

Solution (#1684) The Lotka-Volterra equations read

$$\frac{dF}{dt} = -mF + aFR, \quad \frac{dR}{dt} = bR - kFR.$$

Solution (i) If we set $F(t) = 0$, $R(t) = \varepsilon e^{bt}$ then we have

$$\begin{aligned} \frac{dF}{dt} &= 0, & -mF + aFR &= 0; \\ \frac{dR}{dt} &= b\varepsilon e^{bt}, & bR - kFR &= b\varepsilon e^{bt}. \end{aligned}$$

(ii) If we set $F(t) = \varepsilon e^{-mt}$, $R(t) = 0$ then we have

$$\begin{aligned} \frac{dF}{dt} &= -m\varepsilon e^{-mt}, & -mF + aFR &= -m\varepsilon e^{-mt}; \\ \frac{dR}{dt} &= 0, & bR - kFR &= 0. \end{aligned}$$

It follows that the equilibrium point $(0, 0)$ is not stable. If we were to perturb the point $(0, 0)$ to $(0, \varepsilon)$ (by introducing only rabbits) then the rabbit population would increase exponentially.