

Solution (#29) Let z, w be complex numbers. Then

$$\begin{aligned} |1 - \bar{z}w|^2 - |z - w|^2 &= (1 - \bar{z}w)(1 - z\bar{w}) - (z - w)(\bar{z} - \bar{w}) && \text{[using (1.14), (1.15), (1.16)]} \\ &= 1 - z\bar{z} - w\bar{w} + z\bar{z}w\bar{w} \\ &= (1 - z\bar{z})(1 - w\bar{w}) \\ &= (1 - |z|^2)(1 - |w|^2). \end{aligned}$$