Solution (#1728) The life time T of a particular brand of light bulb is modelled as follows. There is a probability p of the light-bulb blowing immediately (so that $T = 0$); given that the light bulb does not blow immediately, the probability of it having life time $\tau > 0$ or less is $1 - e^{-\lambda\tau}$ (where $\lambda > 0$).

(i) The cumulative distribution function is

$$F(t) = \begin{cases} p + q(1 - e^{-\lambda t}) = 1 - qe^{-\lambda t} & t \geqslant 0 \\ 0 & t < 0 \end{cases}$$

where $q = 1 - p$.

(ii) The (generalized) probability density function $f(t)$ is then

$$f(t) = F'(t) = p\delta(t) + q\lambda e^{-\lambda t}\mathbf{1}_{(0,\infty)}(t)$$

(iii) By the Sifting Property, the expectation of T equals

$$\begin{aligned} & \int_{-\infty}^{\infty} t\left(p\delta(t) + q\lambda e^{-\lambda t}\mathbf{1}_{(0,\infty)}(t)\right)\,\mathrm{d}t \\ = & \ (pt)_{t=0} + \int_0^{\infty} q\lambda t e^{-\lambda t}\,\mathrm{d}t \\ = & \ q\left\{\left[-te^{-\lambda t}\right]_0^{\infty} + \int_0^{\infty} e^{-\lambda t}\,\mathrm{d}t\right\} \\ = & \ \frac{q}{\lambda}. \end{aligned}$$