Solution (#88) For the cubic $z^3 - 12z + 8 = 0$ set $z = w + 4/w$ to find

$$w^3 = -4 \pm 4\sqrt{3}i.$$

Then we have six possible solutions for w, namely

$$\begin{aligned} w_1 &= 2\,\mathrm{cis}\,\frac{2\pi}{9}; \qquad w_2 = 2\,\mathrm{cis}\,\frac{8\pi}{9}; \qquad w_3 = 2\,\mathrm{cis}\,\frac{14\pi}{9}; \\ w_4 &= 2\,\mathrm{cis}\,\frac{4\pi}{9}; \qquad w_5 = 2\,\mathrm{cis}\,\frac{10\pi}{9}; \qquad w_6 = 2\,\mathrm{cis}\,\frac{16\pi}{9}. \end{aligned}$$

This in turn leads to six possible roots z, namely

$$z_1 = 4\cos\frac{2\pi}{9}; \qquad z_2 = 4\cos\frac{8\pi}{9}; \qquad z_3 = 4\cos\frac{14\pi}{9}$$

and, for the three further roots, we find $z_4 = z_3$, $z_5 = z_2$, $z_6 = z_1$ so that there are in fact only three solutions which correspond to those found previously in Example 1.31.