Solution (#1701) The error function erf x can be written as a convolution:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, \mathrm{d}t = \left(\frac{2}{\sqrt{\pi}} e^{-x^2} \right) * (1).$$

Hence its Laplace transform equals

$$\overline{\operatorname{erf}}(s) = \frac{2}{\sqrt{\pi}} \overline{e^{-x^2}} \times \overline{1}.$$

Now by #1696 the Laplace transform of e^{-x^2} equals

$$\frac{\sqrt{\pi}}{2} \exp\left(\frac{s^2}{4}\right) \left(1 - \operatorname{erf}\left(\frac{s}{2}\right)\right),$$

and the Laplace transform of 1 equals s^{-1}. Hence

$$\begin{aligned}
\overline{\operatorname{erf}}(s) &= \frac{2}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2} \exp\left(\frac{s^2}{4}\right) \left(1 - \operatorname{erf}\left(\frac{s}{2}\right)\right) \times \frac{1}{s} \\
&= \frac{1}{s} \exp\left(\frac{s^2}{4}\right) \left(1 - \operatorname{erf}\left(\frac{s}{2}\right)\right).
\end{aligned}$$