Solution (#203) (i) From the definition

$$\begin{aligned}(b,a;d,c) &= \frac{(b-d)(a-c)}{(b-c)(a-d)} = (a,b;c,d)\,;\\ (c,d;a,b) &= \frac{(c-a)(d-b)}{(c-b)(d-a)} = (a,b;c,d)\,,\end{aligned}$$

and using these two identities in turn

$$(a,b;c,d) = (b,a;d,c) = (d,c;b,a)\,.$$

(ii) Further

$$(a,b;d,c) = \frac{(a-d)(b-c)}{(a-c)(b-d)} = \left(\frac{(a-c)(b-d)}{(a-d)(b-c)}\right)^{-1} = \frac{1}{(a,b;c,d)},$$

and

$$\begin{aligned}(a,c;b,d) &= \frac{(a-b)(c-d)}{(a-d)(c-b)}\\ &= -\left[\frac{ac+bd-ad-bc}{(a-d)(b-c)}\right]\\ &= \left[\frac{ab+dc-bd-ac}{(a-d)(b-c)}\right] - \left[\frac{ab+dc-ad-bc}{(a-d)(b-c)}\right]\\ &= 1-\frac{(a-c)(b-d)}{(a-d)(b-c)}\\ &= 1-(a,b;c,d)\,.\end{aligned}$$