

Solution (#1713) Transforming the DE we find

$$(s^2 + s)\frac{d\bar{f}}{ds} + (3s + 2)\bar{f} = 2A.$$

where $A = f(0)$. So

$$s^2(s + 1)\bar{f} = As^2 + B$$

where B is a constant. We then have

$$\bar{f}(s) = \frac{A}{s + 1} + B\left(-\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s + 1}\right).$$

Hence we have

$$f(x) = Ae^{-x} + B(e^{-x} + x - 1) = \alpha e^{-x} + B(x - 1).$$