Solution (#151) Let $a \neq 0$.

Use Ptolemy to show $1, a, -\bar{a}, a^{-1}$ are concyclic in that order when $\operatorname{Re} a \geqslant 0$.

Use Ptolemy to show $1, -\bar{a}, a, a^{-1}$ are concyclic in that order when $\operatorname{Re} a < 0$. We have shown now that $1, a, -\bar{a}, a^{-1}$ are concyclic for any non-zero a.

Replace a with $-\bar{a}$ to show $1, -\bar{a}, a, -\bar{a}^{-1}$ are concyclic.

We can then deduce that all five points $1, -\bar{a}, a, a^{-1}, -\bar{a}^{-1}$ are concyclic.

Finally consider the images of these five points under the map $z \mapsto -\bar{z}$ (which is reflection in the imaginary axis).