

**Solution (#26)**

- $\cos \alpha - i \sin \alpha = 1 (\cos(-\alpha) + i \sin(-\alpha))$ , so this has modulus 1 and argument  $-\alpha$  or  $2\pi - \alpha$  to put the answer in our preferred range.
- $\sin \alpha - i \cos \alpha = -i (\cos \alpha + i \sin \alpha)$ , so this has modulus 1 and argument  $\alpha - \pi/2$  or rather  $\alpha + 3\pi/2$  to put the answer in our preferred range.
- $1 + i \tan \alpha = \sec \alpha (\cos \alpha + i \sin \alpha)$  and so the modulus is  $\sec \alpha$  and the argument is  $\alpha$ .
- $|1 + \cos \alpha + i \sin \alpha| = \sqrt{(1 + \cos \alpha)^2 + \sin^2 \alpha} = \sqrt{2 + 2 \cos \alpha} = \sqrt{4 \cos^2 (\alpha/2)} = 2 \cos (\alpha/2)$ . The argument is

$$\tan^{-1} \left( \frac{\sin \alpha}{1 + \cos \alpha} \right) = \tan^{-1} \left( \frac{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{2 \cos^2 \frac{\alpha}{2}} \right) = \frac{\alpha}{2}.$$