Solution (#136) (i) Let p, q, r be three distinct points in the complex plane. The complex number representing the vector from p to q is $q - p$. Hence the points on the line connecting p and q are represented by complex numbers $p + \lambda (q - p)$ where λ is real. Hence r is collinear with p and q if and only if $(r - p) / (q - p) = \lambda$ is real.

(ii) A complex number w is real if and only if $w = \bar{w}$. So z lies on the line connecting p and q if and only if

$$\frac{z-p}{q-p} = \overline{\left(\frac{z-p}{q-p}\right)} = \frac{\bar{z}-\bar{p}}{\bar{q}-\bar{p}} \qquad \text{[using (1.15) and (1.17)]}$$

which rearranges to

$$(\bar{q} - \bar{p})\, z + (p - q)\, \bar{z} = p\bar{q} - q\bar{p}$$