Solution (#1686) (i) The only equilibrium is $(x,y)=(0,0)$. If $x(0)=\varepsilon_1$ and $y(0)=\varepsilon_2$ then we see

$$(x(t),y(t))=(\varepsilon_1 e^t,\varepsilon_2 e^t)$$

and so this equilibrium is not stable. All perturbations lead to movement away from the origin.
(ii) The only equilibrium is $(x,y)=(0,0)$. If $x(0)=\varepsilon_1$ and $y(0)=\varepsilon_2$ then we see

$$(x(t),y(t))=(\varepsilon_1 e^{-t},\varepsilon_2 e^{-t})$$

and so this equilibrium is stable. All perturbations lead to movement towards the origin.
(iii) The only equilibrium is $(x,y)=(0,0)$. If $x(0)=\varepsilon_1$ and $y(0)=\varepsilon_2$ then we see

$$(x(t),y(t))=(\varepsilon_1 e^{t},\varepsilon_2 e^{-t})$$

and so this equilibrium is not stable – as perturbation with any non-zero x-component will move away from the origin.
(iv) The only equilibrium is we see $(x,y)=(0,0)$. Say $x(0)=\varepsilon_1$ and $y(0)=\varepsilon_2$, then

$$x(t)=\varepsilon_1\cosh t+\varepsilon_2\sinh t,\qquad y(t)=\varepsilon_2\cosh t+\varepsilon_1\sinh t$$

and the equilibrium is not stable. All perturbations lead to movement away from the origin.
(v) The only equilibrium is $(x,y)=(0,0)$. Say $x(0)=\varepsilon_1$ and $y(0)=\varepsilon_2$ then

$$x(t)=\varepsilon_1\cos t+\varepsilon_2\sin t,\qquad y(t)=\varepsilon_2\cos t-\varepsilon_1\sin t.$$

The equilibrium is stable as all perturbations lead to movements near the origin – specifically on elliptical paths (#1165).