Solution (#102) If we take set $w_2 = z_2 - z_1$ and $w_3 = z_3 - z_1$ then the equation

$$\frac{z_2 - z_1}{z_3 - z_1} = \frac{z_1 - z_3}{z_2 - z_3}$$

becomes

$$\begin{aligned} \frac{w_2}{w_3} &= \frac{-w_3}{w_2 - w_3}, \\ \implies (w_2)^2 - w_2 w_3 + w_3^2 &= 0, \\ \implies \left(\frac{w_3}{w_2}\right)^2 - \left(\frac{w_3}{w_2}\right) + 1 &= 0, \\ \implies w_3 = \operatorname{cis}\,(\pm\pi/3)\, w_2, & \qquad \text{[solving this quadratic].} \end{aligned}$$

In either case, $w_3 = \operatorname{cis}\,(\pm\pi/3)\; w_2$ means that the $w_3 = z_3 - z_1 = \overrightarrow{z_1 z_3}$ is of equal length to $w_2 = z_2 - z_1 = \overrightarrow{z_1 z_2}$ and lies at $\pi/3$ from it (anti-clockwise or clockwise). This means that $\triangle z_1 z_2 z_3$ is an equilateral triangle.