I went to secondary school I signed up for all the languages my school taught. It did French and German. It was one of the few comprehensive schools still teaching Latin. There was a course on the BBC teaching Russian. Being a boy of the Cold War I thought that was an ideal language for anyone dreaming

PRESS ASSOCIATION



Mathematical building blocks: the spiral staircase of City Hall, London, designed by Foster and Partners

technological developments permeate modern life, it is essential that society understands what is happening in our world. The Jenkins report in 2000 identified that more dialogue was needed between the scientific community and society. But I believe one can only have genuine dialogue if you've got understanding. How can you have a debate about stem-cell research if you don't understand what a stem cell is? That's not to say that the scientists know all the answers. Many of the interactions between science and society need input from many different perspectives. Scientific research is key to understanding the relative dangers of different drugs but scientists need to understand that drugs are not purely a scientific question, that there is a social and political dimension. But without the understanding of the science, this debate doesn't get going.

The challenging thing for me is that science is not just a single country. It is a continent full of very different cultures. I was out of my depth when I was phoned up by a news channel to explain on the spot the work of the Nobel Prize for Medicine announced that morning. By the following day I'd talked to colleagues enough to download in

Without maths we're lost

It's the glue that binds scientific and artistic cultures. The language o

Marcus
du Sautoy

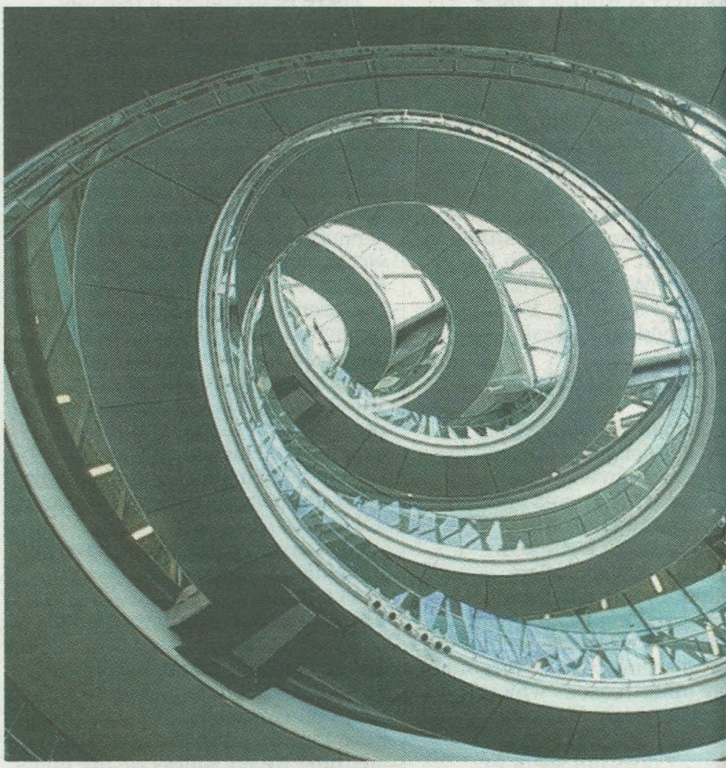
When I was a kid I hadn't wanted to be a mathematician at all. My dream had been to become a spy. This ambition was fuelled by too many visits to see Roger Moore playing 007 at our local cinema combined with the misconception that my mum, who was once in the diplomatic corps, had been a spy. To realise my dream I decided I would follow in my mum's footsteps and join the Foreign Office.

Speaking foreign languages seemed to be the key to fulfilling my dream, so when I went to secondary school I signed up for all the languages my school taught. It did French and German. It was one of the few comprehensive schools still teaching Latin. There was a course on the BBC teaching Russian. Being a boy of the Cold War I thought that was an ideal language for anyone dreaming to become a spy. So I got my French teacher to help me with Russian.

But as I battled away with these languages I became increasingly frustrated with the illogical spellings, the endless irregular verbs that didn't make any sense and which you just had to learn. I've always had a terrible memory and yearned for a sense of order and logic.

At the height of this crisis my maths teacher pulled me aside. Almost conspiratorially he let on that the maths we were doing in the classroom wasn't really what mathematics was about and he suggested a few books that he thought might open up the real world of mathematics to me. One of the books was called *The Language of Mathematics*. I was intrigued. I'd never thought of mathematics as a language. As I read further through the book I realised that this was the language I'd been hankering after.

First, it didn't seem to have any irregular verbs. Everything made logical sense, evolving naturally from a few



Mathematical building blocks: the spiral staircase of City Hall, London,

natural assumptions. That's not to say that there weren't surprising twists and turns throughout the story, but they all made sense. The most exciting discovery was the power of this language to describe the natural world. It had the power to reveal where it had all come from but, more excitingly, to predict what will happen next: for example, to make sense of what is happening (or almost happening) in the Large Hadron Collider, which uses the mathematics of strange symmetrical objects in hyperspace. To assess the potential effect of travel restrictions or vaccinations on the spread of the H1N1 virus requires mathematical modelling. And climate change is a mathematical problem: it's only by understanding the delicate mathematical relationship between different factors in the environment that we can understand why temperatures are rising.

Mathematics brings a transparency to these complex systems. But it isn't only the scientists who are speaking

You can find comple mathematical curve running through the buildings around us

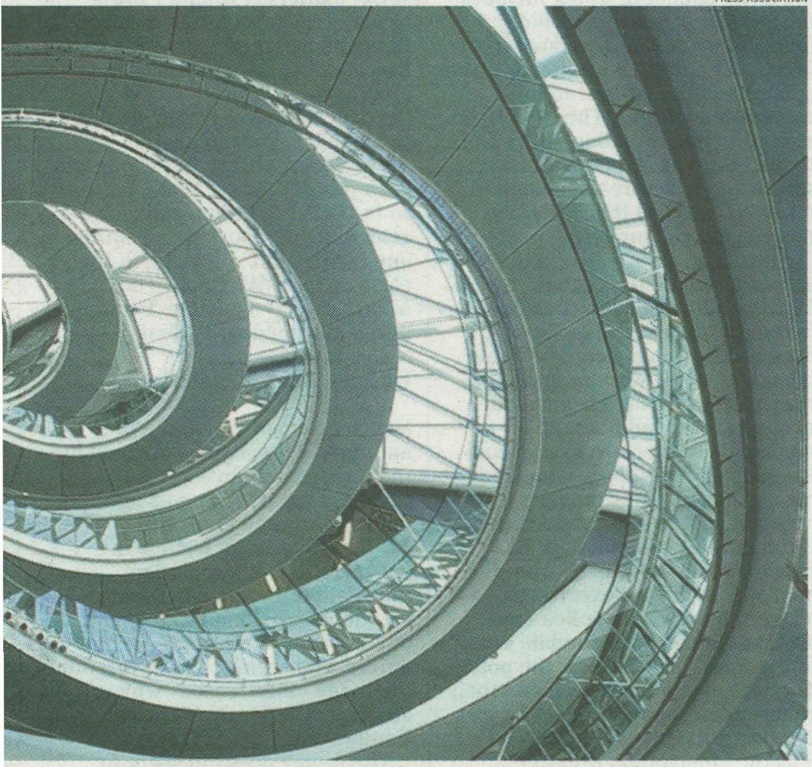
this language. It is extraordinar many interesting mathematical one can find bubbling beneath surface of the work of many art Either consciously or subconsci they are drawn to the same mathematical structures that fascinate me.

Messiaen consciously exploite asynchronicity of the prime nur 17 and 29 to create a sense of timelessness in the *Quartet for End of Time*. In another piece, *I Feu*, I cannot believe he was aw that the two twelve-note seque he uses are the basis for genera one of the strangest symmetrica objects discovered by mathemat in our mathematical journey th

e're lost in a dark labyrinth

tures. The language of number and symmetry is spoken everywhere

PRESS ASSOCIATION



ral staircase of City Hall, London, designed by Foster and Partners

You can find complex mathematical curves running through the buildings around us . . .

this language. It is extraordinary how many interesting mathematical ideas one can find bubbling beneath the surface of the work of many artists. Either consciously or subconsciously they are drawn to the same mathematical structures that fascinate me.

Messiaen consciously exploited the asynchronicity of the prime numbers 17 and 29 to create a sense of timelessness in the *Quartet for the End of Time*. In another piece, *Île de Feu*, I cannot believe he was aware that the two twelve-note sequences he uses are the basis for generating one of the strangest symmetrical objects discovered by mathematicians in our mathematical journey through symmetry. But it is a sensitivity to similar structures that drew him to these two themes. From the magnificence of the Baroque to the modern architecture of Arup, Foster and Hadid, one can find complex mathematical curves running through the buildings that surround us. The writing of Borges is infused with a fascination with infinity and the nature of space.

With mathematics acting like a glue binding all these different scientific and artistic cultures together I believe that mathematics provides a perfect platform for my job as the new Simonyi Professor for the Public Understanding of Science, which I have held for a year. In some strange sense I have found myself realising my dream to join the Foreign Office. I see my role rather like an ambassador for the often alien world of science, trying to provide bridges for a society that is sometimes suspicious of this powerful territory.

Given how much scientific and technological developments permeate modern life, it is essential that society understands what is happening in our world. The Jenkins report in 2000 identified that more dialogue was needed between the scientific community and society. But I believe one can only have genuine dialogue if you've got understanding. How can you have a debate about stem-cell research if you don't understand what a stem cell is? That's not to say that the scientists know all the answers. Many of the interactions between science and society need input from many different perspectives. Scientific research is key to understanding the relative dangers of different drugs but scientists need to understand that drugs are not purely a scientific question, that there is a social and political dimension. But without the understanding of the science, this debate doesn't get going.

The challenging thing for me is that science is not just a single country. It is a continent full of very different cultures. I was out of my depth when I was phoned up by a news channel to explain on the spot the work of the Nobel Prize for Medicine announced that morning. By the following day I'd talked to colleagues enough to download in my Sexy Science column the essence of how telomeres protect chromosomes as they divide and multiply. Just imagine a professorship for the Public Understanding of the Humanities who had to represent everything from philosophy to medieval painting, from South American literature to the music of the Baroque.

It is one of my aspirations during the tenure of my professorship to encourage government, research councils and universities that the more scientific ambassadors we can support the better chance we have of integrating the foreign world of science with the rest of society. Without an understanding of the language of science and mathematics, as Galileo once wrote, we will all be wandering around lost in a dark labyrinth.

