Solution (#140) Let a, b, c be complex numbers representing the vertices of the triangle ABC. From Example 1.50 we know that d, e, f (complex numbers representing D, E, F) satisfy

$$d + \omega b + \omega^2 a = 0, \qquad e + \omega c + \omega^2 b = 0, \qquad f + \omega a + \omega^2 c = 0$$

where $\omega = \operatorname{cis}(2\pi/3)$. The centroid of ADB is $(a + d + b)/3$ (see #136) with similar expressions for the centroid of BEC and CFA. Finally note

$$\begin{aligned}
& \left(\frac{a+d+b}{3}\right) + \omega\left(\frac{b+e+c}{3}\right) + \omega^2\left(\frac{c+f+a}{3}\right) \\
= \;& \frac{1}{3}\left[\left(a - \omega b - \omega^2 a + b\right) + \omega\left(b - \omega c - \omega^2 b + c\right) + \omega^2\left(c - \omega a - \omega^2 c + a\right)\right] \\
= \;& \frac{1}{3}\left[\left(1 - \omega^2 - \omega^3 + \omega^2\right) a + \left(-\omega + 1 + \omega - 1\right) b + \left(-\omega^2 + \omega + \omega^2 - \omega^4\right) c\right] \\
= \;& 0 \qquad [\text{as } \omega^3 = 1].
\end{aligned}$$