

Solution (#99) Let $a = 2$, $b = 1 + i$, $c = 3 + it$. Then

$$\begin{aligned}\angle abc &= \arg\left(\frac{(3 + it) - (1 + i)}{(2 - (1 + i))}\right) \\ &= \arg(2 + i(t - 1)) - \arg(1 - i) \\ &= \tan^{-1}\left(\frac{t - 1}{2}\right) + \frac{\pi}{4}.\end{aligned}$$

So

$$\begin{aligned}\tan^{-1}\left(\frac{t - 1}{2}\right) + \frac{\pi}{4} &= \frac{\pi}{3}, \\ \implies \tan^{-1}\left(\frac{t - 1}{2}\right) &= \frac{\pi}{12}, \\ \implies \frac{t - 1}{2} &= \tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}, \\ \implies t &= 1 + 2(2 - \sqrt{3}) = 5 - 2\sqrt{3}.\end{aligned}$$

If t becomes large and positive then $\tan^{-1}((t - 1)/2) \approx \pi/2$ and so $\angle abc = 3\pi/4$.