Solution (#113) (i)

$$\frac{1}{2+ti}=\frac{2-ti}{t^2+4} \quad \text{so } x=\frac{2}{4+t^2},\ y=\frac{-t}{4+t^2};$$

hence $y/x=-t/2$. So

$$x=\frac{2}{4+4y^2/x^2}=\frac{x^2}{2\left(x^2+y^2\right)} \implies \qquad x^2+y^2=\frac{x}{2}$$

$$\implies \left(x-\frac{1}{4}\right)^2+y^2=\frac{1}{4^2}.$$

Any point in the image of $\operatorname{Re} z=2$ then lies on this circle, but the point $(x,y)=(0,0)$ is not in the image. As $y=mx$ intersects with this circle for all real values of m then every other point on the circle is in the image.

(ii) $i\,(2+it)=-t+2i$ which, as t varies, maps out all of the line $\operatorname{Im} z=2$;

$(2+it)^2=\left(4-t^2\right)+4it$ which as t varies maps out the curve $x=4-(y/4)^2$ which is a parabola

$$\frac{1}{1+it}=\frac{1-it}{1+t^2} \implies x=\frac{1}{1+t^2},\ y=\frac{-t}{1+t^2}$$

so $y/x=-t$ and

$$x=\frac{1}{1+y^2/x^2}=\frac{x^2}{x^2+y^2} \implies x^2+y^2=x \implies \left(x-\frac{1}{2}\right)^2=\frac{1}{2^2},$$

which is all of the circle with centre $(1/2,0)$, radius $1/2$ except for the origin.