Solution (#72) Let $p = m/n$ be a rational in lowest form and z, w be non-zero complex numbers. If $\alpha \in z^p$ and $\beta \in w^p$ then $\alpha^n = z^m$ and $\beta^n = w^m$ and so

$$(\alpha\beta)^n = \alpha^n \beta^n = z^m w^m = (zw)^m$$

showing $\alpha\beta \in (zw)^p$, i.e. $z^p w^p \subseteq (zw)^p$. Conversely say $\gamma \in (zw)^p$ and take any $\alpha \in z^p$. Then we need to show $\gamma/\alpha \in w^p$. We have

$$(\gamma/\alpha)^n = \gamma^n/\alpha^n = (zw)^m/z^m = z^m w^m/z^m = w^m.$$

Hence $\gamma/\alpha \in w^p$ and $(zw)^p \subseteq z^p w^p$ completing the proof.