

Solution (#1639) $y = e^{-2x}$ is a solution. The other is $y = e^{-x/2}$. Make the change of variable $y = e^{-2x}z$ so that

$$(x + 2)z'' = (2x + 3)z'$$

which is a separable DE. The general solution to the DE

$$y(x) = e^{2x} \left(A + B \int_0^x \frac{e^{2t} dt}{t + 2} \right).$$

Given the initial conditions we find

$$y(x) = e^{2x} \left(1 - 2 \int_0^x \frac{e^{2t} dt}{t + 2} \right).$$