Solution (#185) We have $P_c(z) = z^2 + c$ and so

$$Q_c(z) = P_c(P_c(z)) = (z^2 + c)^2 + c = z^4 + 2cz^2 + (c^2 + c).$$

So z is in a period 2 orbit if $Q_c(z) = z$ and $P_c(z) \neq z$. Note that

$$\begin{aligned} Q_c(z) - z &= z^4 + 2cz^2 - z + (c^2 + c) \\ &= (z^2 - z + c)(z^2 + z + c + 1) \\ &= (P_c(z) - z)(z^2 + z + c + 1). \end{aligned}$$

So if z is in a period 2 orbit then

$$z^2 + z + c + 1 = 0.$$

We see that this has distinct roots

$$z = \frac{-1 \pm \sqrt{-3 - 4c}}{2} \qquad \text{provided } c \neq -3/4.$$

If $c \neq -3/4$ these distinct roots make a period 2 orbit. However if $c = -3/4$ then these two roots coincide as $z = -1/2 = P_c(z)$ and so, in this case, z is a fixed point.

We note that 0 is in this period 2 orbit when $c = -1$.