Solution (#1736) (i) Let $\mathbf{e} = (\cos\theta, \sin\theta)$ and $\mathbf{f} = (-\sin\theta, \cos\theta)$ where θ is a function of time t. By the chain rule

$$\dot{\mathbf{e}} = (-\sin\theta\dot{\theta}, \cos\theta\dot{\theta}) = \dot{\theta}\mathbf{f}, \qquad \dot{\mathbf{f}} = (-\cos\theta\dot{\theta}, -\sin\theta\dot{\theta}) = -\dot{\theta}\mathbf{e}.$$

(ii) Let $\mathbf{r} = r\mathbf{e}$. Then by the product rule

$$\dot{\mathbf{r}} = \dot{r}\mathbf{e} + r\dot{\mathbf{e}} = \dot{r}\mathbf{e} + r\dot{\theta}\mathbf{f},$$

and

$$\begin{aligned}
\ddot{\mathbf{r}} &= \ddot{r}\mathbf{e} + \dot{r}\dot{\mathbf{e}} + \dot{r}\dot{\theta}\mathbf{f} + r\ddot{\theta}\mathbf{f} + r\dot{\theta}\dot{\mathbf{f}} \\
&= \ddot{r}\mathbf{e} + \dot{r}(\dot{\theta}\mathbf{f}) + \dot{r}\dot{\theta}\mathbf{f} + r\ddot{\theta}\mathbf{f} + r\dot{\theta}(-\dot{\theta}\mathbf{e}) \\
&= \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\mathbf{f} \\
&= \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\theta}\right)\mathbf{f}.
\end{aligned}$$