From Marcus du Sautoy's inaugural lecture as the Simonyi Professor for the Public Understanding of Science at the University of Oxford

Marcus du Sautoy's sexy science

Shining a light on dark matter

According to scientists, most of the stuff in the universe consists of unseen particles called dark matter. But no one is sure exactly what it is. Douglas Adams suggested that it might actually be the white pellets that fill the boxes that things get packaged in. The script writers of *Futurama* disagree and believe it is the dense excrement deposited by Nibbler, Leela's pet. Or could it be a consciousness connecting multiple worlds and otherwise known as Dust, as Philip Pullman proposes in *His Dark Materials*?

Scientists are not convinced by any of these explanations. Indeed the search for a true explanation of dark matter is one of the holy grails of cosmology. Detecting this elusive stuff is a major goal for the Large Hadron Collider that started smashing particles in earnest last week. The matter making up the stars, planets and galaxies seems to account for only 15 per cent of what is actually out there. Scientists believe that there is a lot more stuff that we can't see. The trouble with this hypothetical dark matter is that it doesn't seem to emit any electromagnetic radiation that could be picked up by a telescope — which is what makes it dark.

So why do we believe there is all this stuff out there? Even though you can't see it, dark matter still has a gravitational pull on the objects that we *can* see.

Gravity has helped us identify a lot of things that at first were obscured from view. For example, the planet Neptune was discovered by mathematical calculations rather than by staring down a telescope. The orbit of Uranus was doing some strange things which could be explained only by the gravitational effect of a large object orbiting the Sun farther out. Sure enough, on September 23, 1846, telescopes picked up the first sighting of this distant planet first predicted "with the point of a pen", as Francois Arago nicely put it.

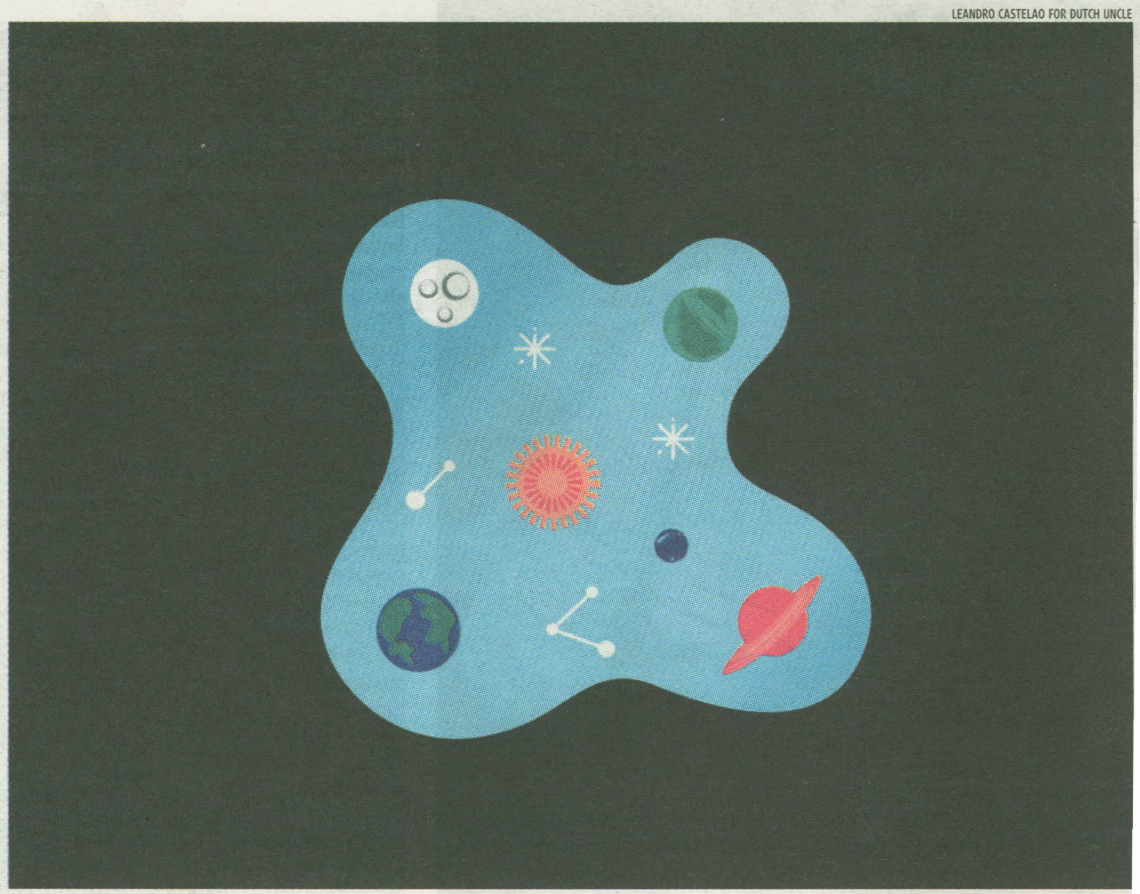
The existence of dark matter has been similarly predicted because astronomers can't find any other explanation for the strange behaviour of galaxies and stars other than the effect of a large amount of unseen matter pushing and pulling them about. The first hint of something hiding out there came as early as 1933, when the Swiss astronomer Fritz Zwicky couldn't explain how the Coma cluster of galaxies could be spinning so fast given the mass of stars he could see. The only explanation he could come up with is that there was a lot more stuff sitting like a halo around these galaxies that was pushing them like a helping hand spinning a merry-go-round. Further evidence began to mount for the existence of something out there that, although unseen, was heavy enough to bend light as it travelled through space (so-called gravitational lensing).

We can't see dark matter because it emits no observable radiation, so that until we can actually detect this stuff directly it will remain just a hypothesis to explain the weird behaviour of certain astronomical phenomena.

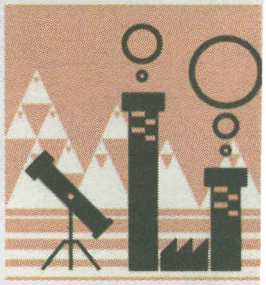
Or can dark matter be seen interacting with objects here on Earth? One team in Italy believes it has managed to do precisely that. By placing a 250kg (551lb) lump of ultra-pure sodium iodide crystal 1,400m (4,600ft) beneath the Gran Sasso mountain, which screens out possible interaction with background cosmic radiation, the Italian team has picked up periodic flashes of light coming from the nuclei in the crystals. The scientists believe that this is being caused by the crystal interacting with a huge cloud of dark matter that the earth is passing through. Crucially, the number of flashes seems to depend on the time of year, which would be consistent with the variation of the Earth's motion through this cloud — the same phenomena that causes you to hit more raindrops when cycling into the wind than when the wind is behind you.

So why isn't the scientific community cheering in recognition of the Italian team's achievement? Science depends on being able to reproduce experiments, and currently no one has been able to repeat the Italian team's claim. Not through want of trying but because, according to the journal *Nature*, the only company that makes pure enough sodium iodide crystals, Saint Gobain in Paris, has signed an intellectual property agreement with the Italian team and is therefore unable to supply the crystals to anyone else.

Whether or not this is a sorry example of scientific progress being held back because of legal and commercial concerns, for the time being dark matter continues to keep its elusive identity obscured from view.



LEANDRO CASTELAO FOR DUTCH UNCLE



Cool fact

Boulby Mine in the North York Moors is the second deepest mine in Europe and is home to the UK's attempts to detect dark matter.

Marcus du Sautoy's sexy science

Another heated debate

LEANDRO CASTELAO FOR DUTCH UNCLE

This week, those dignitaries meeting in Copenhagen will announce a protocol to reduce emissions. But the fear is that it could be too little too late. Carbon dioxide levels in the atmosphere are 35 per cent greater than they have been for most of the past 650,000 years, and the potential increase in global temperatures caused by this extra CO_2 have serious implications for the future of the planet.

Tipping points have been identified that could make it impossible to reverse the change in climate through reducing CO_2 emissions.

The ocean floors contain deposits of ice filled with methane; similar deposits are trapped in the Siberian permafrost. If temperatures rise, these ice crystals could melt, releasing the trapped gas into the atmosphere. Methane is a far worse greenhouse gas than CO_2; its sudden release could substantially accelerate the rise in global temperatures.

Those who think we can acclimatise to a gradual increase in temperature are sorely deluded. A dramatic reduction in CO_2 emissions is clearly the best solution but there may also be a place for doing something about the CO_2 already in the atmosphere through "geo-engineering".

Are there ways in which we can extract the CO_2 or reduce the effect of the sun's rays that are trapped by the greenhouse gases? Nature has been using a method to absorb CO_2 ever since things started growing on the planet: photosynthesis in plants converts it into sugars.

However, when the plants die and decompose, most of the carbon that they stored is returned to the atmosphere. Halting that release by burying the degrading plant material in the land or deep ocean may reduce the natural emissions, but research shows the benefits do not outweigh the energy consumed in transport, burial and processing of the plant material.

There is one intriguing proposal to exploit the photosynthesis performed by algae floating on the surface of the ocean. Some of the carbon that they absorb sinks to the bottom of the ocean, where it is released without any human intervention. If we increased the algae, could we absorb more carbon into the deep oceans?

Promoting algae growth requires certain nutrients: nitrogen, phosphate and iron. The amount of algae present in the sea is currently limited by the lack of iron in the water. Seeding the oceans with iron could increase the algae growth and hence increase the absorption of carbon from the atmosphere.

Small-scale experiments on patches of the ocean have borne out the theory, but it is not clear that the same results would be achieved on a larger scale. Tinkering with the ecosystem could be disastrous. An increase in algae would rob the oceans of other essential nutrients, such as nitrogen and phosphate. There would also be an associated depletion of oxygen levels that could have a devastating effect on the fish stocks.

What about methods that could reduce the absorption of solar radiation? Whitewashing all our roofs, pavements and roads would reflect the sunlight, but not enough of the world's surface is urbanised to make this a viable option. Since more of the surface of the Earth is dedicated to agriculture, clever choices of crops whose surface reflects the sunlight is considered a more effective alternative. Hot deserts make up 2 per cent of the Earth's surface; covering them in a reflective polyethylene-aluminium sheet would have a significant effect on the radiation absorbed, but could have a serious impact on global weather systems.

An alternative proposal is to put reflective materials higher in the atmosphere. This could be in the form of particles released into the stratosphere or on the tops of clouds, which would scatter sunlight back to space. The most dramatic version consists of placing huge sunshields out in space like a pair of sunglasses for the Earth.

Some of these methods may be feasible, but require further research and funding — and all entail unknown risks and side-effects. So far, science has failed to come up with a silver bullet to mitigate global warming. The best option is to attempt to return the planet to an ecosystem that we know works: one with less CO_2 in the atmosphere. Let's hope that those in Copenhagen can make it happen.

