

Solution (#1580) Let

$$y(x) = \begin{cases} \frac{1}{2}e^x - 1 & \text{for } x \leq \ln 2, \\ 1 - 2e^{-x} & \text{for } \ln 2 \leq x. \end{cases}$$

Then

$$\begin{aligned} y'(x) &= \frac{1}{2}e^x = 1 - (1 - \frac{1}{2}e^x) = 1 - |y| & \text{for } x < \ln 2; \\ y'(x) &= 2e^{-x} = 1 - (1 - 2e^{-x}) = 1 - |y| & \text{for } x > \ln 2. \end{aligned}$$

Note also that

$$y'(\ln 2) = \frac{1}{2}e^{\ln 2} = 2e^{-\ln 2} = 1 = 1 - |y(\ln 2)|$$

is well-defined. Finally

$$y(0) = \frac{1}{2}e^0 - 1 = -\frac{1}{2}.$$

A sketch of the solution is given below:

