Solution (#100) Let a, b be non-zero complex numbers representing points A, B in the Argand diagram. For real numbers λ, μ we have

$$\begin{aligned} |\lambda a+\mu b| \geqslant |\lambda a| \iff & \quad |\lambda a+\mu b|^2 \geqslant |\lambda a|^2 \\ \iff & \quad (\lambda a+\mu b)\left(\lambda \bar{a}+\mu \bar{b}\right) \geqslant \lambda^2 |a|^2 \\ \iff & \quad \lambda^2 |a|^2 + \lambda\mu\left(a\bar{b}+\bar{a}b\right) + \mu^2 |b|^2 \geqslant \lambda^2 |a|^2 \\ \iff & \quad 2\lambda\mu \operatorname{Re}\left(a\bar{b}\right) + \mu^2 |b|^2 \geqslant 0. \end{aligned}$$

If this holds for all real λ, μ then it must be the case that $\operatorname{Re}\left(a\bar{b}\right) = 0$. Conversely if $\operatorname{Re}\left(a\bar{b}\right) = 0$ then the inequality clearly holds for all real λ, μ.

Now $\operatorname{Re}\left(a\bar{b}\right) = 0$ if and only if

$$\pm\frac{\pi}{2} = \arg\left(a\bar{b}\right) = \arg\left(\frac{a\bar{b}}{|b|^2}\right) = \arg\left(a/b\right)$$

so that AOB is a right angle.