Not-so-cool fact

A mass extinction of life on Earth occurred 250 million years ago. One theory blames a 5C increase in global temperatures. This released methane from the Siberian and ocean bed traps, which in turn led to another 5C rise in temperature, wiping out 95 per cent of life on Earth.

Another heated debate

This week, those dignitaries meeting in Copenhagen will announce a protocol to reduce emissions. But the fear is that it could be too little too late. Carbon dioxide levels in the atmosphere are 35 per cent greater than they have been for most of the past 650,000 years, and the potential increase in global temperatures caused by this extra CO_2 have serious implications for the future of the planet.

Tipping points have been identified that could make it impossible to reverse the change in climate through reducing CO_2 emissions.

The ocean floors contain deposits of ice filled with methane; similar deposits are trapped in the Siberian permafrost. If temperatures rise, these ice crystals could melt, releasing the trapped gas into the atmosphere. Methane is a far worse greenhouse gas than CO_2; its sudden release could substantially accelerate the rise in global temperatures.

Those who think we can acclimatise to a gradual increase in temperature are sorely deluded. A dramatic reduction in CO_2 emissions is clearly the best solution but there may also be a place for doing something about the CO_2 already in the atmosphere through "geo-engineering".

Are there ways in which we can extract the CO_2 or reduce the effect of the sun's rays that are trapped by the greenhouse gases? Nature has been using a method to absorb CO_2 ever since things started growing on the planet: photosynthesis in plants converts it into sugars.

However, when the plants die and decompose, most of the carbon that they stored is returned to the atmosphere. Halting that release by burying the degrading plant material in the land or deep ocean may reduce the natural emissions, but research shows the benefits do not outweigh the energy consumed in transport, burial and processing of the plant material.

There is one intriguing proposal to exploit the photosynthesis performed by algae floating on the surface of the ocean. Some of the carbon that they absorb sinks to the bottom of the ocean, where it is released without any human intervention. If we increased the algae, could we absorb more carbon into the deep oceans?

Promoting algae growth requires certain nutrients: nitrogen, phosphate and iron. The amount of algae present in the sea is currently limited by the lack of iron in the water. Seeding the oceans with iron could increase the algae growth and hence increase the absorption of carbon from the atmosphere.

Small-scale experiments on patches of the ocean have borne out the theory, but it is not clear that the same results would be achieved on a larger scale. Tinkering with the ecosystem could be disastrous. An increase in algae would rob the oceans of other essential nutrients, such as nitrogen and phosphate. There would also be an associated depletion of oxygen levels that could have a devastating effect on the fish stocks.

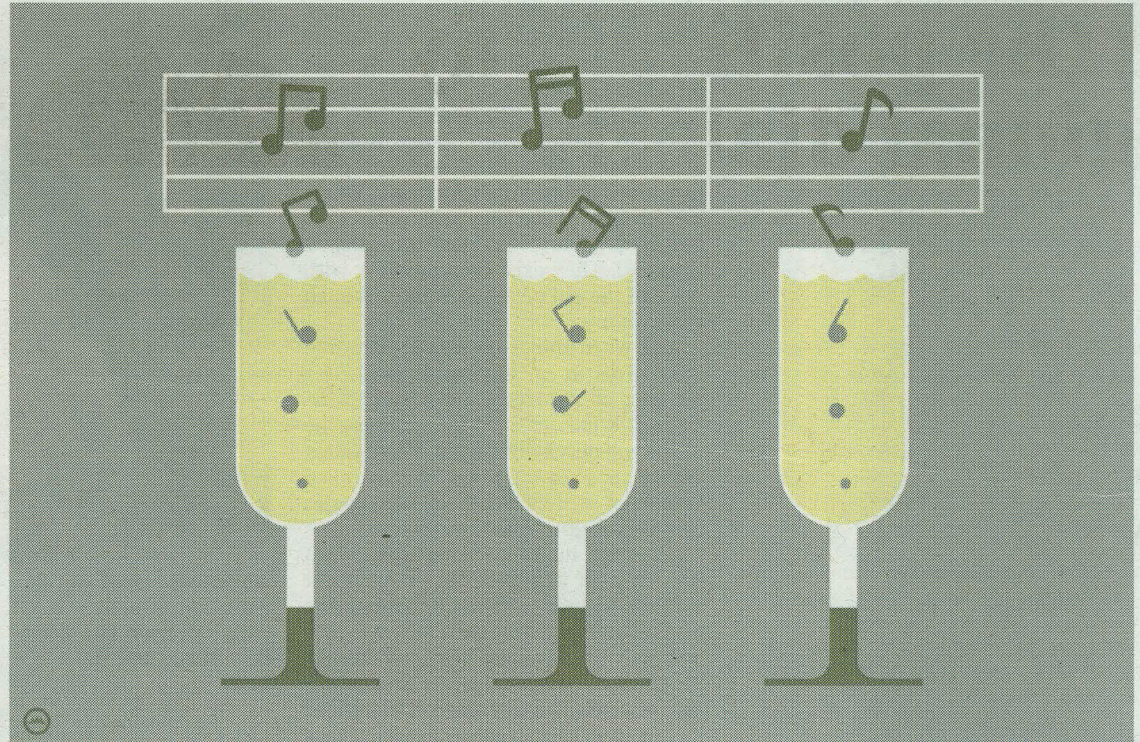
What about methods that could reduce the absorption of solar radiation? Whitewashing all our roofs, pavements and roads would reflect the sunlight, but not enough of the world's surface is urbanised to make this a viable option. Since more of the surface of the Earth is dedicated to agriculture, clever choices of crops whose surface reflects the sunlight is considered a more effective alternative. Hot deserts make up 2 per cent of the Earth's surface; covering them in a reflective polyethylene-aluminium sheet would have a significant effect on the radiation absorbed, but could have a serious impact on global weather systems.

An alternative proposal is to put reflective materials higher in the atmosphere. This could be in the form of particles released into the stratosphere or on the tops of clouds, which would scatter sunlight back to space. The most dramatic version consists of placing huge sunshields out in space like a pair of sunglasses for the Earth.

Some of these methods may be feasible, but require further research and funding — and all entail unknown risks and side-effects. So far, science has failed to come up with a silver bullet to mitigate global warming. The best option is to attempt to return the planet to an ecosystem that we know works: one with less CO_2 in the atmosphere. Let's hope that those in Copenhagen can make it happen.

Marcus du Sautoy's sexy science

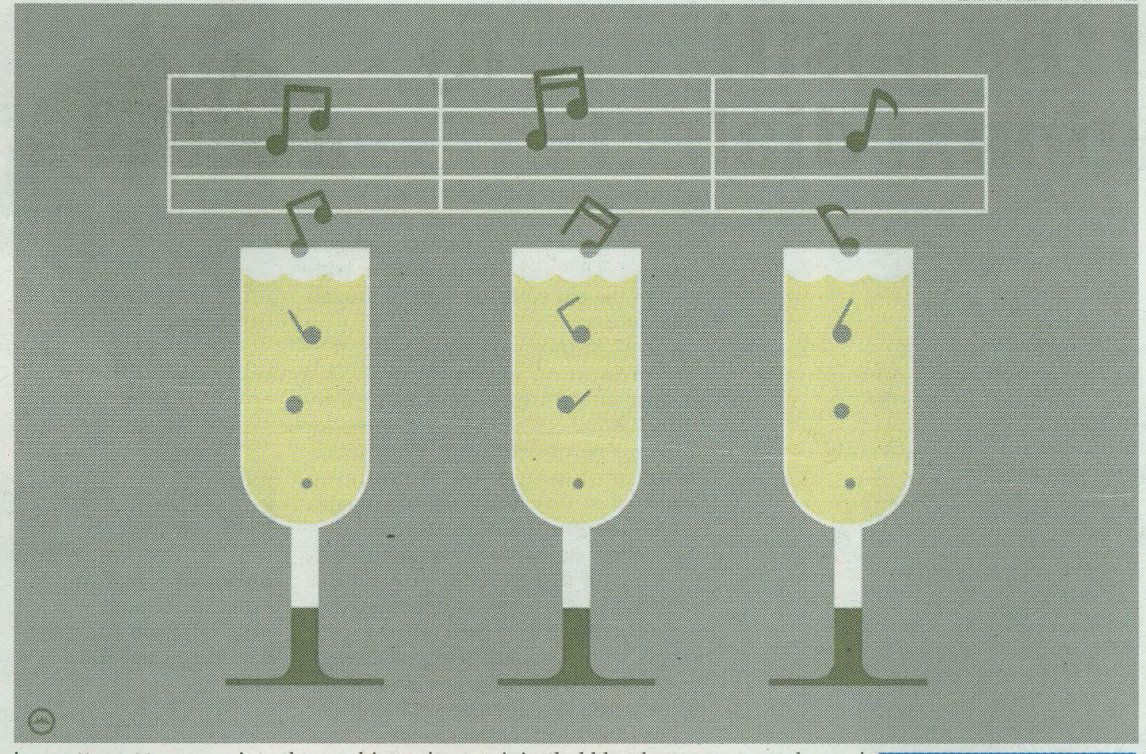
LEANDRO CASTELAO FOR DUTCH UNCLE



Bubbles to block out the bawls

Marcus du Sautoy's sexy science

LEANDRO CASTELAO FOR DUTCH UNCLE



Bubbles to block out the bawls

I have never particularly been a fan of the rather strained excitement of New Year's Eve. I just don't find the idea of numbers ticking over to start a new cycle of counting that thrilling. So this year I came up with a cunning plan to minimise its effect. I flew west across the dateline from London to New Zealand.

Setting off on December 30, I arrived in Auckland on the morning of January 1. It took all my mathematical powers and the aid of the Airshow option of the inflight entertainment, showing our flight path, to work out where December 31 went. I calculated that I had to endure an hour of New Year's Eve before I was propelled into the new decade as we crossed the dateline at 2am on January 1, 2010 — although, looking at a map of time zones relative to the dateline in the Pacific, I could be wrong. The whole thing looks something of a dog's breakfast.

Unfortunately, a dog's breakfast is what I ended up getting on the plane, because my drastic course of action to avoid new year festivities started to backfire as, first, a couple with a baby sat down next to me for the 28-hour flight, followed by a woman with a dog in her handbag on the other side (how did she get *that* past security?). I braced myself for a long night of screaming and barking, and ordered several bottles of sparkling wine from the drinks trolley as it passed by.

