Solution (#1682) The system in (6.63) reads

$$\phi' = (ab/k)\,\rho, \qquad \rho' = (-km/a)\,\phi.$$

So

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(k^2m\phi^2 + a^2b\rho^2\right) &= 2k^2m\phi\phi' + 2a^2b\rho\rho' \\ &= 2k^2m\phi\left(\frac{ab}{k}\right)\rho + 2a^2b\rho\left(-\frac{km}{a}\right)\phi \\ &= 2kmab\phi\rho - 2kmab\phi\rho = 0.\end{aligned}$$

It follows that $k^2m\phi^2 + a^2b\rho^2$ is a constant of the system.

Call this constant K (which is necessarily positive). The curve

$$k^2m\phi^2 + a^2b\rho^2 = K$$

is an ellipse in the $\phi\rho$-plane. Recall

$$F = \phi + \frac{b}{k} \qquad \text{and} \qquad R = \rho + \frac{m}{a}$$

and so the (F, R) describes a translation of this ellipse about the equilibrium $(b/k, m/a)$.