Solution (#1716) The DE transforms to

$$s(1-s)\frac{\mathrm{d}\bar{y}}{\mathrm{d}s}+(n+1-s)\bar{y}=0$$

so that

$$\bar{y}(s)=A\frac{(s-1)^n}{s^{n+1}}$$

for some constant A. We noted previously that $\overline{L_n}(s)=(s-1)^n s^{-n-1}$.

Inverting we see

n	$\overline{L_n}(s)$	$L_n(x)$
0	s^{-1}	1
1	$s^{-1}-s^{-2}$	$1-x$
2	$s^{-1}-2s^{-2}+s^{-3}$	$1-2x+\frac{1}{2}x^2$