Gulping back a glass of bubbles, I buried myself in the latest copy of *Nature* in an attempt to escape into the soothing world of science. As the noise on both sides mounted, it became increasingly hard to concentrate — until I turned the page to find an article that immediately grabbed my attention. It seemed to vindicate my choice of sparkling wine as a way to ward off the noise of my neighbours, for it turns out that the bubbles in the liquid, rather than the effect of the alcohol, could be the key to perfect soundproofing.

It was discovered in the 1930s that bubbles of air "sing" as they rise through the liquid. Just as a balloon filled with water wobbles when you hold it, a tiny bubble of air tends to vibrate as it rubs against the liquid on its passage to the surface. This friction is like a violin bow rubbing over a string, except that it is the bubble that vibrates. As with the violin string, the note that you hear depends on the resonant frequency of the bubble.

In 1933 Marcel Minnaert, a Belgian scientist, formulated the equation for calculating resonant frequency. It depends on several factors: the resistance of the air to compression, the density of the water and the radius of the bubble.

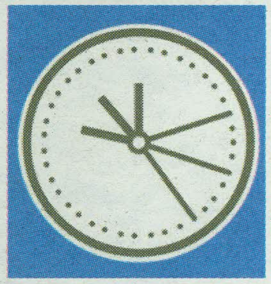
Usually, for something to vibrate and produce a note in the audio frequency it needs to be large. But Minnaert's formula, surprisingly, implied that the note emitted by a small bubble could be within the range of human hearing.

The recent research reported in *Nature* suggested a reversal of this process. If a sound wave is passed through a liquid containing bubbles that resonate at the frequency of the sound wave, the bubbly liquid will absorb the energy of the wave — rather like the transfer of energy from someone pushing a swing to the person on the swing, or like a singer hitting a note that shatters a glass. Materials have already been manufactured that use this idea to absorb waves in the electromagnetic spectrum, such as light. The wavelength is sufficiently small for crystals to be manufactured that resonate at the frequency of the light to be absorbed. But the same principle applied to sound, which has a much larger wavelength, would require structures several metres across.

This is where the clever property of bubbles comes into play. Although they are small, their resonant frequency is in the audio range. So the proposal is to capture bubbles of different sizes in a rubber-like material that will allow the bubbles to vibrate without escaping. Arranged cleverly, the bubbles can then absorb the energy of the sound waves passing through the material, so blocking transmission of the sound.

Unfortunately, research into the power of bubbles to silence noise is still in its infancy — so, rather than putting a couple of glasses to my ears, I was reduced to exploiting the alcoholic effect of my bubbly liquid to survive the flight.

As he was flying, the author was appointed OBE in the New Year Honours for services to science



Cool fact

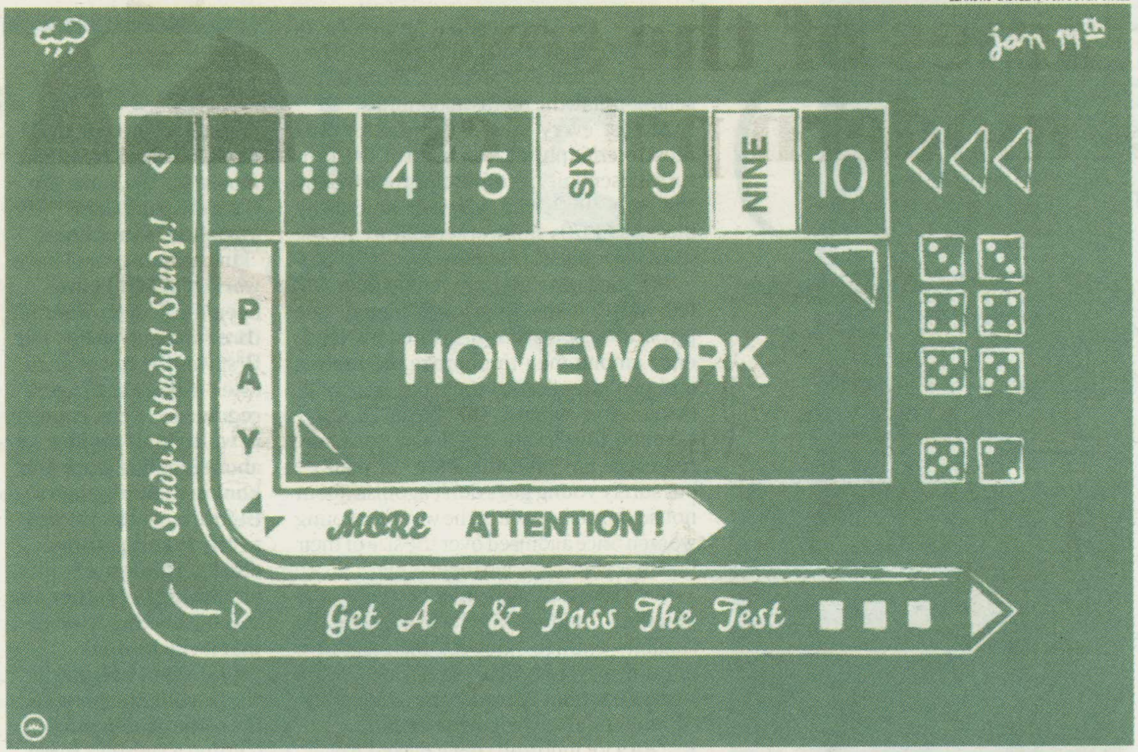
Because of the strange arrangement of time zones round the dateline, for two hours a day there are actually three days being observed at the same time. For example, 23.15 on a Wednesday in Samoa is 23.15 on a Thursday in Tonga — but 00.15 on a Friday in nearby Kiritimati (otherwise known as Christmas Island).

The science of fear

In Eureka, free with The Times tomorrow

Marcus du Sautoy's sexy science

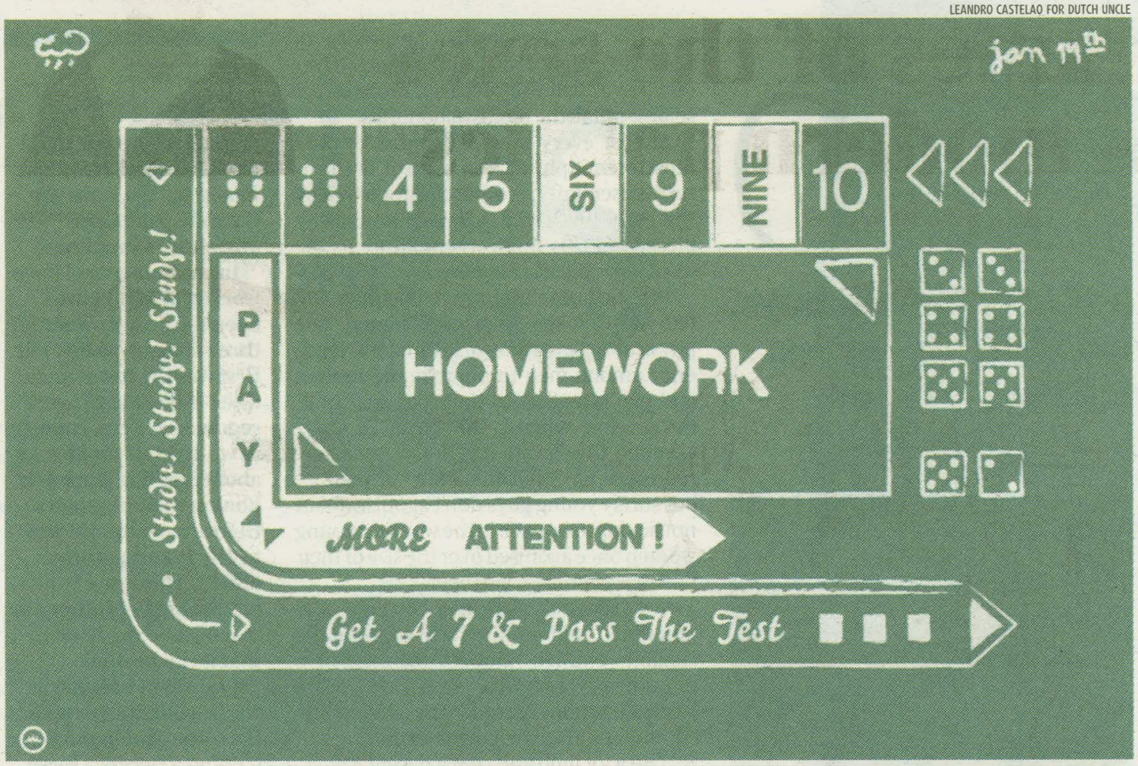
LEANDRO CASTELAO FOR DUTCH UNCLE



So, punks, do you feel lucky?

Marcus du Sautoy's sexy science

LEANDRO CASTELAO FOR DUTCH UNCLE



So, punks, do you feel lucky?

Last week, when bookmakers announced that they were considering replacing traditional odds-based betting in the UK with a decimal system, the television racing pundit John McCririck told *The Times* that change was inevitable. "I love the old tic-tac prices and the 'top of the head, 9-4' and all that," he said, "but we need to bring young people into racing and they don't understand these odds."

The comment reminded me of season four of the HBO TV cop series *The Wire*, which centres on the education system in Baltimore. It focuses on the stories of several young schoolboys and a teacher's attempt to lure them away from illegal drug-dealing on the corner.

Roland "Prez" Pryzbylewski, a former cop turned maths teacher, fails to interest the kids in learning anything at all until, one break-time, he discovers the students gambling.

It turns out that the game on the street is craps. When Prez reveals that maths can give them an edge in betting on it, the kids suddenly start paying more attention in class. They raid the stock cupboard for dice and soon the classroom is full of students shooting craps against the classroom wall.

When another teacher enters Prez's classroom, she is horrified to find the kids gambling. But, as Prez retorts: "You trick 'em into thinking they aren't learning, and they do."

For those unfamiliar with the game, craps is played with two dice. If the person throwing the dice gets 7 or 11, he wins. If he gets 2, 3 or 12, then you win. However, if any of the other scores are thrown — 4, 5, 6, 8, 9 or 10 — then the shooter has to hit this score again to win the bet. If he throws a 7 before this happens, though, then you win. As Prez explains to the kids, getting 7 is twice as likely as getting 4. So if the first throw is a 4, you are twice as likely to win than the person throwing the dice.

