Solution (#1678) The system can be written

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 & -1 & -1 \\ 1 & 3 & 1 \\ -2 & 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

The characteristic polynomial of the above 3×3 matrix is $(4 - \lambda)(2 - \lambda)(6 - \lambda)$. The system's general solution is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = Ae^{2t}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + Be^{4t}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + Ce^{6t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$