

Solution (#162) Considering the primitive roots of unity in each case we have

$$\Phi_1(x) = x - 1; \quad \Phi_2(x) = x - (-1) = x + 1; \quad \Phi_4(x) = (x - i)(x - (-i)) = x^2 + 1.$$

For the third and fifth cases, 1 is the only root that is not primitive and so

$$\Phi_3(x) = \frac{x^3 - 1}{x - 1} = x^2 + x + 1; \quad \Phi_5(x) = \frac{x^5 - 1}{x - 1} = x^4 + x^3 + x^2 + x + 1.$$

Finally

$$\Phi_6(x) = (x - \operatorname{cis}(2\pi/6))(x - \operatorname{cis}(5\pi/6)) = x^2 - 2\cos(\pi/3)x + 1 = x^2 - x + 1.$$