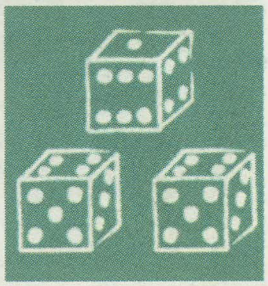
While 7 wins the thrower the bet on the first throw, it will lose him the bet in subsequent throws. This gives craps an interesting mathematical dynamic. A 7 is the most popular score when rolling two dice because there are six different ways that the dice can land to score 7 (1+6, 2+5, 3+4, 4+3, 5+2, 6+1). Throwing a 2 is much harder because both dice must land showing 1s (which is why scoring 2 is called "snake eyes"). So you are twice as likely to lose on the first throw as the shooter because there are four ways to get 2, 3 or 12 compared with eight ways to get 7 or 11. If none of these scores comes up, then the tide turns in your favour on subsequent throws.

Armed with this knowledge, Prez's kids go out and make a killing on the street — so I'm sure his class would have been well prepared to assess the bookies' plan to go decimal and its potential pitfalls for punters. Yes, it makes the maths easier, but the new system will also be doing them out of their small change.

Take, for example, the experiment that is to be given a trial at race meetings in the spring, when the bet of 7-3 will be replaced by a simple multiplier — a number by which you multiply your bet to find out how much you'll receive if the horse should win. Odds of 7-3 mean that if you bet £3, then you get back your £3 plus a profit of £7. So in decimal, the multiplier will be 3.33.

Certainly, if you have a fiver to bet it is much easier to whip out your mobile phone and use the calculator function to work out a payout of 3.33 x £5 = £16.65 than to work out the odds at 7-3. But actually you would win more at 7-3 because 5 + (7-3) x 5 comes out at £16.67 when it is rounded. The effect of rounding 7-3 + 1 before multiplying by 5 is different to multiplying by 7-3 + 1 and then rounding. True, 2p may not be a big difference to the punter — but when you are a bookie, those thousands of 2ps quickly mount up.

Although perfectly legal, the result is called "salami slicing", where many thin slices combine to make a big, fat salami. The key test will be how a bet of 8-3 gets rounded. If you see bets with the multiplier 3.67, you will know that the bookies are giving you back the salami. Bets of 3.66 mean that they are keeping it. If the average punter had attended Prez's class, he or she would know whether it was just the jockeys who were being taken for a ride.

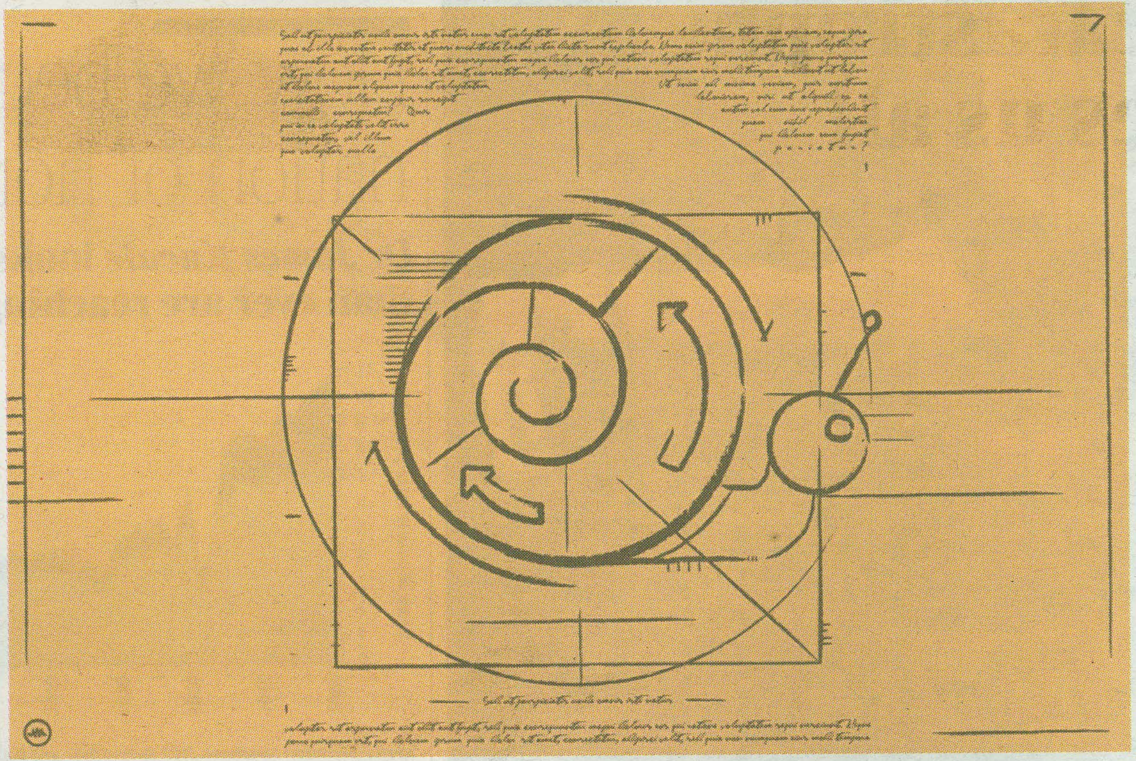


Conundrum

Throw three dice. If a six appears, you win £1. If no six appears, you pay me £1. Should you take the bet?

Answer: No. There are 6x6x6=216 different ways in which the three dice can land, of which 5x5x5=125 have no sixes, which means that there is a 125/216 = 58 per cent chance that I'll win.

Marcus du Sautoy's sexy science



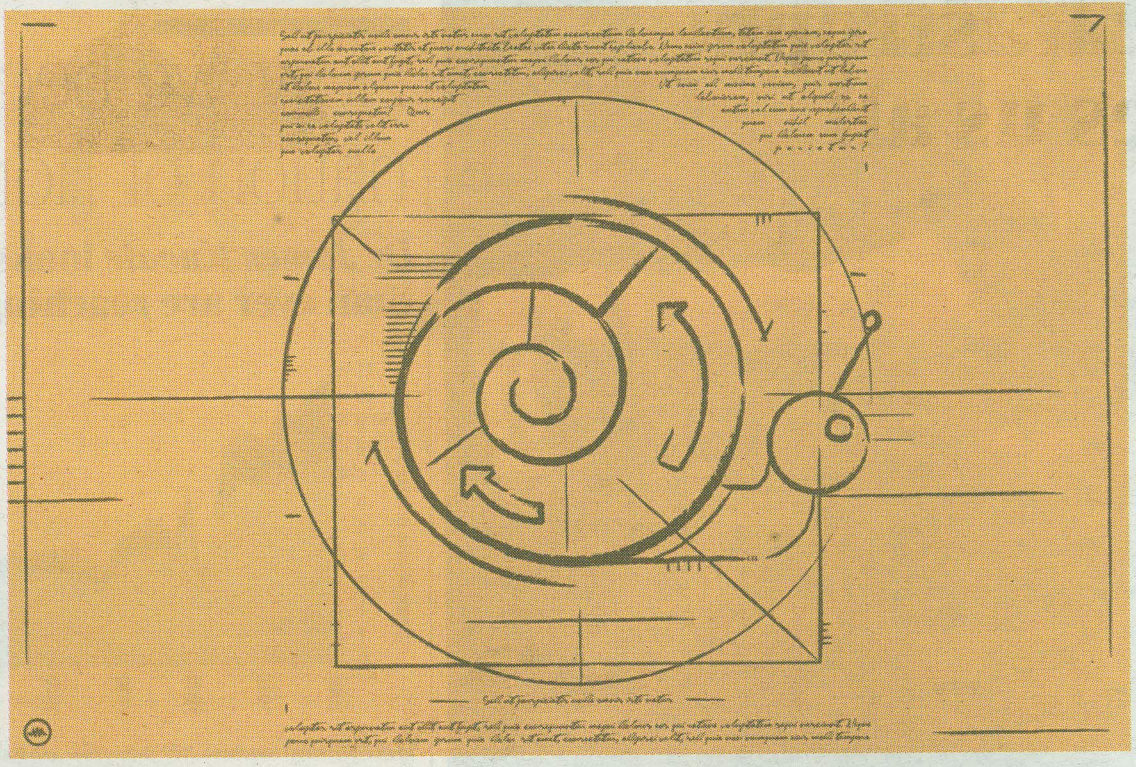
LEANDRO CASTELAO FOR DUTCH UNCLE

Spiral tap gives snails a new twist

In Ian Fleming's novel *Dr No*, James Bond's nemesis Julius No survives a murder attempt thanks to a rare medical condition. The shot that is fired into the left side of his body misses its mark because No's heart is located on the right. *Situs inversus* is a congenital condition

Marcus du Sautoy's sexy science

LEANDRO CASTELAO FOR DUTCH UNCLE



Spiral tap gives snails a new twist

In Ian Fleming's novel *Dr No*, James Bond's nemesis Julius No survives a murder attempt thanks to a rare medical condition. The shot that is fired into the left side of his body misses its mark because No's heart is located on the right. *Situs inversus* is a congenital condition that reverses the location of the internal organs in the body and affects fewer than one person in 10,000. It is strange how, given the symmetry of our external bodies, we are so asymmetrical on the inside. It is one of the intriguing challenges of foetal development how this combination of symmetry and asymmetry evolves as an embryo grows. Recent research, though, into the way a snail's shell spirals may throw new light on this puzzle.

A new embryo starts as a single cell with perfect symmetry. As it begins its growth it divides into two cells whose combination has mirror symmetry. Two cells double to four cells, which tend to arrange themselves into a square formation. This dividing and doubling continues as an extraordinarily complex organism evolves. Some of that early symmetry has been retained as the mirror symmetry we display externally. But internally our bodies are asymmetrical: our heart is on the left, appendix on the right, the left lung has two lobes while the right has three.

How the simple symmetry and its breaking evolves during the growth of the embryo is intriguing. Clearly our genetic code is an important part of managing the growth — as the cellular structure of the outcome bares a striking resemblance to the people whose DNA combined to begin the process of dividing that first cell.

But recent research from Japan on the growth of snails gives some insight into how the environment can have a big influence on what happens at each stage of development. The previous arrangement of cells at one stage will have an important bearing on how the cells are configured when they next double. Change that arrangement and you can dramatically alter the resulting organism.

If you place a spiralling shell down on the table with the spiral facing up then in most shells the spiral evolves in a clockwise orientation from the centre of the shell out. But this is not always the case. Some species of snail have shells that spiral out in an anticlockwise direction, but it is much rarer. However there is one species of pond snail, *Lymnaea stagnalis*, which demonstrates examples of both clockwise or anticlockwise spiralling shells. The orientation of the spiral is determined by the genetic make-up of the snail's mother.

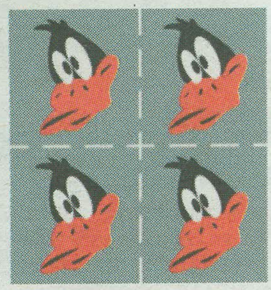
Professor Reiko Kuroda and her colleagues at Tokyo University were intrigued to find out if they could cause a genetically clockwise shell to reverse its direction during embryonic development. Remarkably, by giving the cells a little push in the right direction, they could overcome the genetic programme. The crucial stage of development in which you can change the direction of the spiral turned out to be surprisingly early on. It seems that when four cells divide into eight the asymmetry sets in and the shell begins to spiral one way or the other. The four new "daughter" cells, which appear at this stage, sit on top of the first four cells arranged in a square. The genetic code decides whether they sit with a slight twist towards clockwise or anticlockwise.

But by giving the four cells sitting on top a little push with two glass rods, the Japanese team found that it could alter the orientation of the cells and later development of the snail would spiral in the opposite direction to the one determined by the genetic code. It seems that the development of the shell from this point is determined more by the previous arrangement of cells as they divide, rather than information coming from the DNA of the cells.

It wasn't only the spiral of the shell that was altered. As with human beings, the internal organs of the snail are also arranged asymmetrically. If the spiral of the shell is changed at the eight-cell stage then the orientation of the snail's heart, stomach, liver coiling and gut looping are similarly flipped.

Of course, this prod with a stick has not changed the genetic make-up of the snail so that mother snails that were genetically meant to be clockwise and give birth to clockwise prodigy would still do so, even though they might have been pushed by the Japanese team into developing an anticlockwise shell.

The research on the humble pond snail is likely to have important implications beyond perceiving the spiralling of shells. It could help us to understand the moment when asymmetry sets in during the development of other species similar to ourselves.

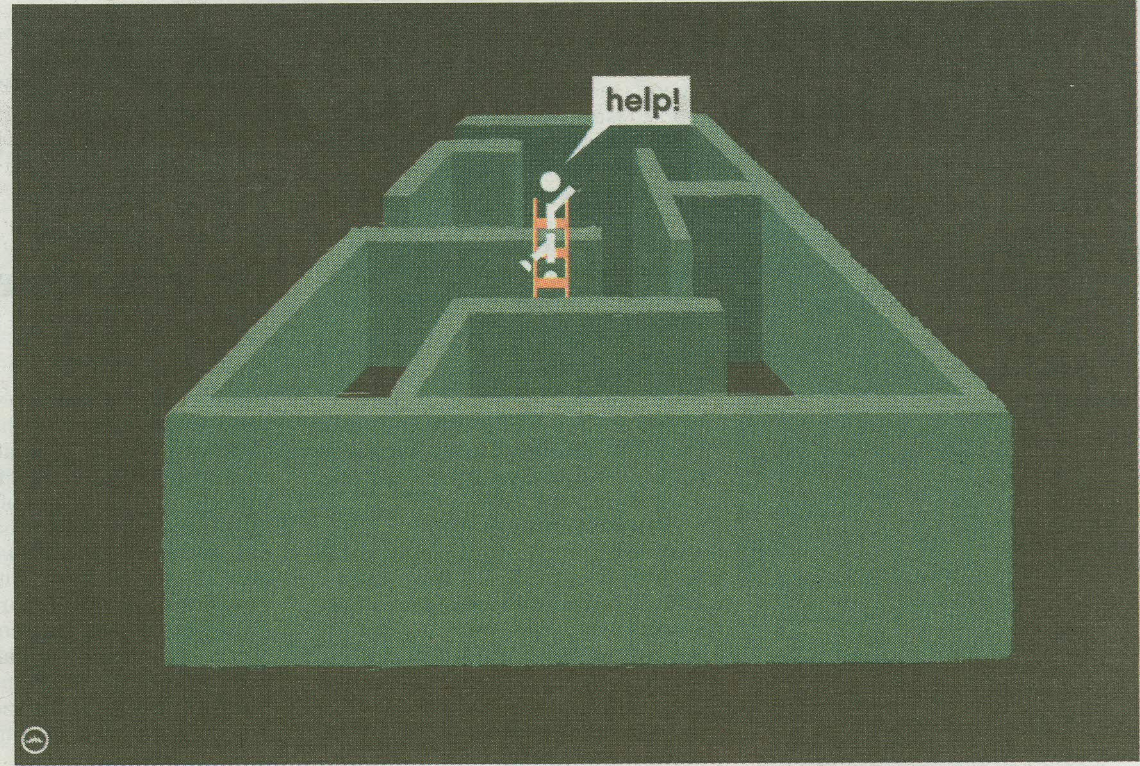


Cool fact

If you take the molecular structure of the drug LSD and manufacture a mirror image of the molecule then the resulting chemical has no hallucinogenic effect on the body.

Marcus du Sautoy's sexy science

LEANDRO CASTELAO FOR DUTCH UNCLE



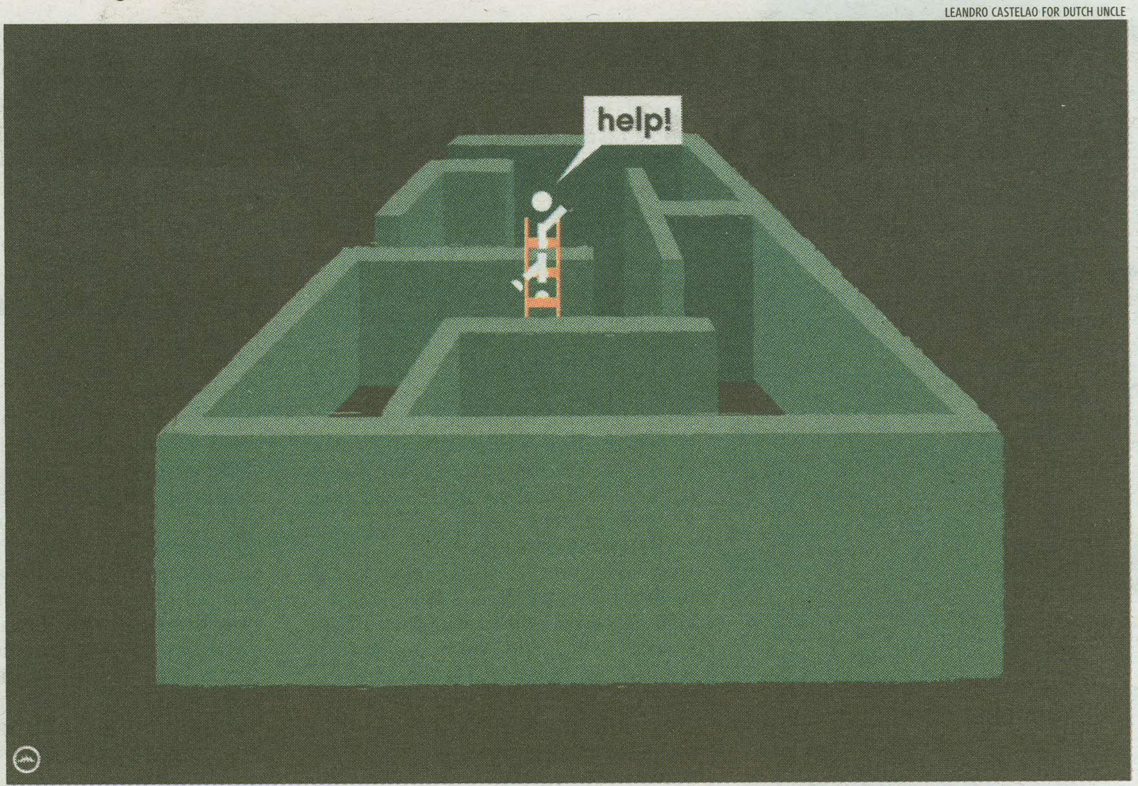
No need to be dazed by mazes

Despite seeing Hampton Court many times from the air when flying into Heathrow, last weekend was the first time I'd visited. As with all these

Marcus du Sautoy's sexy science

No need to be dazed by mazes

LEANDRO CASTELAO FOR DUTCH UNCLE



Despite seeing Hampton Court many times from the air when flying into Heathrow, last weekend was the first time I'd visited. As with all these iconic landmarks you need visitors from abroad to give you a reason to go. Our guests were very excited by the enactment of Henry VIII's marriage to his sixth wife Catherine Parr. But it was the famous hedge maze that I was looking forward to exploring, especially since mazes have been in the scientific news this week.

A recent article in the *Journal of the American Chemical Society* reports on the extraordinary ability of a blob of oil to find the quickest path through any maze. Video footage of the experiment is amazing. The red blob looks almost intelligent as at each junction it makes the correct decision to get to the exit of the maze as quickly as possible. Of course the blob isn't intelligent. Professor Bartosz Grzybowski, of Northwestern University in Evanston, Illinois, who set up the experiment, has done the clever bit.

He gets the oil to find the fastest route by flooding the maze with an alkaline solution of potassium hydroxide. At the exit he adds hydrochloric acid. This acid then leaks through the alkaline solution in the maze setting up a kind of chemical hill. The longer the path from the entrance to the maze to the exit, the shallower the gradient of the hill. Just like a ball, the blob of oil falls down the hill with the steepest gradient.

This isn't just some recreational bit of science. Grzybowski is interested in how to target cancer tumours with drugs, but the maze of blood vessels can result in drugs getting lost as they navigate through the body. The hope is that the same game that turns the oil into a clever chemical rat could be used to deliver drugs quickly and efficiently to targets in the body.

Flooding the maze at Hampton Court with potassium hydroxide was probably not going to endear me to the palace staff. But navigating mazes is of importance not only to the medical industry. Sophisticated mathematical analysis of mazes has helped Google to navigate the internet and the telecommunication industry to route calls efficiently. So I decided to resort to mathematics rather than chemistry to get us through the maze.

Anyone who has read Jerome K. Jerome's *Three Men in a Boat* might remember the visit to Hampton Court when Harris, one of the men in the boat, proposes a theory to get them to the centre: walk along keeping your right hand always in contact with the wall of the maze. When I applied Harris's strategy it worked perfectly. For some reason, Harris has trouble with it but he was probably distracted by the crowds who start following him desperate for help to get out.

Harris's strategy doesn't work for all mazes. If part of the maze is disconnected from the rest of the maze this plan fails because you can never get into the disconnected bit of the maze. Rather a different strategy is needed which owes more to the Brothers Grimm than Jerome K. Jerome and depends on a mathematical theorem proved by the great 18th-century Swiss mathematician Leonhard Euler.

