Solution (#85) Our aim is to find x and y so that

$$z^3 + mz + n = z^3 + (-3xy)z + x^3 + y^3.$$

From the given identity we then know that $z = -x - y$ is a root of the cubic $z^3 + mz + n = 0$.

So we have two simultaneous equations

$$-3xy = m, \qquad x^3 + y^3 = n.$$

We can use the first equation to eliminate y so that the second becomes a quadratic in x^3 which we can solve (taking some care that $xy = -m/3$ still holds).

We see now that this method is essentially Cardano's method with $x = -T$ and $y = U$, where the found root is

$$z = T - U = -x - y$$

and

$$xy = -m/3 = (-T)U, \qquad x^3 + y^3 = n = -t + u = (-T)^3 + U^3.$$