Solution (#1747) Let

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

We need to find matrices

$$M(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix}$$

such that

$$M'(t) = AM(t), \qquad M(0) = I_2.$$

So

$$a'(t) = c(t), \qquad b'(t) = d(t), \qquad c'(t) = 0, \qquad d'(t) = 0.$$

Thus c and d are constant, and as $c(0) = 0$ and $d(0) = 1$ we have

$$c(t) = 0 \qquad \text{and} \qquad d(t) = 1 \qquad \text{for all } t.$$

We then have

$$a(t) = \text{const.} \qquad \text{and} \qquad b(t) = t + \text{const.}.$$

Given $a(0) = 1$ and $b(0) = 0$ we have

$$a(t) = 1 \qquad \text{and} \qquad b(t) = t \qquad \text{for all } t.$$

So

$$M(t) = e^{At} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}.$$