

Solution (#1696) Let $r > 0$ and set

$$f(x) = e^{-rx^2}.$$

Then for any s ,

$$\begin{aligned}\bar{f}(s) &= \int_0^\infty \exp\left(-r\left(x + \frac{s}{2r}\right)^2 + \frac{s^2}{4r}\right) dx \\ &= \exp\left(\frac{s^2}{4r}\right) \int_{s/2\sqrt{r}}^\infty \exp(-v^2) \frac{dv}{\sqrt{r}} \\ &= \frac{1}{2}\sqrt{\frac{\pi}{r}} \exp\left(\frac{s^2}{4r}\right) \left(1 - \operatorname{erf}\left(\frac{s}{2\sqrt{r}}\right)\right).\end{aligned}$$