Solution (\#43) Let $a, b$ be distinct complex numbers and $r$ a real number. If $|r|>|a-b|$ then the triangle inequality shows

$$
|z-a|-|z-b|=r
$$

has no solutions.
If $r=|a-b|$ then the solutions $z$ comprise half of the line connecting $a$ and $b$, specifically the half which starts at $b$ and moves away from $a$.

If $r=-|a-b|$ then the equation is solved by those $z$ which lie on the half-line connecting $a$ and $b$, which starts at $a$ and moves away from $b$.

