Solution (#1607) Consider the DE

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+y-1}{2x+2y-1}.$$

If we substitute $x = X + a$, $y = Y + b$ then the RHS becomes

$$\frac{X+Y+a+b-1}{2X+2Y+2a+2b-1}.$$

For this function to be homogeneous polar we need that

$$a+b=1 \qquad \text{and} \qquad 2a+2b=1$$

which clearly are contradictory equations.

Alternatively if we set $z = x + y$ then the DE becomes

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 1 = \frac{z-1}{2z-1}.$$

We can rearrange this into a separable DE

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{3z-2}{2z-1};$$

separating the variables and integrating we find

$$x + c = \int \mathrm{d}x = \int \frac{2z-1}{3z-2}\,\mathrm{d}z = \int \frac{2}{3} + \frac{1/3}{3z-2}\,\mathrm{d}z = \frac{2}{3}z + \frac{1}{9}\ln|3z-2|$$

where c is a constant. Recalling that $z = x + y$, we find

$$x + c = \frac{2}{3}x + \frac{2}{3}y + \frac{1}{9}\ln|3x+3y-2|$$

which rearranges to

$$\ln|3x+3y-2| + 6y - 3x = \text{const.}.$$