Solution (#12) Let $z_i = x_i + iy_i$ for $i = 1, 2, 3$. Then

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + x_1 y_2).$$

As $x_1 x_2 - y_1 y_2$ and $y_1 x_2 + x_1 y_2$ are symmetric in 1 and 2, i.e. invariant when we swap 1 and 2, then $z_1 z_2 = z_2 z_1$.

Likewise

$$\begin{aligned} (z_1 z_2) z_3 &= [(x_1 x_2 - y_1 y_2) + i(y_1 x_2 + x_1 y_2)](x_3 + iy_3) \\ &= [x_1 x_2 x_3 - (y_1 y_2 x_3 + y_1 y_3 x_2 + y_2 y_3 x_1)] + i[(x_2 x_3 y_1 + x_1 x_3 y_2 + x_1 x_2 y_3) - y_1 y_2 y_3]. \end{aligned}$$

As

$$x_1 x_2 x_3, \qquad y_1 y_2 x_3 + y_1 y_3 x_2 + y_2 y_3 x_1, \qquad x_2 x_3 y_1 + x_1 x_3 y_2 + x_1 x_2 y_3, \qquad y_1 y_2 y_3$$

are symmetric in $1, 2$ and 3, and in particular don't change when change $1, 2, 3$ for $2, 3, 1$ respectively, then

$$(z_1 z_2) z_3 = (z_2 z_3) z_1 = z_1 (z_2 z_3)$$

with the last equality following from the first part.