Arm yourself with a packet of pebbles in one pocket and breadcrumbs in the other, and here are your instructions for navigating any maze you might encounter. A new path trodden is marked with pebbles. A path walked for the second time gets breadcrumbs. You must never go down a path more than twice, ie, if it's covered in pebbles and breadcrumbs don't go down.

Entering the maze take any path and start leaving a trail of pebbles behind you. If you come to a new junction just choose any new path and continue trailing the pebbles. If you come to a junction you've been to before (you'll see pebbles crossing it) then turn back down the path you've come from and trail the breadcrumbs to show this is the second time you've been down this path. If you come to a junction you've visited already and you're trailing breadcrumbs then go down a new path if it exists, swapping to trailing pebbles, otherwise go down an old path with just pebbles on it and trail breadcrumbs to indicate that it's the second time you've gone down that path. If you hit a dead end, turn around and trail breadcrumbs back to the last junction you were at.

Thanks to the power of mathematics this will ensure that you visit every bit of the maze and get out again. You just have to hope the birds don't eat all the breadcrumbs before you are out.

View videos of the oil racing through the maze at: http://is.gd/6Vvsb

Cool fact

The biggest permanent hedge maze in the world is the Pineapple Garden Maze in Hawaii. The path is nearly two-and-a-half miles (4km) long and the maze covers an area of just over three acres (12,700m²).

Marcus du Sautoy's sexy science

Earth at the mercy of a restless Sun

On January 19 a huge blast of light and energy erupted from the surface of the Sun, the equivalent of millions of atomic bombs being detonated. This event, called an M-class solar flare, was followed quickly by four more bursts of increasing magnitude from the same source — a sunspot.

Solar flares are so powerful that they can wreak havoc with electricity grids on Earth and scramble GPS equipment. Some have further suggested that the Sun's activity can affect the world's climate. These recent flares herald the beginning of an increase in solar activity that is due to peak in the summer of 2013 as part of solar cycle number 24.

The solar cycle is a pattern that occurs as the Sun's magnetic field flips over, reversing its magnetic poles. The Sun consists of liquid plasma which spins faster around its equator than at its poles, causing the magnetic field to get "squashed up". It's a bit like the isobars that you see on weather maps — the closer the bars are squashed together, the higher the winds. At various points the magnetic field gets pinched, causing a sunspot to appear and a burst of magnetic activity. The increased magnetism reduces the temperature, which, in turn, leads to a reduction of light emitted, making the Sun look darker at these points.

Sunspots were first observed in Ancient China, when desert storms filtered the Sun's glare enough for people to see the black spots on its surface. But it was not until 1843 that Heinrich Schwabe noted the cyclic activity, after making observations of the Sun for two decades. Using detailed records that had been kept of previous sunspots, astronomers were then able to trace activity back to 1755, which is traditionally called solar cycle number 1. Each new cycle begins when the sunspot activity is at its minimum.

The different rates of rotation of the magnetised plasma on the surface of the Sun are like turning cogs which return to their starting point every 11 years. But because the Sun is a fluid rather than a solid body, this cycle is open to variations. Sometimes it is as short as nine years. The last solar cycle was actually one of the longest, lasting for more than 12 and a

LEANDRO CASTELAO/DUTCH UNCLE

Hot fact

Between 1645 and 1715 the Sun experienced very little sunspot activity. Some have proposed that this could explain the Little Ice Age, when the River Thames would regularly freeze over.

half years. Some experts were so worried by the late appearance of solar cycle 24 that they feared it might not happen.

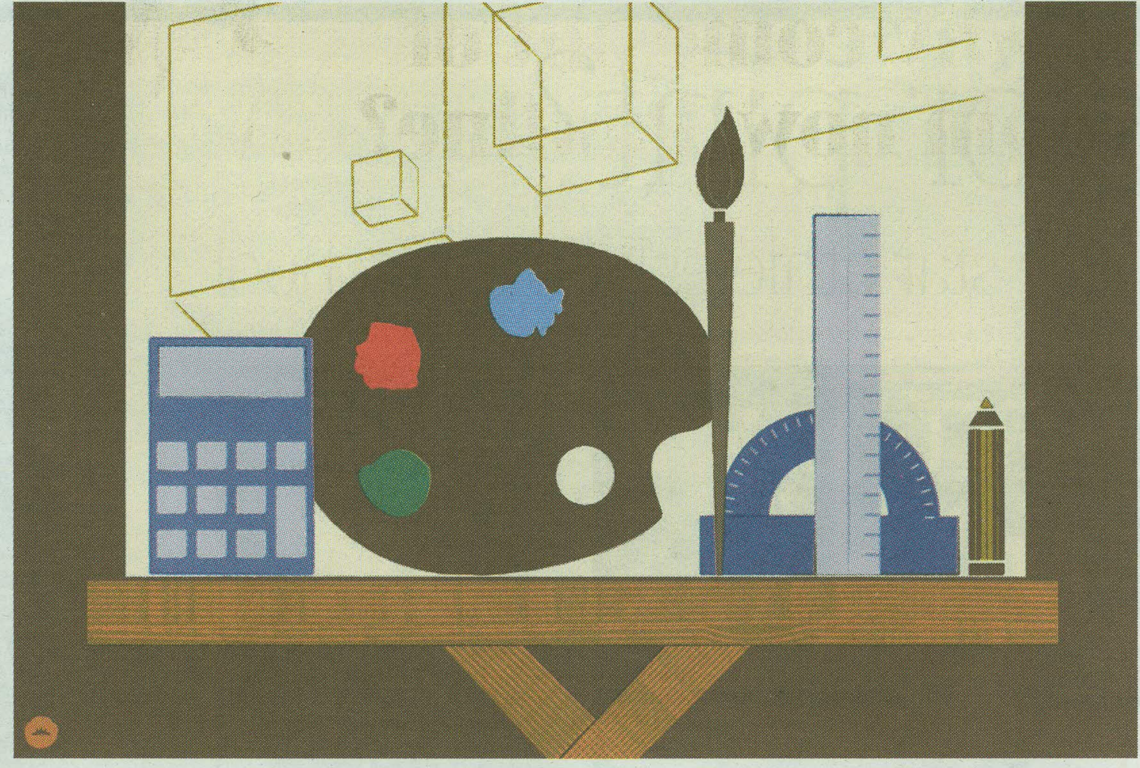
Given that the Sun's variation can cause a difference of 1.4 watts for every square metre on Earth, studying solar activity could be very important in understanding our planet's changing climate. So on February 9, Nasa is planning to launch the Solar Dynamics Observatory, to collect unprecedented data on what is happening in the Sun.

The new observatory will provide images every 10 seconds for five years, giving an almost continuous view of the Sun's activity. To keep in contact with the ground station in New Mexico while maintaining an uninterrupted view of the Sun, its orbit has been plotted to trace an unusual figure of eight in the sky. The data that the observatory will send back to Earth is the equivalent of downloading half a million songs a day from iTunes.

With so many telescopes pointing at stars in the distant recesses of the Universe, it's odd that only now will we obtain a detailed view of our nearest star — the one that we orbit every 365 days.

Marcus du Sautoy's sexy science

LEANDRO CASTELAO FOR DUTCH UNCLE



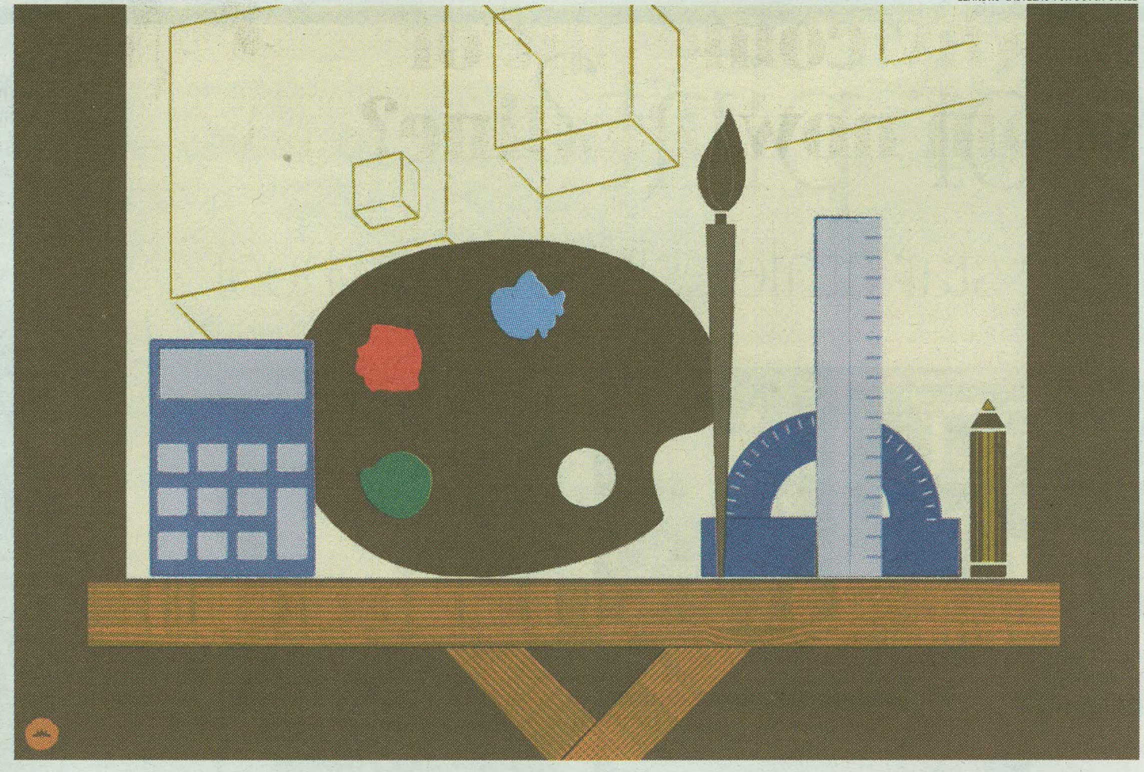
Portrait of the artist as scientist

Traditionally, as you progress through your education, you are expected to make a choice: Shakespeare or the second law of thermodynamics,

Marcus du Sautoy's sexy science

Portrait of the artist as scientist

LEANDRO CASTELAO FOR DUTCH UNCLE



Traditionally, as you progress through your education, you are expected to make a choice: Shakespeare or the second law of thermodynamics, Debussy or DNA, Rubens or relativity. Art or science. But isn't this a false dichotomy? Artists and scientists are often drawn to the same cardinal structures. Frequently artists have consciously chosen to be inspired by scientific ideas but sometimes they have unknowingly recreated the same structures that science has also singled out as important.

