Solution (#1680) The system can be rewritten in terms of $\mathbf{r}(t) = (x(t), y(t))^T$ as

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix} \mathbf{r} + \begin{pmatrix} 2 \\ e^{2t} \end{pmatrix}, \qquad \mathbf{r}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The homogeneous system has a general solution of

$$\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + Be^{6t} \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

A particular solution is

$$x = -\frac{4}{3} - \frac{1}{4}e^{2t}, \qquad y = 2 + \frac{1}{4}e^{2t}.$$

Finally given the initial conditions we have

$$x = \frac{9}{5}e^{t} + \frac{47}{60}e^{6t} - \frac{4}{3} - \frac{1}{4}e^{2t}, \qquad y = -\frac{18}{5}e^{t} + \frac{47}{20}e^{6t} + 2 + \frac{1}{4}e^{2t}.$$