Music is often recognised as the creative art that closely resonates with those fascinated by scientific, or more precisely mathematical, structures. As the German philosopher and mathematician Gottfried Leibniz noted: music is the sensation of counting without being aware that you are counting. But the connection runs much deeper than a simple bond between number and beat.

The French composer Olivier Messiaen chose to exploit properties of prime numbers in his piece *Quartet for the End of Time*. The piano part consists of a 17-note rhythmic sequence repeated over and over, but the chord sequence that is played on top of this rhythm consists of 29 chords. As the 17-note rhythm starts for the second time, the chords are just coming up to two thirds of the way through its sequence. The choice of the prime numbers 17 and 29 mean that the rhythmic and chord sequences won't repeat themselves until 17 x 29 notes through the piece. The mathematics of primes provides Messiaen with the key to create a sense of timelessness in the piece.

Messiaen's creativity led him to home in on structures that he was unaware had independent mathematical significance. In *Ile de Feu II*, the two 12-note sequences that Messiaen uses are the basis for generating one of the oddest symmetrical objects discovered by mathematicians.

For those practising the visual arts there is one aspect of the mathematical canon that they will rub up against: geometry. The language of mathematics was key to the painters of the Renaissance creating a sense of 3-D on a 2-D canvas. Painters such as Picasso and Dali in the 20th century were fascinated by mathematicians' ability to conjure shapes in four dimensions, shapes that can never be realised in our physical world. In his painting *Crucifixion (Corpus Hypercubus)* Dali depicts Christ being crucified on the 3-D net of a 4-D cube. The idea of the fourth dimension existing beyond our material world resonated for Dali with the spiritual world, transcending our physical universe. Dali described himself as a carnivorous fish swimming in two waters: the cold water of art and the hot water of science.

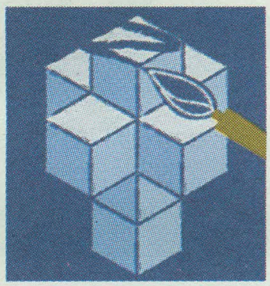
To find mathematical structures beneath literary works is more surprising, but a writer such as Jorge Luis Borges was obsessed with infinity and space. In his story *The Library of Babel*, the narrator tries to conceive the overall shape of the library in which he lives by exploring the network of interconnected hexagonal rooms. He concludes: the library is unlimited and cyclical. If an eternal traveller were to cross it in any direction, after centuries he would see the same volumes were repeated in the same disorder.

Written in the 1940s, Borges' description of the library shares much in common with what science is proposing for the shape of the Universe: a 3-D structure that is finite but unbounded. Just as the 2-D surface of the Earth is wrapped up in three dimensions to create a shape with no boundary, we believe that the 3-D Universe is the surface of some finite four-dimensional shape. Without any scientific training, Borges was trying to capture the same shapes that are at the heart of modern geometry.

Ideas of geometry are at the core of one of the central schools of choreography to emerge from the 20th century. Rudolf Laban's theory of dance grew out of his belief that the human body moves as if its limbs are tracing a network of lines along fundamental symmetrical shapes. Dancers are encouraged to imagine themselves encased inside a cube, or an icosahedron, like a three-dimensional version of Da Vinci's *Vitruvian Man*.

Artists are always seeking interesting new structures to frame their creative process and the mathematician's palette of shapes, patterns and numbers has proved an inspiration. Messiaen and Dali, Borges and Laban are not just creative artists. They are a band of secret mathematicians.

Marcus du Sautoy gives the Faraday Lecture tonight at the Royal Society entitled *The Secret Mathematicians*

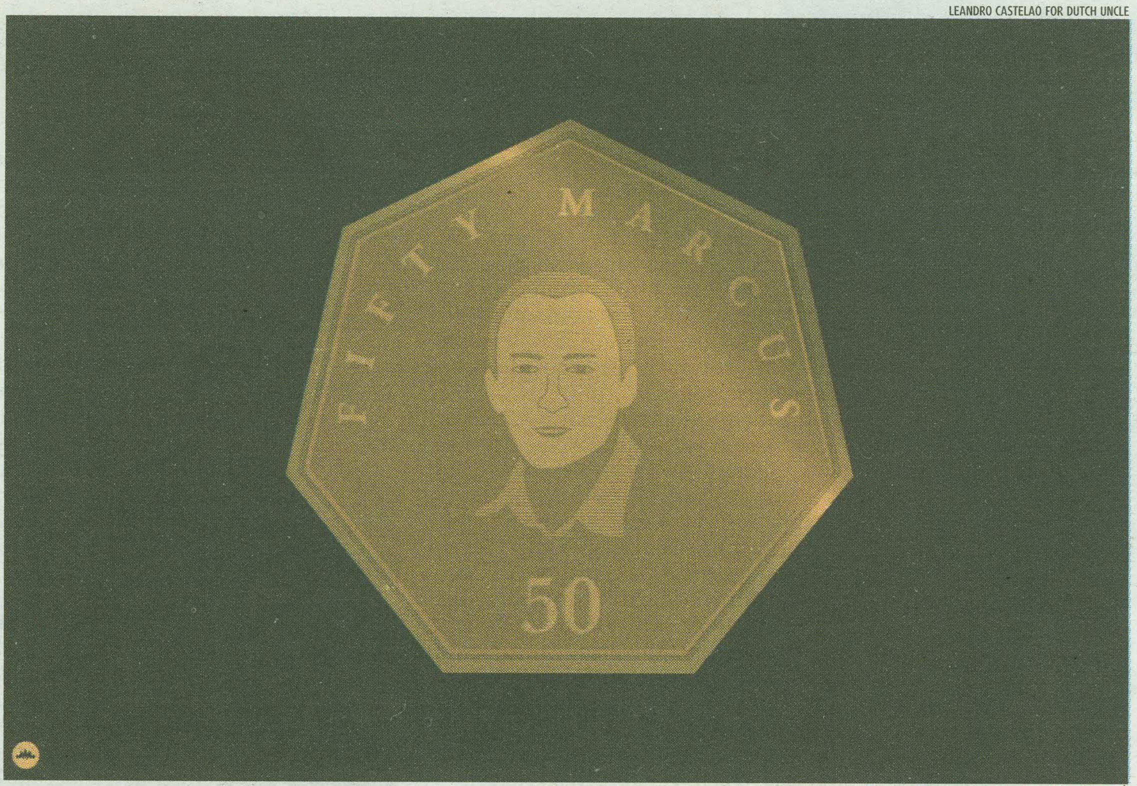


Cool fact

Unwrap a 3-D cube into its 2-D net and you see six squares arranged in the shape of a cross. The net of a 4-D cube used by Dali consists of eight cubes, four stacked in a column and four glued to the sides of one of the cubes in this central column.

Marcus du Sautoy's sexy science

Satisfying symmetry



LEANDRO CASTELAO FOR DUTCH UNCLE

I am going to be indulgent today and talk about my research into the mathematics of symmetry. The concept is fundamental across all the sciences: chemists use symmetry to understand crystal structures; physicists can predict what they may see in the Large Hadron Collider at CERN because of a strange symmetrical object in multidimensional space; biologists discovered that the virulence of many viruses is down to their symmetrical shape.

But to tell you the truth, despite all these wonderful applications of the mathematics of symmetry, my motivation is purely selfish. I love the buzz of discovering some new eternal truth about the mathematical world. The adrenaline rush of creating a strange symmetrical object never seen before, with interesting new properties, is addictive. Of course, it's wonderful if there is some killer application of your discovery, but that is rarely the motivation for hiking to the extremes of the mathematical landscape.

One question that I am trying to answer is: how many symmetrical objects are there? That question is too vague, as the answer is infinitely many. But if I ask how many symmetrical objects there are with a given number of symmetries — say 343 symmetries — then the question becomes more tangible.

The answer is quite simple if an object has a prime number of symmetries. For example, how many symmetrical objects are there with seven symmetries? The answer: only one. Place a seven-sided 50p piece on a piece of paper and draw round it. The rotational symmetries of the coin are the different ways that you can spin it so that it fits back inside its outline. For example, you can spin it by 1/7 of a turn clockwise or 2/7 of a turn anticlockwise. You'll find that there are six essentially different spins that you can make. Mathematicians like to add a seventh symmetry, where you just leave the coin where it is. This is like the zeroth symmetry. Just as with numbers, having a zero is incredibly useful. So the 50p piece has seven rotational symmetries. This is essentially the only object with seven symmetries.

What if I now consider the number of objects with 49=7x7 symmetries? Now there are two different objects. A 49-sided coin has 49 rotational symmetries. But there is another object with 49 symmetries that is essentially different from the coin. Consider a combination lock consisting of two wheels with the numbers 1-7 on each wheel. A symmetry of the combination lock consists of all the different ways that I can spin the wheels. Starting with the wheels set at (1,1) there are 49 different settings of the wheels, so this is also an object with 49 symmetries.

So there is one object with seven symmetries, two with 49=7x7 symmetries. How many are there with 343=7x7x7, or 7x7x7x7? Is there a pattern to the way the number of symmetrical objects increases the more sevens I multiply by?

At the moment we know that the sequence goes 1,2,5,15,83,860. In a stunning act of calculation, Mike Vaughan-Lee in Oxford and Eamonn O'Brien in Auckland, came up with a formula which calculated that there are 113,147 different symmetrical objects with 77 symmetries. But, beyond this, things are a mystery.

Part of my research has tried to shed light on whether there is a pattern to the way this sequence evolves. If you are faced with a sequence such as 1,2,3, 5, 8, 13 . . . then most people will spot that you arrive at the next number by adding the previous two numbers, giving you the famous Fibonacci sequence. My research indicates that there is a similar but more complicated rule governing the behaviour of the numbers in counting symmetrical objects. The trouble is pinning down what the exact rule is.

It is challenges such as these that make mathematics a living subject. What is exciting is what we don't know.

You might say, who cares? But it was discoveries of strange new symmetrical objects in multidimensional space that led to new error-correcting codes that are at the heart of the telecommunications industry. Millions of people use this maths every day to talk on the phone, download films or send e-mails without interference corrupting the data.

But these symmetrical objects were never discovered with such uses in mind. Anyone who chooses to fund research hoping to chase applications will miss so much of the mathematics that underpins the modern world. That was discovered by people who just loved the thrill of exploring the mathematical landscape for its own sake.

As part of Jewish Book Week, Marcus du Sautoy is representing England in a match against the Israel Writers Football Team on Sunday at 11.30am. See *jewishbookweek.com*

Cool Fact

For a donation to Common Hope, a charity supporting education in Guatemala, I will name a new symmetrical object in your honour. Go to firstgiving.com/findingmoonshine